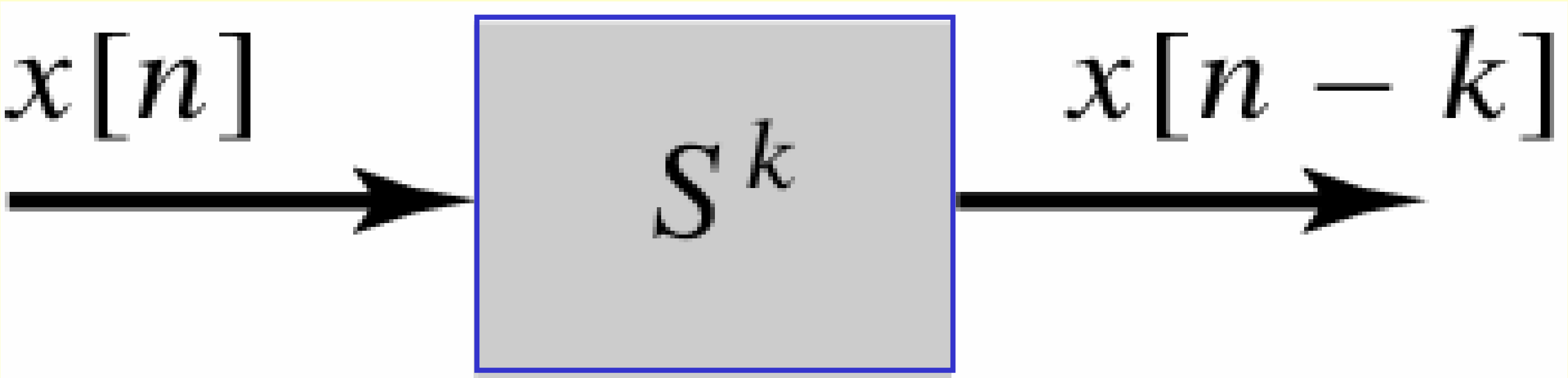
Signals and Systems
信號與系統

Lecture 1-3

Cascading System

(系統串接方式)

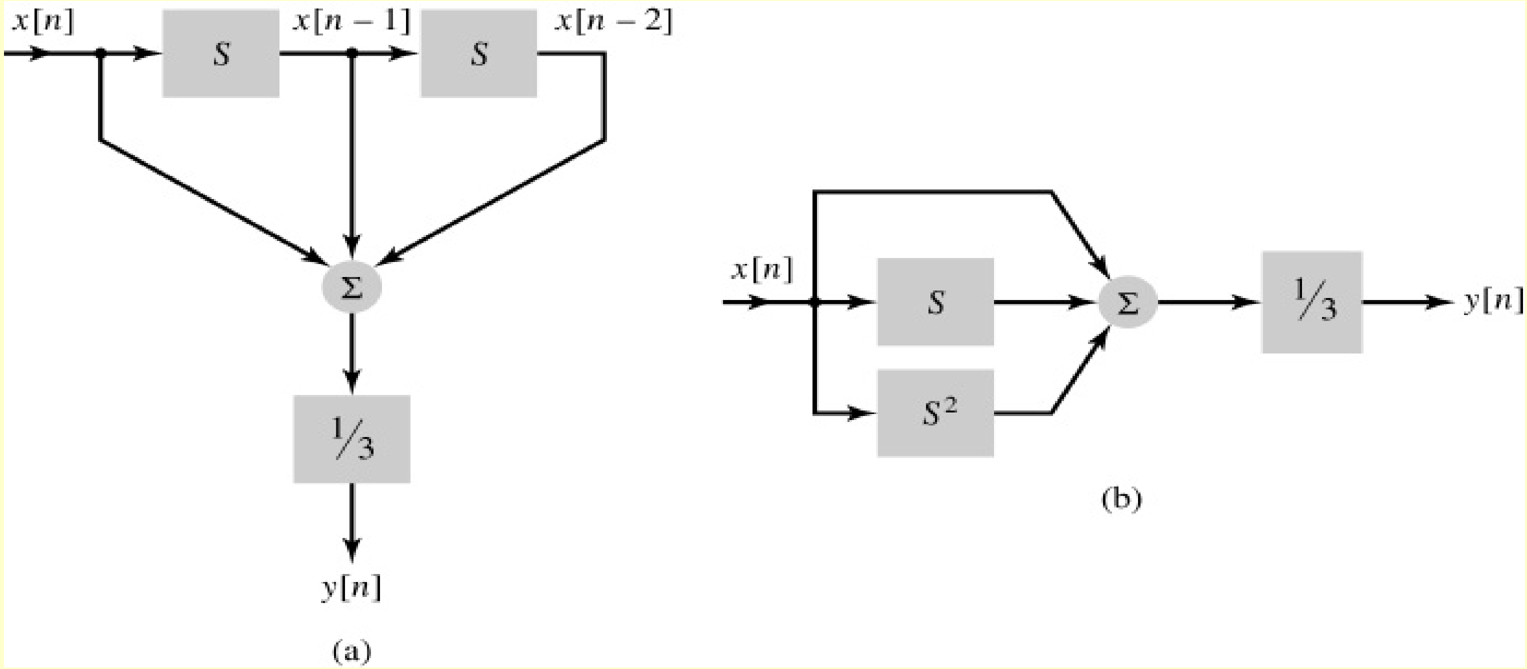
Discrete-time-shift operator S^k, operating on the discrete-time signal $x[n]$ to produce $x[n-k]$.



(離散時間延遲系統與離散時間時間平移運算元 S^k)

Two implementations of the moving-average system

(a) cascade form of implementation and
(b) parallel form of implementation.



Properties of Systems

- Stability 穩定性
- Memory 記憶性
- Causality 因果性
- Invertibility 可逆性
- Time Invariance 非時變性
- Linearity 線性

Stability

- **B**ounded **I**nput causes **B**ounded **O**utput (BIBO).
- 輸入與輸出需滿足:

$$|y(t)| \le M_y < \infty$$

$$|x(t)| \le M_x < \infty$$

for M_x, M_y are finite and positive numbers.

