



## Lecture 2-4

### Linear Time-Invariant System (LTI System) 線性非時變系統

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### 解微分方程式基本複習

一階微分方程式：
$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = 0$$

解：
$$y(t) = ce^{rt}$$

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系統特徵方程式：
$$a_1 r + a_0 = 0$$

解系統特徵方程式：
$$r = -\frac{a_0}{a_1}$$

解：
$$y(t) = ce^{-\frac{a_0}{a_1}t}$$

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二階微分方程式：
$$\frac{d^2 y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t) = 0$$

解：
$$y(t) = c_1 e^{r_1 t} + c_2 e^{r_2 t}$$

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系統特徵方程式：
$$r^2 + r + 1 = 0$$

解系統特徵方程式：
$$r_1, r_2 = \frac{-1 \pm j\sqrt{3}}{2}$$

解：
$$y(t) = c_1 e^{\left(\frac{-1+j\sqrt{3}}{2}\right)t} + c_2 e^{\left(\frac{-1-j\sqrt{3}}{2}\right)t}$$



## Solving Differential Equation

略過本節

齊次方程式 (Homogeneous Equation) :

所有輸入相關項設為零 (無輸入激發情況下)

$$\sum_{k=0}^N a_k \frac{d^k}{dt^k} y^{(h)}(t) = 0$$

齊次解 (Homogeneous Solution):

$$y^{(h)}(t) = \sum_{i=1}^N c_i e^{r_i t}$$



$r_i$  : 系統特徵方程式(Characteristic Equation) 的根:

$$\sum_{k=0}^N a_k r^k = 0$$

只要上式成立，則前頁所示齊次解即成立。



## Solving Difference Equation

略過本節

齊次方程式 (Homogeneous Equation) :

所有輸入相關項設為零 (無激發情況)

$$\sum_{k=0}^N a_k y^{(h)}[n-k] = 0$$

齊次解 (Homogeneous Solution):

$$y^{(h)}[n] = \sum_{i=1}^N c_i r_i^k$$



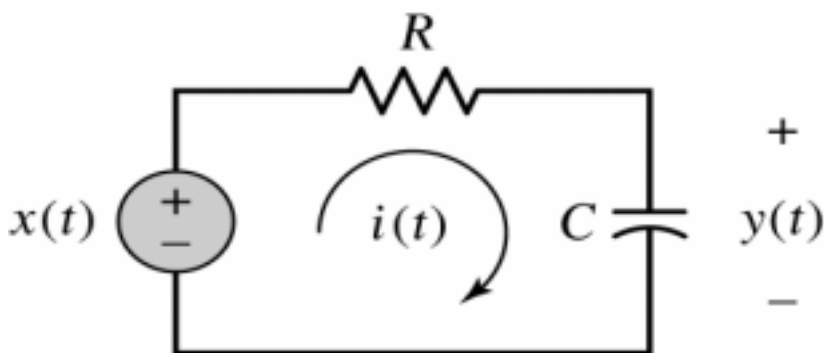
$r_i$  : 系統特徵方程式(Characteristic Equation) 的根:

$$\sum_{k=0}^N a_k r^{N-k} = 0$$

只要上式成立，則前頁所示齊次解即成立。



EX2.17 請用微分方程式描述下述RC circuit，其 $x(t)$ 表輸入電壓、 $y(t)$ 表輸出電壓，並求出此方程式的齊次解。





$$\sum_{k=0}^N a_k \frac{d^k}{dt^k} y^{(h)}(t) = 0$$

*Solution :*

$$\therefore y(t) + R \left( C \frac{d}{dt} y(t) \right) = x(t),$$

*Homogeneous Equation :*

$$y(t) + R \left( C \frac{d}{dt} y(t) \right) = 0, \Rightarrow a_0 = 1, \quad a_1 = RC$$

*Characteristic Equation :*

$$\sum_{k=0}^1 a_k r^k = a_0 + a_1 r = 1 + RCr = 0, \quad \therefore r = -\frac{1}{RC}$$

*Homogeneous Solution :*

$$y^{(h)}(t) = c_1 e^{rt} = c_1 e^{-\frac{t}{RC}}$$

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EX2.18 請用差分方程式描述一階遞迴系統，並求出其齊次解。

$$\sum_{k=0}^N a_k y^{(h)}[n-k] = 0$$

$$y[n] - \rho y[n-1] = x[n]$$

齊次方程式：  $y[n] - \rho y[n-1] = 0, \quad a_0 = 1, a_1 = -\rho$

特徵方程式：  $a_0 r + a_1 = r - \rho = 0,$

$$r = \rho.$$

一階齊次解：  $y^{(h)}[n] = c r^n = c \rho^n$

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## 如何獲得特殊解? Particular Solution

- 特殊解  $y^{(p)}$  代表微分或差分方程式對某已知輸入的任何一個解。
- 特殊解  $y^{(p)}$  不是唯一的。



EX2.19 請用差分方程式描述一階遞迴系統，並求出其特殊解。  $y[n] - \rho y[n-1] = x[n]$ , if  $x[n] = (1/2)^n$

*Solution:*

假設  $y^{(p)}[n] = c_p \left(\frac{1}{2}\right)^n$ , 代入原差分方程式中:

$$c_p \left(\frac{1}{2}\right)^n - \rho c_p \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n, \text{ multiplying } \left(\frac{1}{2}\right)^{-n} \text{ to all terms,}$$

$$c_p - \rho c_p \left(\frac{1}{2}\right)^{-1} = 1, \Rightarrow c_p (1 - 2\rho) = 1, \therefore c_p = \frac{1}{1 - 2\rho}$$

$$\therefore y^{(p)}[n] = \frac{1}{1 - 2\rho} \left(\frac{1}{2}\right)^n,$$



if  $\rho = 1/2$  case,  $y^{(p)}[n] = c_p \left(\frac{1}{2}\right)^n = \frac{1}{1-2\rho} \left(\frac{1}{2}\right)^n$ ,

we cannot find a  $c_p$  to satisfy the above condition.