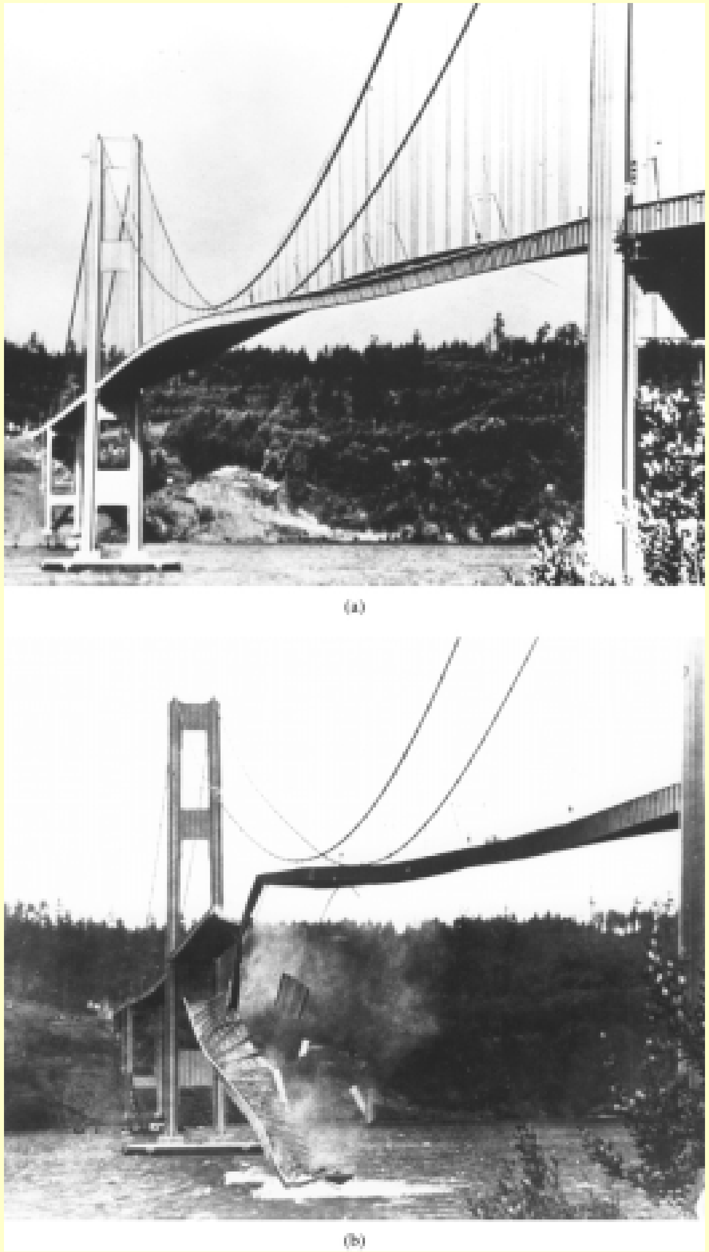
- Unstable Example:
  - "*Tacoma Narrows Suspension Bridge*"

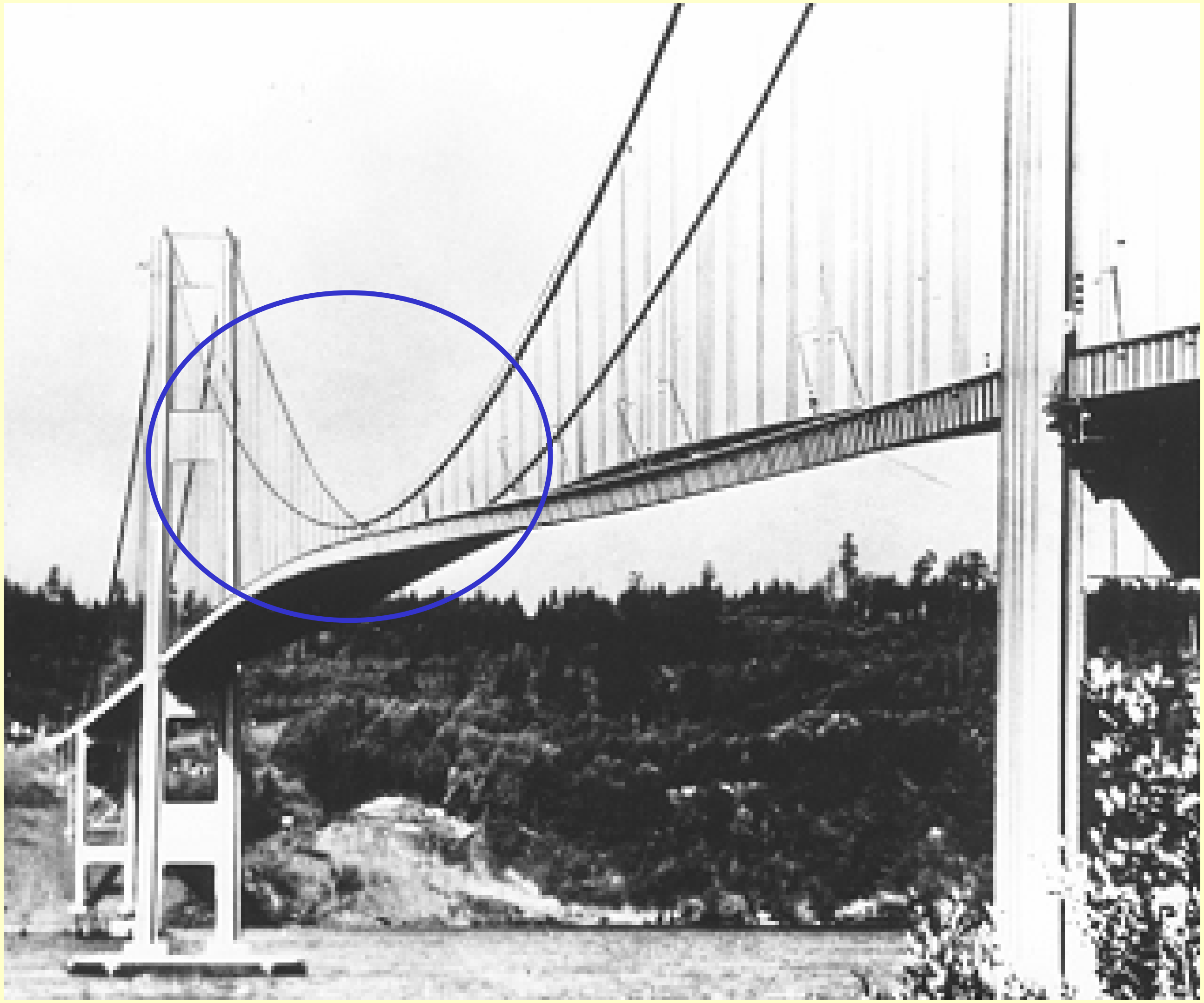
Dramatic photographs showing the collapse of the *Tacoma* Narrows suspension bridge on November 7, 1940.