必須要假設另一種形態：

假設  $y^{(p)}[n] = c_p n \left(\frac{1}{2}\right)^n$ ，代入原差分方程式中



*Solution:*

假設  $y^{(p)}[n] = c_p n \left(\frac{1}{2}\right)^n$ ，代入原差分方程式中：

$$c_p n \left(\frac{1}{2}\right)^n - \rho c_p (n-1) \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n,$$

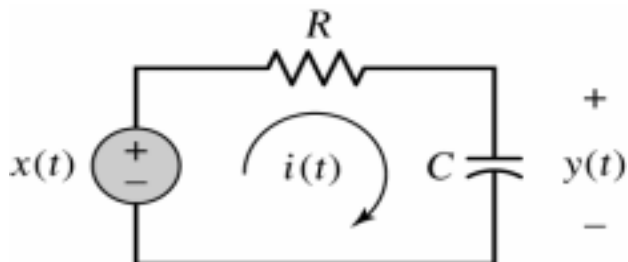
$$\therefore c_p n - 2\rho (n-1)c_p = 1, \Rightarrow c_p (n - 2n\rho + 2\rho) = 1,$$

$$\Rightarrow c_p [n(1-2\rho) + 2\rho] = 1, \therefore c_p = \frac{1}{n(1-2\rho) + 2\rho}$$

$$\therefore y^{(p)}[n] = \frac{1}{(1-2\rho) + 2\rho} \left(\frac{1}{2}\right)^n$$



EX2.20 請求出下列RC 電路的特殊解，已知輸入為  $x(t) = \cos(\omega_0 t)$  伏特。



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$$\therefore y(t) + RC \frac{d}{dt} y(t) = x(t), \quad \text{where } \underline{x(t) = \cos(\omega_0 t)}$$

*Particular Solution:*

$y^{(p)}(t) = c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t)$ , 代入上式可得:

$$c_1 \cos(\omega_0 t) + c_2 \sin(\omega_0 t) + RC[-c_1 \omega_0 \sin(\omega_0 t) + c_2 \omega_0 \cos(\omega_0 t)] = \cos(\omega_0 t)$$

$$\Rightarrow \cos(\omega_0 t)[c_1 + RCc_2 \omega_0] + \sin(\omega_0 t)[c_2 - RCc_1 \omega_0] = \cos(\omega_0 t),$$

$$\therefore \begin{cases} c_1 + RCc_2 \omega_0 = 1 \\ c_2 - RCc_1 \omega_0 = 0 \end{cases}$$

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$$\therefore \begin{cases} c_1 + RCc_2\omega_0 = 1 \\ c_2 - RCc_1\omega_0 = 0 \end{cases}$$

$$\therefore c_1 = \frac{1}{1+(RC\omega_0)^2}, \quad c_2 = \frac{RC\omega_0}{1+(RC\omega_0)^2}$$

$\therefore$  *Particular Solution* :

$$y^{(p)}(t) = \frac{1}{1+(RC\omega_0)^2} \cos(\omega_0 t) + \frac{RC\omega_0}{1+(RC\omega_0)^2} \sin(\omega_0 t) \text{ 伏特}$$



## 如何獲得完整解? Complete Solution

Procedure:

1. 從特徵方程式的根找出齊次解的形式  $y^{(h)}$ 。
2. 假設形式和輸入一樣，找出特殊解  $y^{(s)}$ ，而且和齊次解所有項目不同。
3. 決定齊次解的係數，使其完整解  $y=y^{(p)}+y^{(h)}$  可滿足初始條件。



EX2.21 請用差分方程式描述一階遞迴系統，並求出其完整解。

$$y[n] - \frac{1}{4}y[n-1] = x[n], \quad \text{if } x[n] = (1/2)^n u[n], \quad \text{and } y[-1] = 8$$

$$\text{齊次方程式： } y[n] - \frac{1}{4}y[n-1] = 0, \quad a_0 = 1, a_1 = -\frac{1}{4}$$

$$\text{特徵方程式： } a_0 r + a_1 = r - \frac{1}{4} = 0, \quad \therefore r = \frac{1}{4}.$$

$$\text{一階齊次解： } y^{(h)}[n] = c r^n = c \left(\frac{1}{4}\right)^n$$

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特殊解：

假設  $y^{(p)}[n] = c_p \left(\frac{1}{2}\right)^n$ ，代入原差分方程式中：

$$c_p \left(\frac{1}{2}\right)^n - \frac{1}{4}c_p \left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n, \quad \text{multiplying } \left(\frac{1}{2}\right)^{-n} \text{ to all terms,}$$

$$c_p - \frac{1}{4}c_p \left(\frac{1}{2}\right)^{-1} = 1, \quad \Rightarrow c_p \left(1 - \frac{1}{2}\right) = 1, \quad \therefore c_p = \frac{1}{1/2} = 2$$

$$\therefore y^{(p)}[n] = c_p \left(\frac{1}{2}\right)^n = 2 \left(\frac{1}{2}\right)^n,$$

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### 完整解(Complete Solution):

$$y[n] = y^{(h)}[n] + y^{(p)}[n] = c\left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n$$

加入初始值:

$$y[0] = x[0] + \frac{1}{4}y[-1] = (1/2)^0 + \frac{1}{4}(8) = 1 + 2 = 3,$$

$$= c\left(\frac{1}{4}\right)^0 + 2\left(\frac{1}{2}\right)^0 = c + 2$$

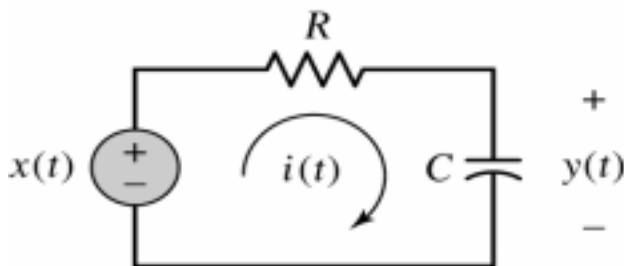
$$\therefore c = 1,$$

$$\therefore y[n] = \left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n, \quad \forall n \geq 0$$

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EX2.22 請求出下列RC 電路的完整解，已知輸入為  $x(t) = \cos(t)u(t)$  伏特，假設  $R=1 \Omega$ 、 $C=1F$ ，且電容初始電壓為  $y(0^-) = 2$  伏特。



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### Homogeneous Solution

$$y^{(h)}(t) = c e^{rt} = c e^{-\frac{1}{RC}t}$$

### Particular Solution:

$$y^{(p)}(t) = \frac{1}{1+(RC)^2} \cos(t) + \frac{RC}{1+(RC)^2} \sin(t)$$

### Complete Solution:

$$R = 1, C = 1, \omega_0 = 1$$

$$y(t) = c e^{-t} + \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t)$$



$$\therefore \text{initial voltage: } y(0^-) = y(0^+) = 2,$$

$$2 = c e^{-(0^+)} + \frac{1}{2} \cos(0^+) + \frac{1}{2} \sin(0^+) = c + \frac{1}{2},$$

$$\therefore c = \frac{3}{2}$$

### Complete Solution:

$$y(t) = \frac{3}{2} e^{-t} + \frac{1}{2} \cos(t) + \frac{1}{2} \sin(t)$$



## 如何用 Differential and Difference Equations 描述系統特性?

自然響應 (Natural Response):

- 零輸入時的系統輸出，描述任何儲存得能量並滿足初始條件。
- 類似 齊次解 :  $y^{(n)}$

強迫響應 (Forced Response):

- 假設初始為零時，由輸入訊號造成的系統輸出。
- 類似 特殊解 :  $y^{(f)}$

完整輸出 :  $y = y^{(n)} + y^{(f)}$

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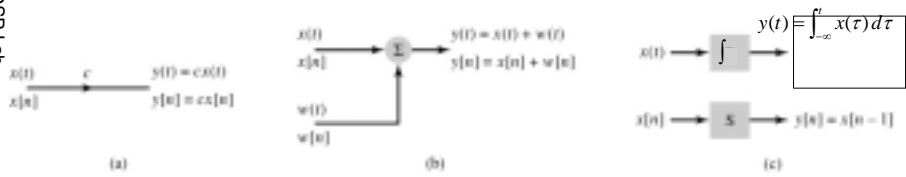


## 如何用方塊圖(Block Diagram) 描述系統

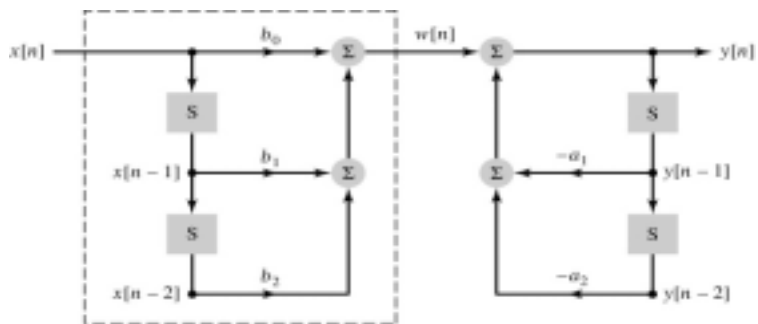
本節新開始

- 方塊圖表示輸入訊號基本運算的互連狀態
- 方塊圖描述系統內部計算或操作如何排列
- 方塊圖表示運算
  - 純量乘法
  - 加法
  - 積分
  - 時間平移
  - ...

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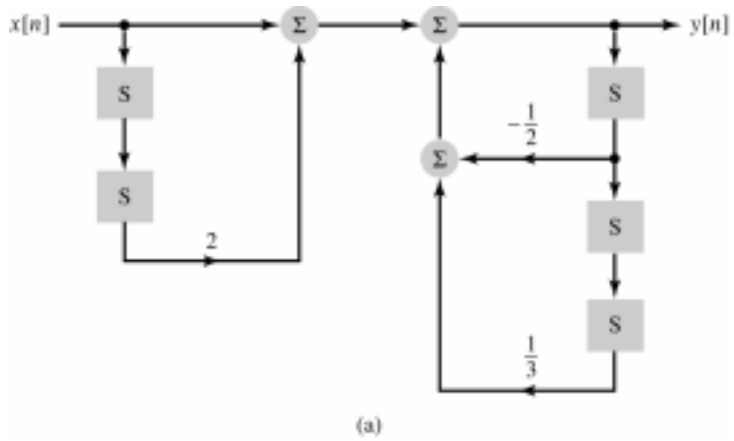
Symbols for elementary operations in block diagram descriptions of systems.  
 (a) Scalar multiplication.  
 (b) Addition.  
 (c) Integration for continuous-time systems and time shifting for discrete-time systems.



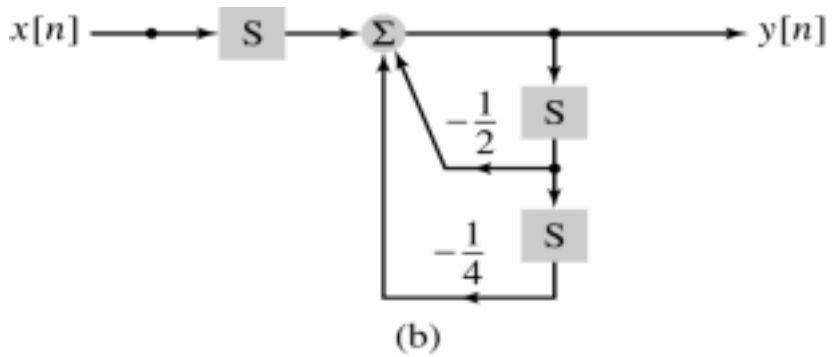
Block diagram of A second-order difference equation. (Direct form I : needs 4 memory locations:  $x[n-1]$ ,  $x[n-2]$ ,  $y[n-1]$ ,  $y[n-2]$  )



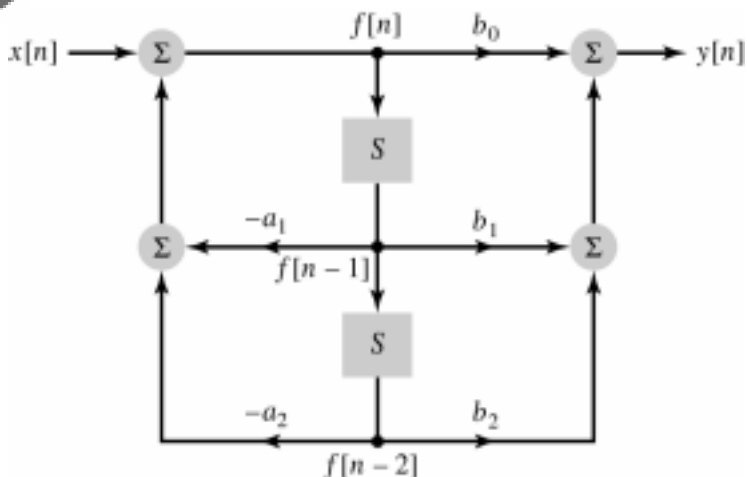
Block diagram representation for Problem 2.33 (2.34b in next slide)



(a)



(b)



Block diagram of A second-order difference equation.  
(Direct form II : needs 2 memory locations:  
 $f[n-1]$ ,  $f[n-2]$  )