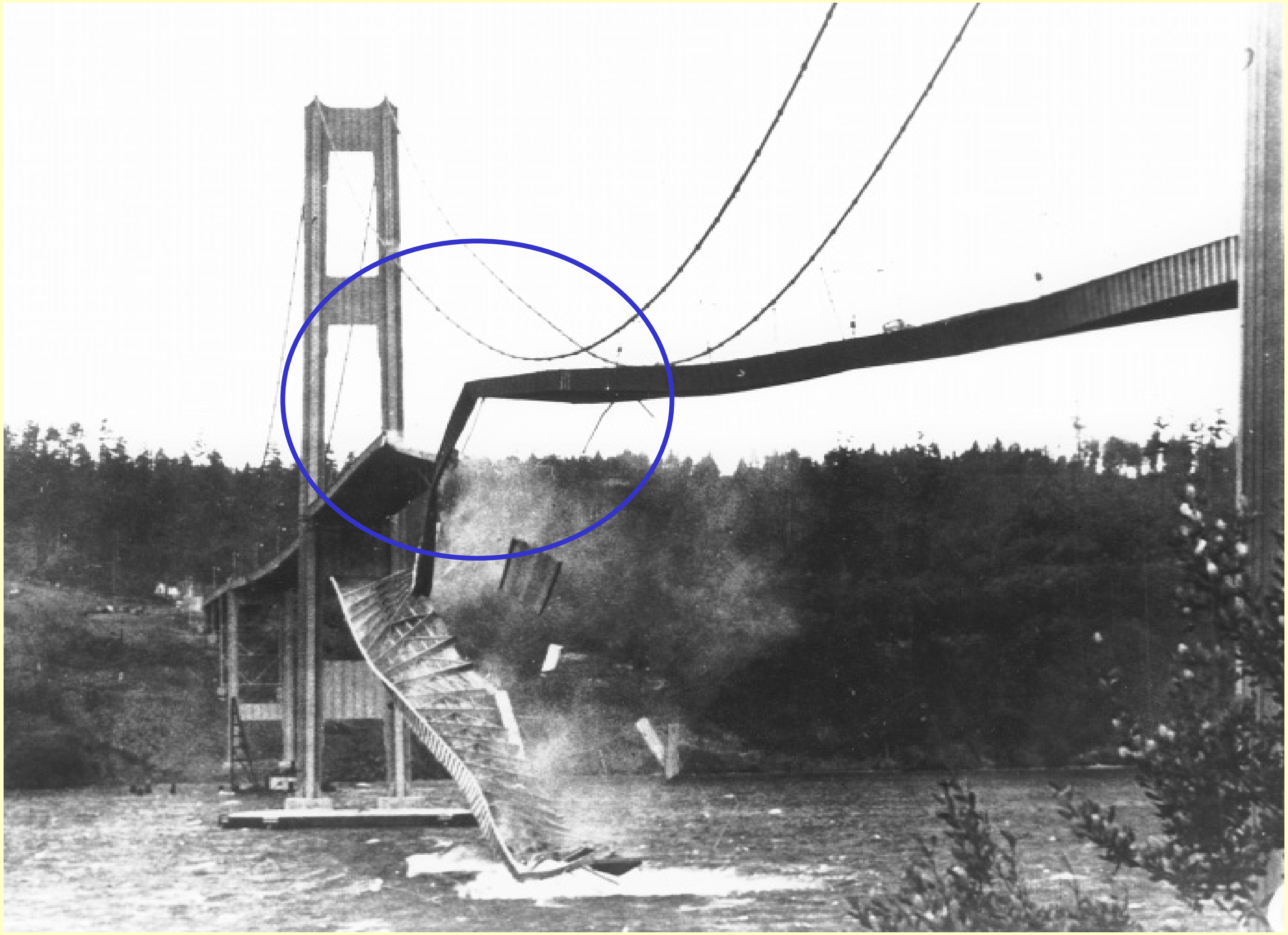
(a) Photograph showing the twisting motion of the bridge's center span just before failure.

(b) A few minutes after the first piece of concrete fell, this second photograph shows a 600-ft section of the bridge breaking out of the suspension span and turning upside down as it crashed in Puget Sound, Washington. Note the car in the top right-hand corner of the photograph.

(Courtesy of the Smithsonian Institution.)