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*Direct form I :*

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$\therefore \sum_{k=0}^2 a_k y[n-k] = \sum_{p=0}^2 b_p x[n-p]$$

$$\text{define } \sum_{k=0}^2 a_k f[n-k] = x[n], \quad \therefore x[n-p] = \sum_{k=0}^2 a_k f[n-k-p]$$

$$\therefore \sum_{k=0}^2 a_k y[n-k] = \sum_{p=0}^2 b_p \sum_{k=0}^2 a_k f[n-k-p] = \sum_{k=0}^2 a_k \sum_{p=0}^2 b_p f[n-k-p]$$

$$\therefore y[n-k] = \sum_{p=0}^2 b_p f[(n-k)-p]$$

$$\therefore y[n] = \sum_{p=0}^2 b_p f[n-p]$$

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*Direct form II :*

$$a_0 = 1$$

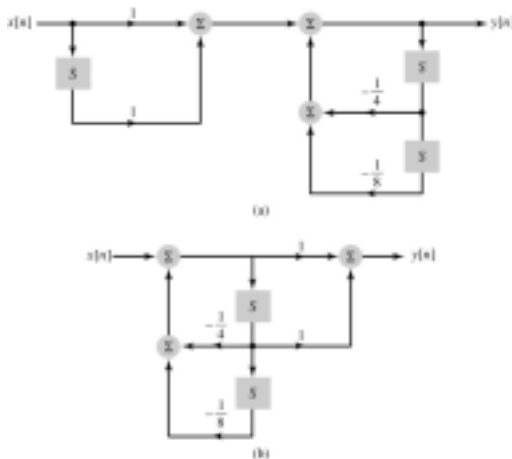
$$\sum_{k=0}^2 a_k f[n-k] = x[n]$$

$$f[n] = x[n] - a_1 f[n-1] - a_2 f[n-2]$$

$$y[n] = \sum_{p=0}^2 b_p f[n-p] = b_0 f[n] + b_1 f[n-1] + b_2 f[n-2]$$

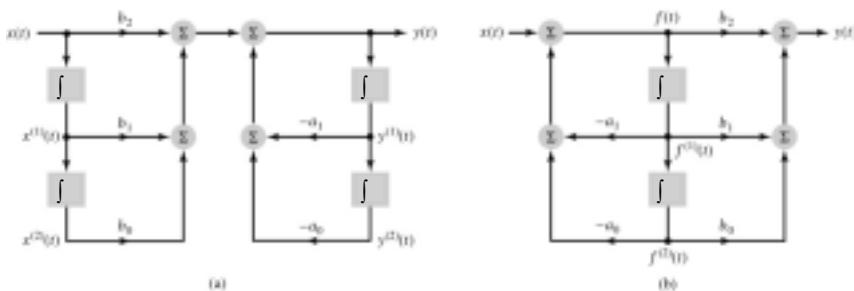


Solution to Problem 2.24. (a) Direct form 1,  
(b) Direct form 2.

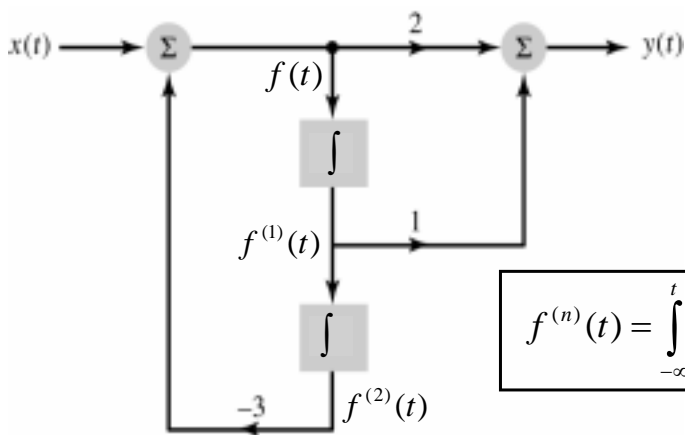




Continuous-time LTI system described by a second-order integral equation. (a) Direct form I. (b) Direct form II.



Block diagram representation for Problem 2.25.



$$f^{(n)}(t) = \int_{-\infty}^t f^{(n-1)}(\tau) d\tau$$



$$y(t) = 2f(t) + f^{(1)}(t)$$

$$f(t) = x(t) - 3f^{(2)}(t)$$

∴

$$y(t) = 2(x(t) - 3f^{(2)}(t)) + f^{(1)}(t) = 2x(t) - 6f^{(2)}(t) + f^{(1)}(t)$$

$$\frac{dy(t)}{dt} = 2 \frac{dx(t)}{dt} - 6f^{(1)}(t) + f^{(1)}(t) = 2 \frac{dx(t)}{dt} - 6f^{(1)}(t) + x(t) - 3f^{(2)}(t)$$

$$\frac{d^2y(t)}{dt^2} = 2 \frac{d^2x(t)}{dt^2} - 6f(t) + \frac{dx(t)}{dt} - 3f^{(1)}(t)$$

$$= 2 \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 3(2f(t) + f^{(1)}(t)) = 2 \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 3y(t)$$

∴

$$\boxed{\frac{d^2y(t)}{dt^2} + 3y(t) = 2 \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt}}$$



## State-Variable Description of LTI System

### 以狀態變數描述 LTI 系統

- 描述系統輸出與目前系統狀態和輸入之間的關係。
- 狀態變數(state variables) 描述以矩陣的形式表示。
- 多種輸入選擇暗示系統狀態不是唯一。

- 狀態可能會改變  $q_1[n] \rightarrow q_2[n]$

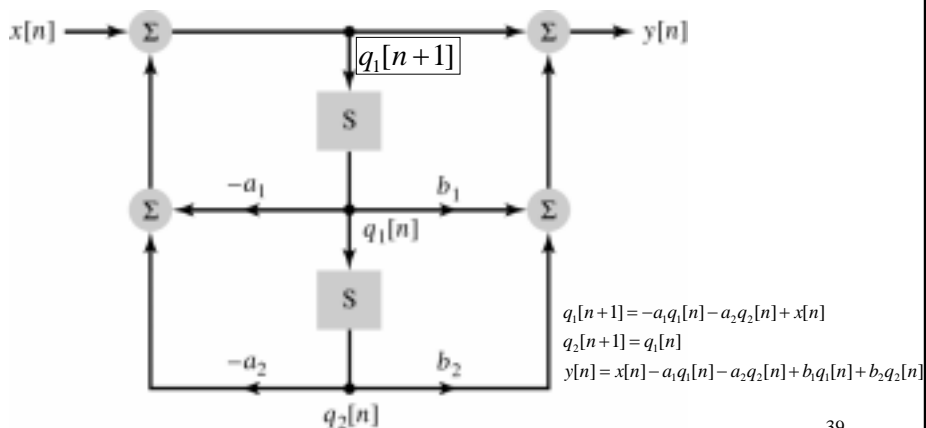
• 範例： $q_1[n+1] = -a_1q_1[n] - a_2q_2[n] + x[n]$

$$q_2[n+1] = q_1[n]$$

$$y[n] = x[n] - a_1q_1[n] - a_2q_2[n] + b_1q_1[n] + b_2q_2[n]$$



Direct form II representation of a second-order discrete-time LTI system depicting state variables  $q_1[n]$  and  $q_2[n]$ .



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$$q_1[n+1] = -a_1q_1[n] - a_2q_2[n] + x[n]$$

$$q_2[n+1] = q_1[n]$$

$$y[n] = x[n] - a_1q_1[n] - a_2q_2[n] + b_1q_1[n] + b_2q_2[n]$$

$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} b_1 - a_1 & b_2 - a_2 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + [1]x[n]$$

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$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} b_1 - a_1 & b_2 - a_2 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + [1]x[n]$$

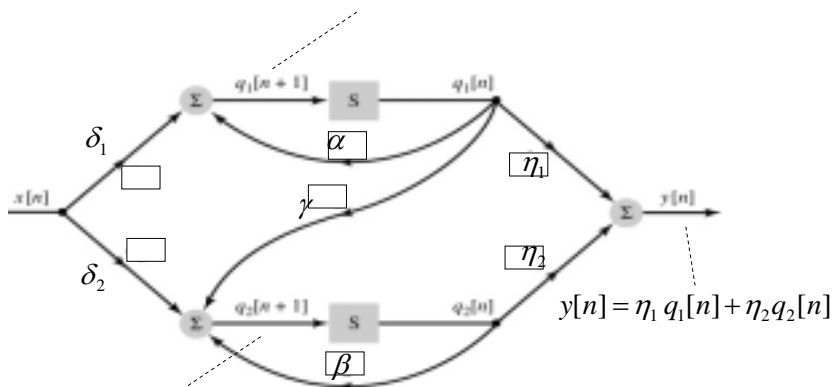
$$\Rightarrow \underline{q}[n+1] = \underline{A}\underline{q}[n] + \underline{b}x[n]$$

$$y[n] = \underline{C}\underline{q}[n] + \underline{D}x[n]$$



### Block diagram of LTI system for Example 2.28.

請用狀態變數描述下圖： $q_1[n+1] = \alpha q_1[n] + \delta_1 x[n]$



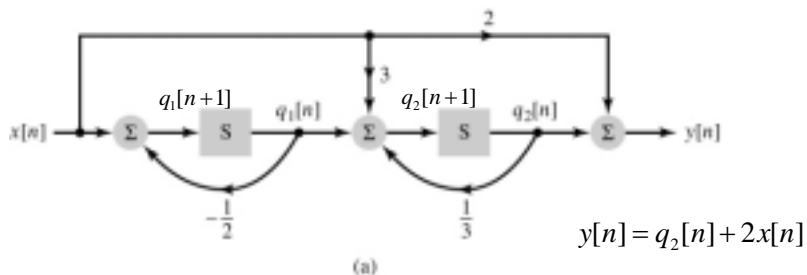
$$q_2[n+1] = \gamma q_1[n] + \beta q_2[n] + \delta_2 x[n]$$



Block diagram of LTI system for Problem 2.26 (2.41b on next slide).

請用狀態變數描述下圖：

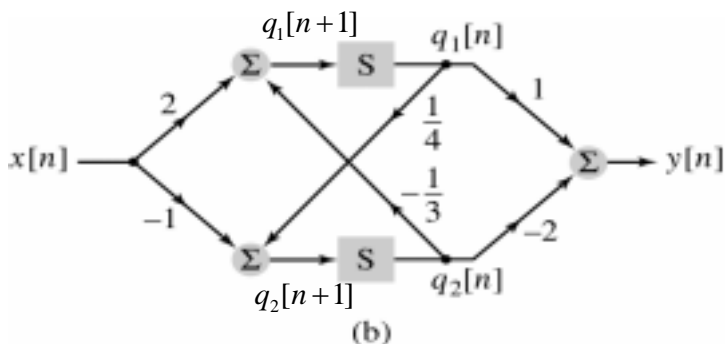
$$q_1[n+1] = -\frac{1}{2}q_1[n] + x[n] \qquad q_2[n+1] = q_1[n] + \frac{1}{3}q_2[n] + 3x[n]$$



請用狀態變數描述下圖：  
 $q_1[n+1] = a_{11}q_1[n] + a_{12}q_2[n] + b_1x[n]$   
 $q_2[n+1] = a_{21}q_1[n] + a_{22}q_2[n] + b_2x[n]$

找出 這些參數值？

$$y[n] = c_1q_1[n] + c_2q_2[n] + d x[n]$$



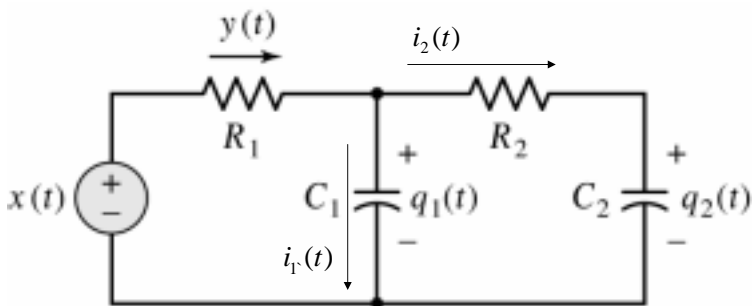


### Circuit diagram of LTI system for Example 2.29.

請用狀態變數描述下圖：

$$x(t) = y(t)R_1 + q_1(t)$$

$$q_1(t) = i_2(t)R_2 + q_2(t)$$



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Solution:

$$x(t) = y(t)R_1 + q_1(t) \rightarrow y(t) = \frac{1}{R_1}x(t) - \frac{1}{R_1}q_1(t)$$

$$q_1(t) = i_2(t)R_2 + q_2(t) \rightarrow i_2(t) = \frac{1}{R_2}q_1(t) - \frac{1}{R_2}q_2(t)$$

$$\therefore q_1 = \frac{1}{C_1} \int_{-\infty}^t i_1(\tau) d\tau \rightarrow i_1(t) = C_1 \frac{dq_1(t)}{dt}$$

$$\therefore q_2 = \frac{1}{C_2} \int_{-\infty}^t i_2(\tau) d\tau \rightarrow i_2(t) = C_2 \frac{dq_2(t)}{dt}$$