Example of a Stable System

$Example:$ [Moving-average case]

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right).$$

$$\left|y[n]\right| = \frac{1}{3}\left(\left|x[n] + x[n-1] + x[n-2]\right|\right)$$

$$\leq \frac{1}{3}\left(\left|x[n]\right| + \left|x[n-1]\right| + \left|x[n-2]\right|\right) \leq \frac{1}{3}\left(M_x + M_x + M_x\right) = M_x$$

Example of an Unstable System

$Example:$

$$y[n] = r^n \ x[n], \quad \forall\ r > 1$$

$$even\ if\ \left|x[n]\right| \le M_x < \infty, \quad \forall all\ n$$

$$\left|y[n]\right| = \left|r^n \ x[n]\right| = \left|r^n\right| \cdot \left|x[n]\right|$$

$$if\ \ r > 1, \quad r^n\ divergences$$

Memory or Memory-less

Memory :

若系統輸出取決於過去的訊號輸入時

Memory-less :

若系統輸出僅取決於現在的訊號輸入時

Causality

Causal :

若系統輸出取決於目前或過去的訊號輸入時

$$Ex:\quad y[n]=x[n]+x[n-1]$$

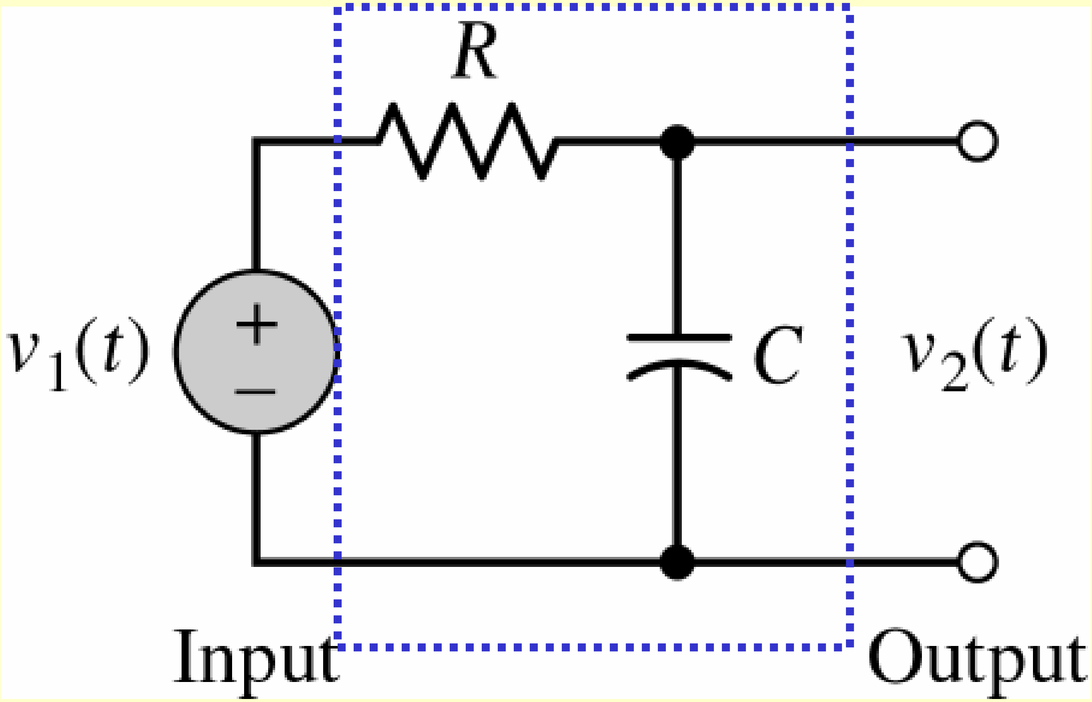
Non-causal :

若系統輸出取決於一個或以上未來的訊號輸入時

$$Ex:\quad y[n]=x[n+1]+x[n]$$

Example: Causal System ?

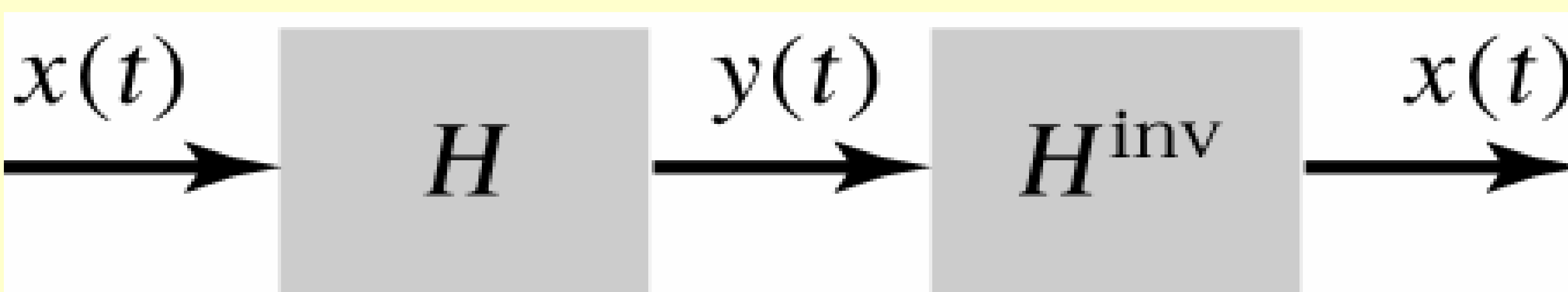
Series *RC* circuit driven from an ideal voltage source $v_1(t)$, producing output voltage $v_2(t)$



System Invertibility

The second operator H^{inv} is the inverse of the first operator H. Hence, the input $x(t)$ is passed through the cascade correction of H and H^{-1} completely unchanged.

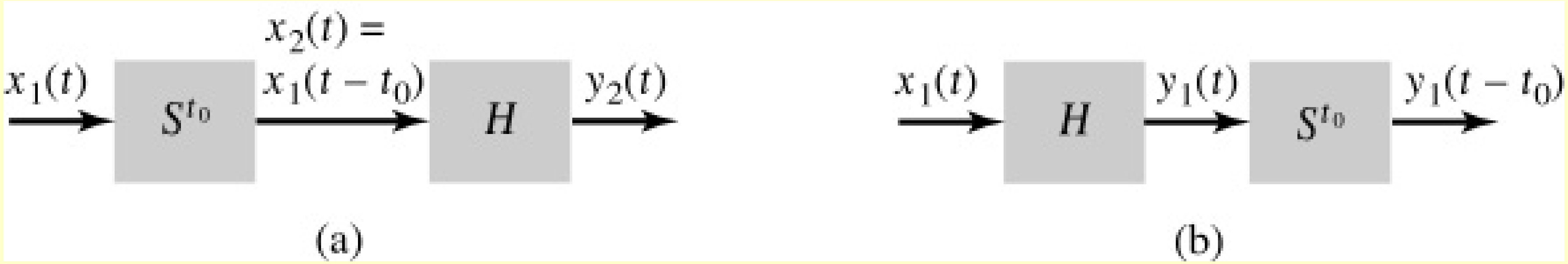
$$H^{inv} = H^{-1}$$



Time Invariance

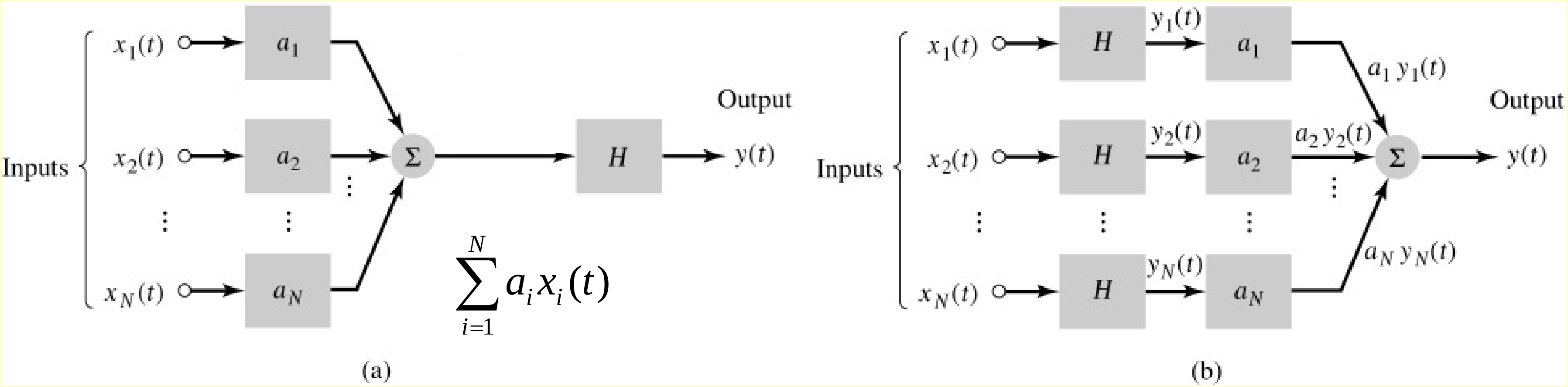
These two situations are equivalent, provided that *H* is time invariant.

$$y_2(t) = y_1(t - t_0)$$



The Linearity Properties

If these two configurations produce the same output *y*(*t*), the operator *H* is linear.

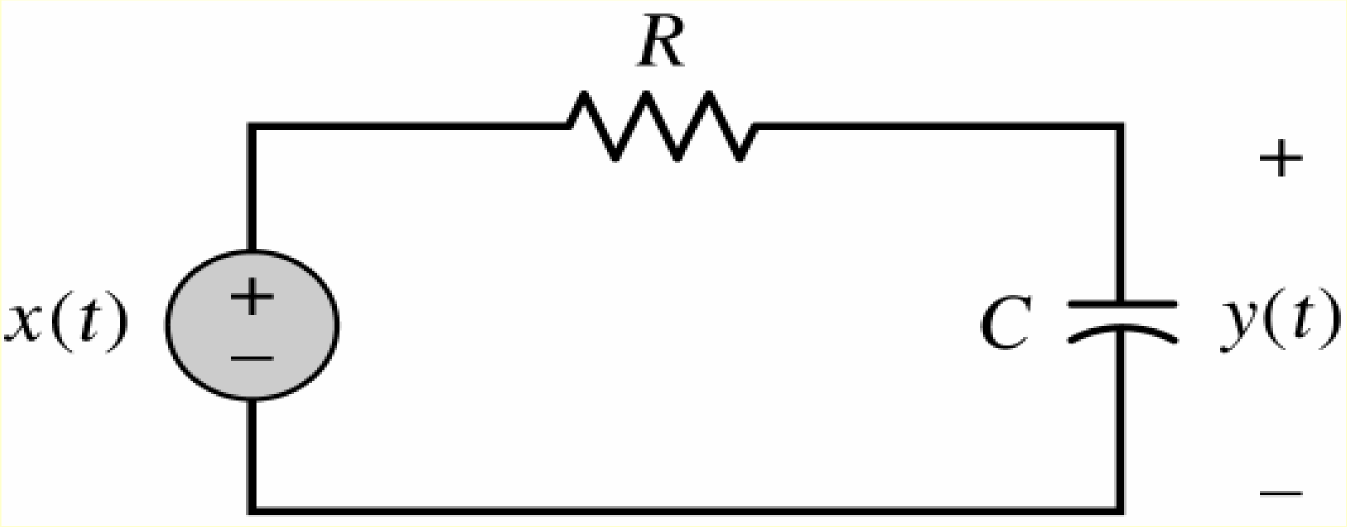


$$\sum_{i=1}^{N} a_i x_i(t)$$

EX: RC circuit : *y*(*t*) = ? in response to the unit-impulse input *x*(*t*) = δ(*t*).