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$$\therefore i_2(t) = \frac{1}{R_2} q_1(t) - \frac{1}{R_2} q_2(t) = C_2 \frac{dq_2(t)}{dt}$$

$$\therefore \frac{dq_2(t)}{dt} = \frac{1}{R_2 C_2} q_1(t) - \frac{1}{R_2 C_2} q_2(t)$$

$$\begin{aligned} \therefore i_1(t) = y(t) - i_2(t) &= \left( \frac{1}{R_1} x(t) - \frac{1}{R_1} q_1(t) \right) - \left( \frac{1}{R_2} q_1(t) - \frac{1}{R_2} q_2(t) \right) \\ &= C_1 \frac{dq_1(t)}{dt} \end{aligned}$$

$$\therefore \frac{dq_1(t)}{dt} = \left( \frac{1}{C_1 R_1} x(t) - \frac{1}{C_1 R_1} q_1(t) \right) - \left( \frac{1}{C_1 R_2} q_1(t) - \frac{1}{C_1 R_2} q_2(t) \right)$$

$$= \frac{1}{C_1 R_1} x(t) - \left( \frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} \right) q_1(t) + \frac{1}{C_1 R_2} q_2(t)$$

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$$\frac{dq_2(t)}{dt} = \frac{1}{R_2 C_2} q_1(t) - \frac{1}{R_2 C_2} q_2(t)$$

$$\frac{dq_1(t)}{dt} = \frac{1}{C_1 R_1} x(t) - \left( \frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} \right) q_1(t) + \frac{1}{C_1 R_2} q_2(t)$$

$$\begin{bmatrix} \frac{dq_1}{dt} \\ \frac{dq_2}{dt} \end{bmatrix} = \begin{bmatrix} -\left( \frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} \right) & \frac{1}{R_2 C_1} \\ \frac{1}{R_2 C_2} & -\frac{1}{R_2 C_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 C_1} \\ 0 \end{bmatrix} x(t)$$

$$= \underline{A} \underline{q} + \underline{b} x$$

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$$y(t) = (x(t) - q_1(t)) / R_1 = -\frac{1}{R_1} q_1(t) + \frac{1}{R_1} x(t)$$

$$= \begin{bmatrix} -\frac{1}{R_1} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1} \end{bmatrix} x(t)$$

$$= \underline{c} \underline{q} + dx$$

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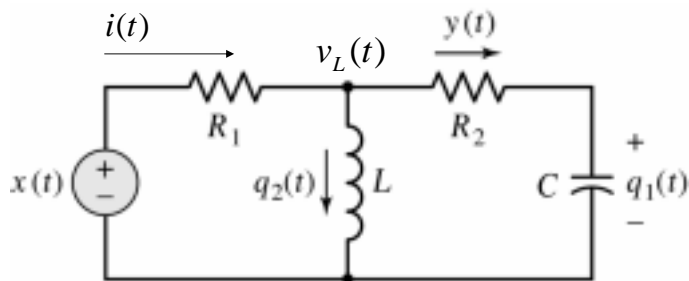


### Circuit diagram of LTI system for Problem 2.27.

請用狀態變數描述下圖：

$$\frac{dq_1(t)}{dt} = ?$$

$$\frac{dq_2(t)}{dt} = ?$$



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$$\begin{aligned}\therefore x(t) &= R_1(y(t) + q_2(t)) + y(t)R_2 + q_1(t) \\ &= y(t)(R_1 + R_2) + q_1(t) + R_1q_2(t) \\ \therefore y(t) &= \frac{x(t)}{R_1 + R_2} - \frac{1}{R_1 + R_2}q_1(t) - \frac{R_1}{R_1 + R_2}q_2(t)\end{aligned}$$

$$\therefore q_1(t) = \frac{1}{C} \int_{-\infty}^t y(\tau) d\tau,$$

$$\begin{aligned}\therefore \frac{dq_1(t)}{dt} &= \frac{1}{C} y(t) \\ &= \frac{x(t)}{C(R_1 + R_2)} - \frac{1}{C(R_1 + R_2)}q_1(t) - \frac{R_1}{C(R_1 + R_2)}q_2(t)\end{aligned}$$



$$\begin{aligned}\therefore x(t) &= R_1(y(t) + q_2(t)) + L \frac{dq_2(t)}{dt} \\ \therefore \frac{dq_2(t)}{dt} &= \frac{x(t)}{L} - \frac{R_1}{L} y(t) - \frac{R_1}{L} q_2(t) \\ &= \frac{x(t)}{L} - \frac{R_1}{L} \left( \frac{x(t)}{R_1 + R_2} - \frac{1}{R_1 + R_2} q_1(t) - \frac{R_1}{R_1 + R_2} q_2(t) \right) - \frac{R_1}{L} q_2(t) \\ &= \left( \frac{1}{L} - \frac{R_1}{L(R_1 + R_2)} \right) x(t) + \frac{R_1}{L(R_1 + R_2)} q_1(t) + \left( \frac{R_1^2}{L(R_1 + R_2)} - \frac{R_1}{L} \right) q_2(t) \\ &= \frac{R_2}{L(R_1 + R_2)} x(t) + \frac{R_1}{L(R_1 + R_2)} q_1(t) - \frac{R_1 R_2}{L(R_1 + R_2)} q_2(t)\end{aligned}$$



$$y(t) = \frac{x(t)}{R_1 + R_2} - \frac{1}{R_1 + R_2} q_1(t) - \frac{R_1}{R_1 + R_2} q_2(t)$$

$$\frac{dq_1(t)}{dt} = \frac{x(t)}{C(R_1 + R_2)} - \frac{1}{C(R_1 + R_2)} q_1(t) - \frac{R_1}{C(R_1 + R_2)} q_2(t)$$

$$\frac{dq_2(t)}{dt} = \frac{R_2}{L(R_1 + R_2)} x(t) + \frac{R_1}{L(R_1 + R_2)} q_1(t) - \frac{R_1 R_2}{L(R_1 + R_2)} q_2(t)$$