回憶： $step\ response: \quad y(t) = \left(1 - e^{-t/RC}\right)u(t)$

(電容充電 *y*(t) 電壓緩緩上升)

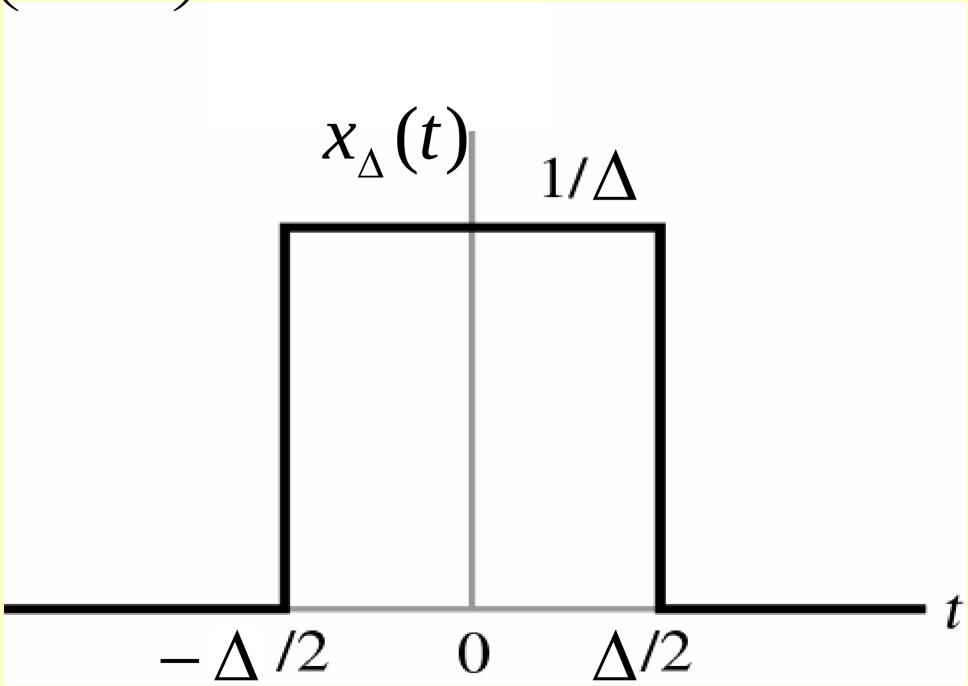


Rectangular pulse of unit area approaches a unit impulse as $\Delta \to 0$

$$x_\Delta(t) = \frac{1}{\Delta} u\left(t + \frac{\Delta}{2}\right) - \frac{1}{\Delta} u\left(t - \frac{\Delta}{2}\right)$$

$$= x_1(t) - x_2(t)$$

$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$



System Output y(t), applying Linear property:

$$y_1(t) = H\{x_1(t)\}; \quad y_2(t) = H\{x_2(t)\};$$

$$y(t) = y_1(t) \pm y_2(t) = H\{x_1(t) \pm x_2(t)\}$$ (應用線性特性)

$$solution:$$

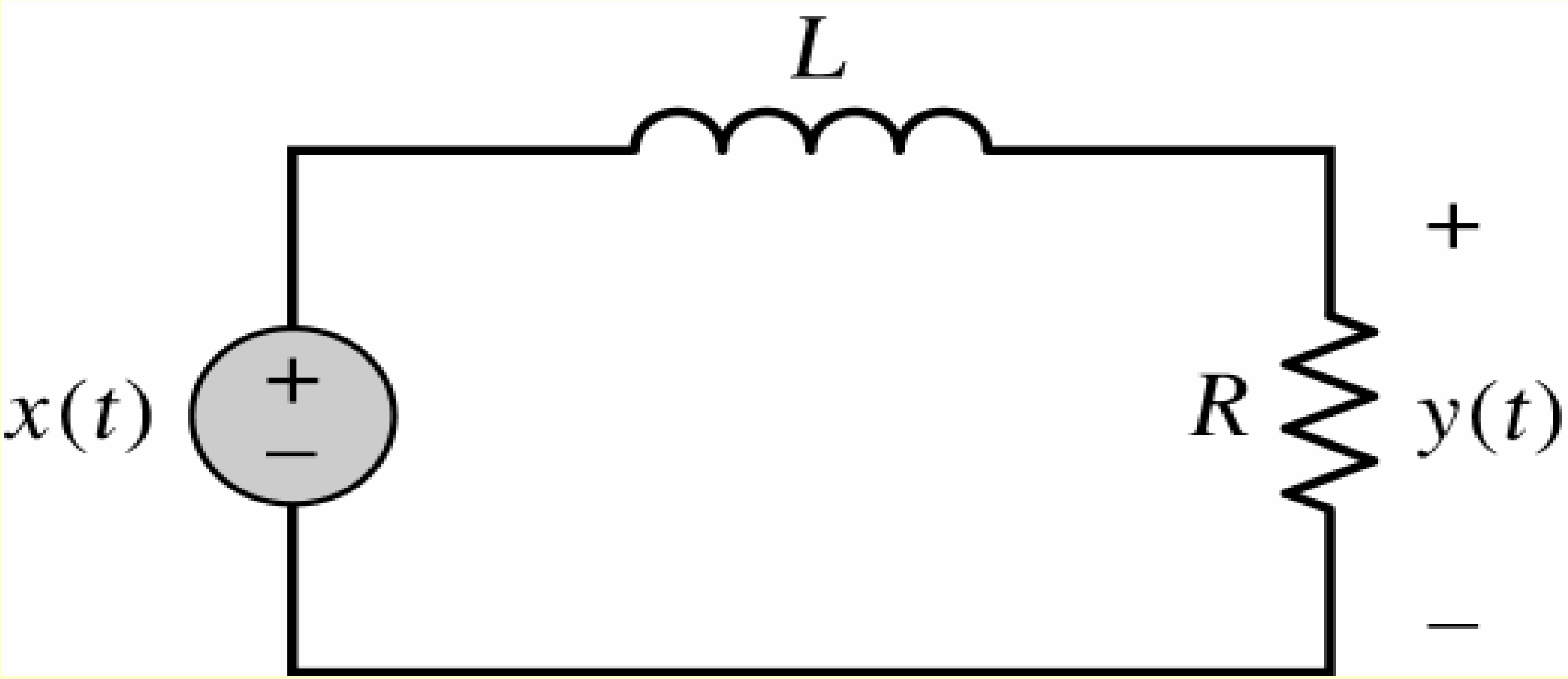
$$y_1(t) = \frac{1}{\Delta}\left(1 - e^{-(t+\Delta/2)/(RC)}\right)u(t + \Delta/2)$$

$$y_2(t) = \frac{1}{\Delta}\left(1 - e^{-(t-\Delta/2)/(RC)}\right)u(t - \Delta/2)$$