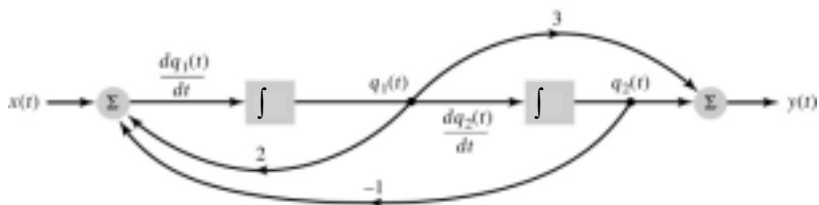
$$\begin{aligned} \begin{bmatrix} \frac{dq_1(t)}{dt} \\ \frac{dq_2(t)}{dt} \end{bmatrix} &= \begin{bmatrix} \frac{-1}{C(R_1 + R_2)} & \frac{-R_1}{C(R_1 + R_2)} \\ \frac{R_1}{L(R_1 + R_2)} & \frac{-R_1 R_2}{L(R_1 + R_2)} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C(R_1 + R_2)} \\ \frac{R_2}{L(R_1 + R_2)} \end{bmatrix} x(t) \\ &= A \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + b x(t) \end{aligned}$$

$$\begin{aligned} y(t) &= \begin{bmatrix} \frac{1}{R_1 + R_2} & \frac{-R_1}{R_1 + R_2} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1 + R_2} \end{bmatrix} x(t) \\ &= C \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + D x(t) \end{aligned}$$



## Block diagram of LTI system for Example 2.30.

請用狀態變數描述下圖：



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## Convolution Sum using MATLAB.

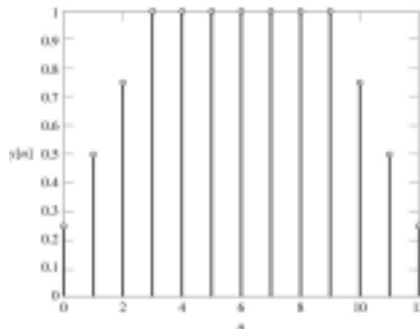
```
>> h = [-1, 0.5];
>> x = [2, 4, -2];
>> y = conv(x, h)
y =
    2    5    0   -1
```

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## Convolution sum computed using MATLAB.

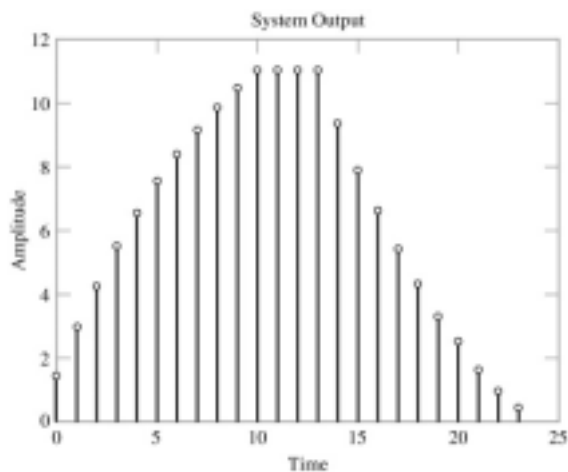
```
>> h = 0.25 * ones(1, 4);  
>> x = ones(1, 10);  
>> n = 0 : 12;  
>> y = conv(x, h);  
>> stem(n, y);  
>> xlabel('n');  
>> ylabel('y[n]')
```



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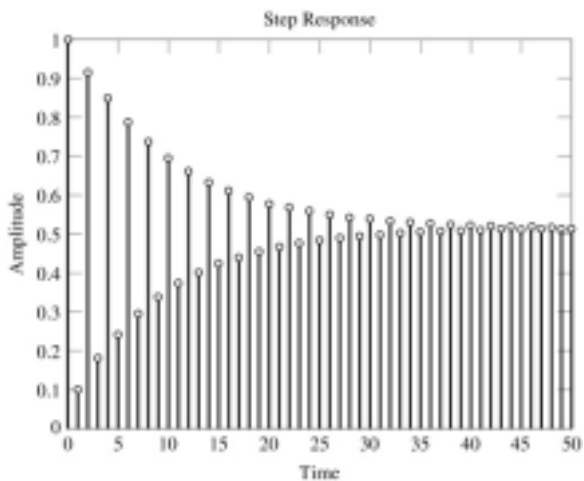
## Solution to Problem 2.29.



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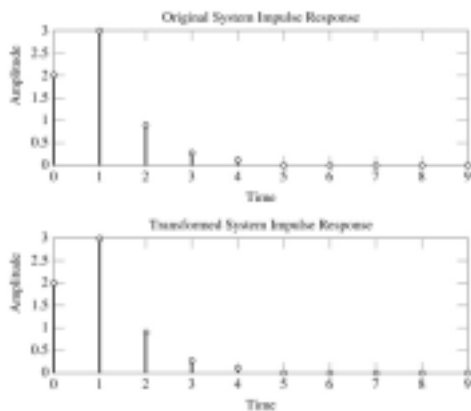
## Step response computed using MATLAB.



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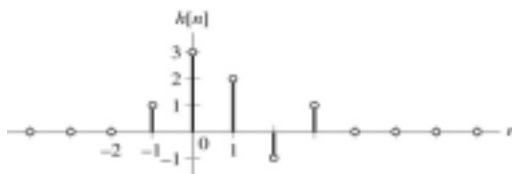
## Impulse responses associated with the original and transformed state-variable descriptions computer using MATLAB.



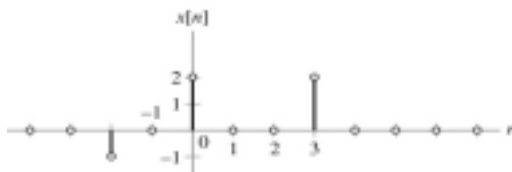
60



P2.32:  $y[n] = x[n] * h[n] = ?$



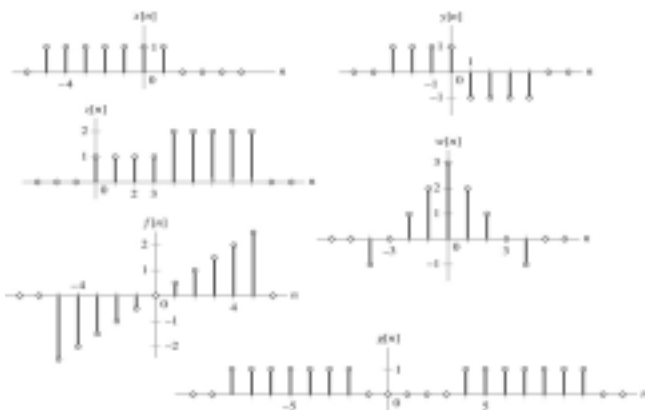
(a)