$\therefore$ (中間步驟請自行參考教科書並加以推導)

$$y(t) = y_1(t) - y_2(t) = \frac{1}{RC} e^{-t/RC} u(t)$$

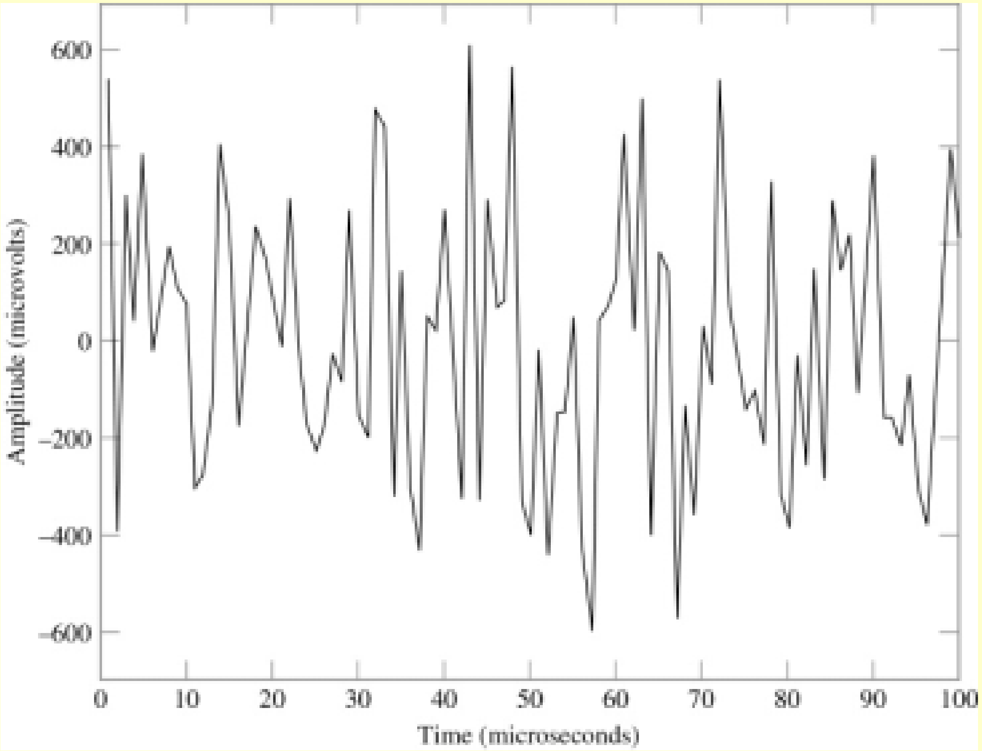
EX: RL circuit : $y(t)$ in response to the unit-impulse input $x(t) = \delta(t)$?



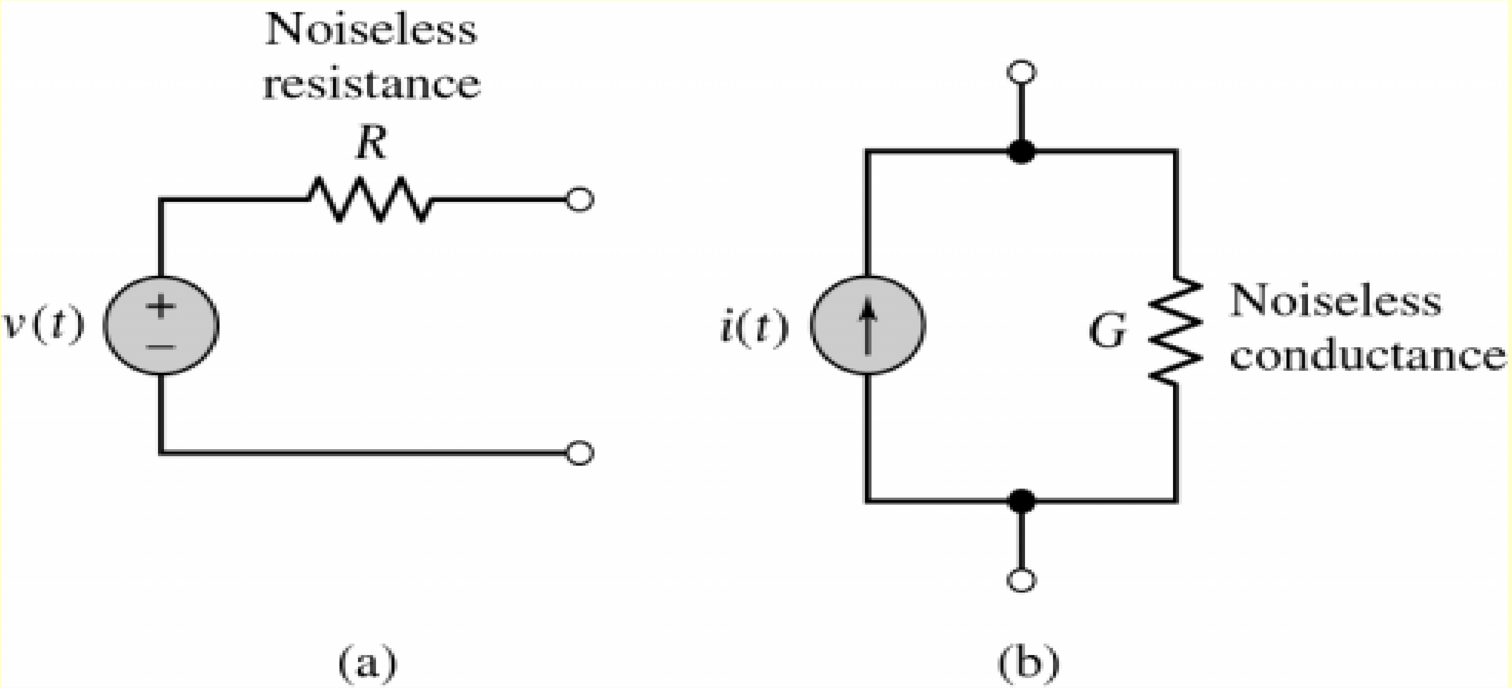
類似前題請自行參考並推導

Waveform of electrical noise generated by a thermionic diode with a heated cathode.