(b)



P2.33:  $x[n] * y[n] = ?$   $z[n] * w[n] = ?$   $f[n] * g[n] = ?$

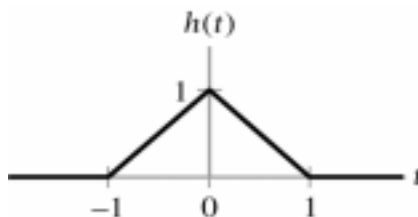




P2.38: LTI 系統脈衝響應如下，若輸入為：

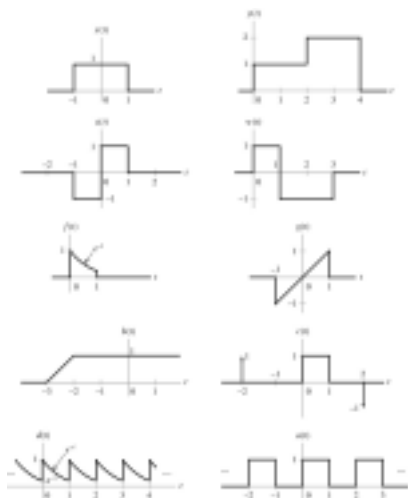
(a)  $x(t) = 2\delta(t+2) + \delta(t-2)$

(b)  $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$



P2.40:  $x(t) * y(t) = ?$   $z(t) * w(t) = ?$   $f(t) * g(t) = ?$

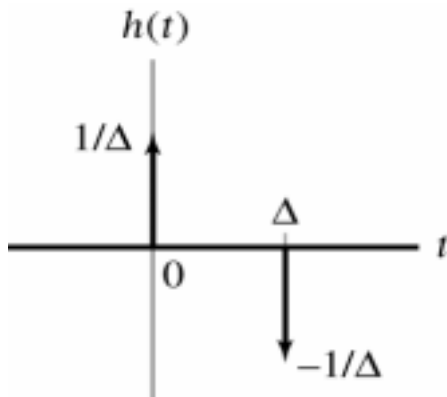
$b(t) * c(t) = ?$   $d(t) * a(t) = ?$



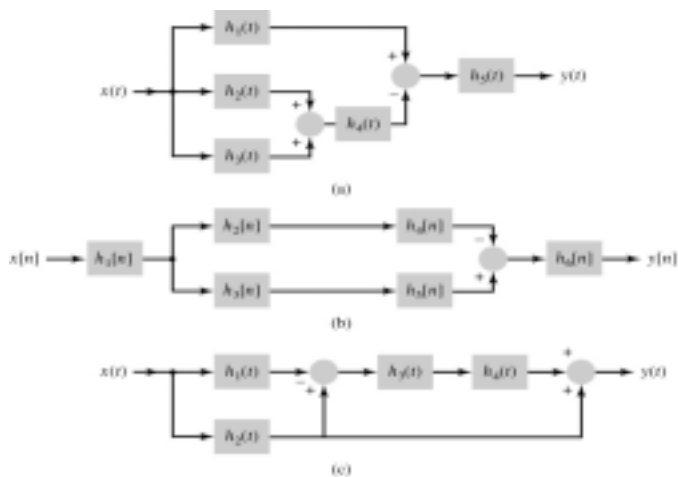




P2.43: 利用下圖系統脈衝響應，寫出系統輸出與輸入關係式 = ?

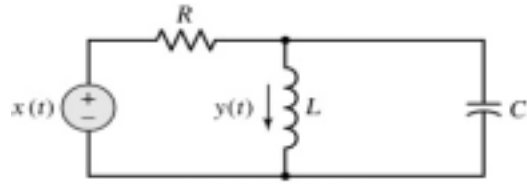


P2.46: 找出下列各圖總系統脈衝響應 = ?

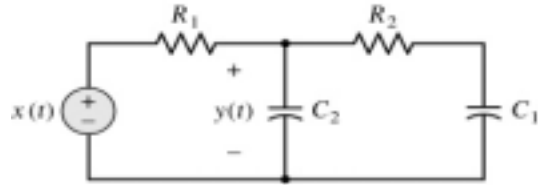




P2.52: 以微分方程式描述下列電路 = ?



(a)



(b)



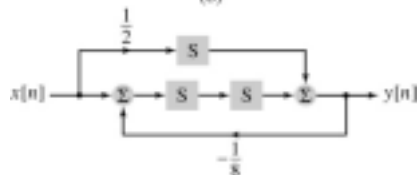
P2.65: 以差分方程式描述下列電路 = ?



(a)



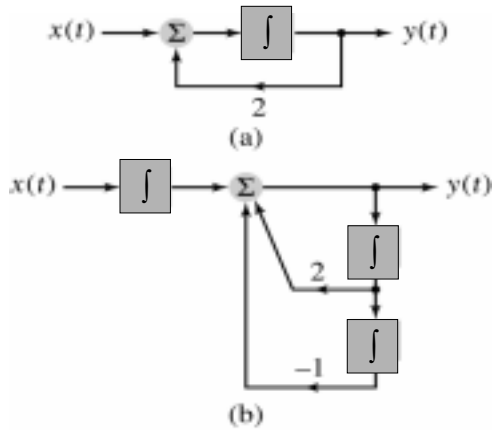
(b)



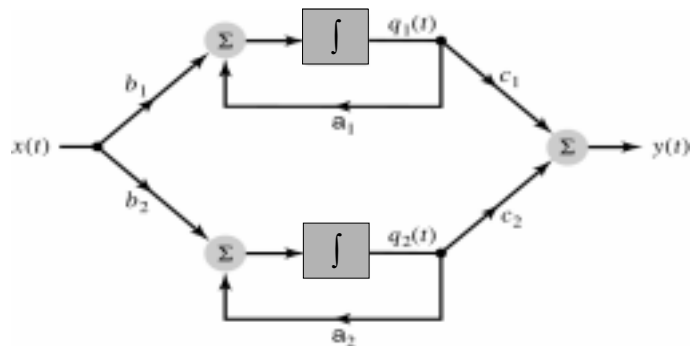
(c)



P2.68: 以微分方程式描述下列電路 = ?



P2.74: 以狀態變數方式描述下列電路 = ?





P2.75: 進階習題