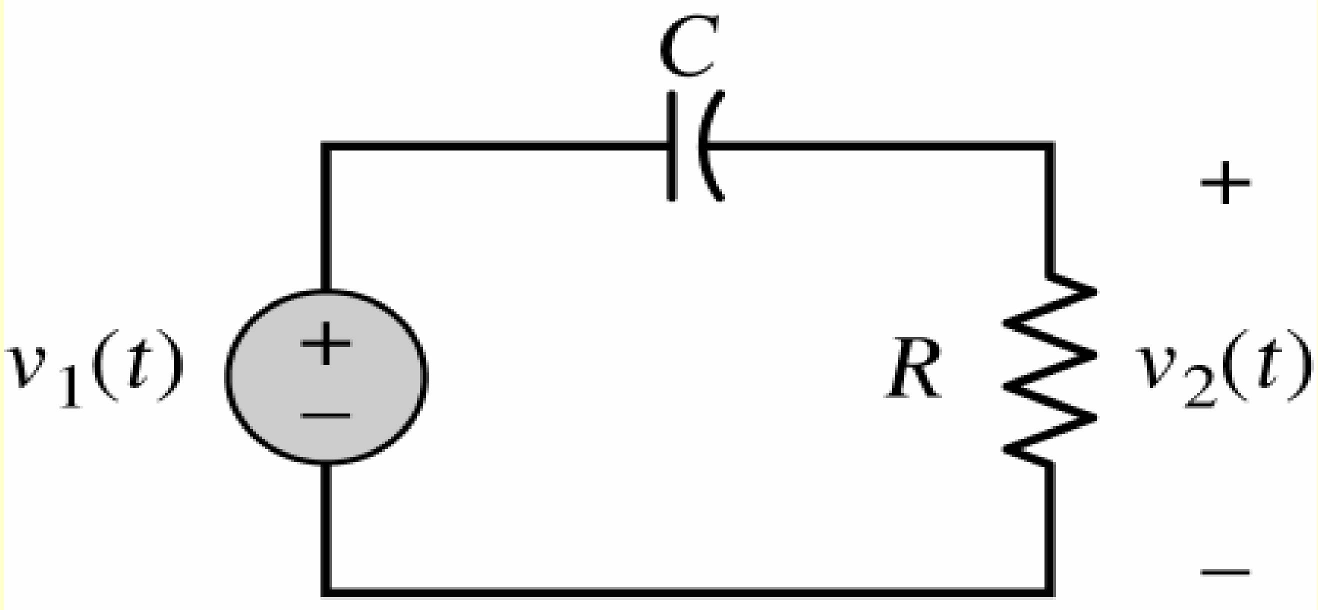
Note that the time-averaged value of the noise voltage displayed is approximately zero.



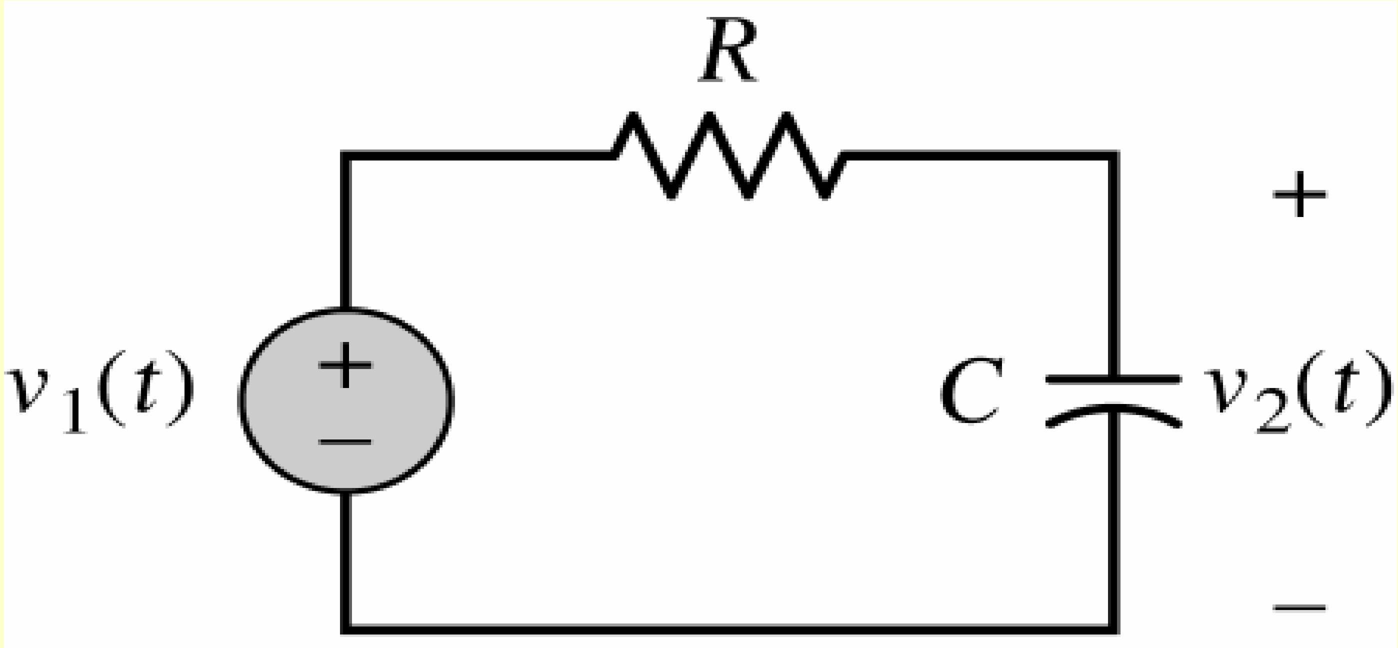
(a) Thévenin equivalent circuit of a noisy resistor.
(b) Norton equivalent circuit of the same resistor.



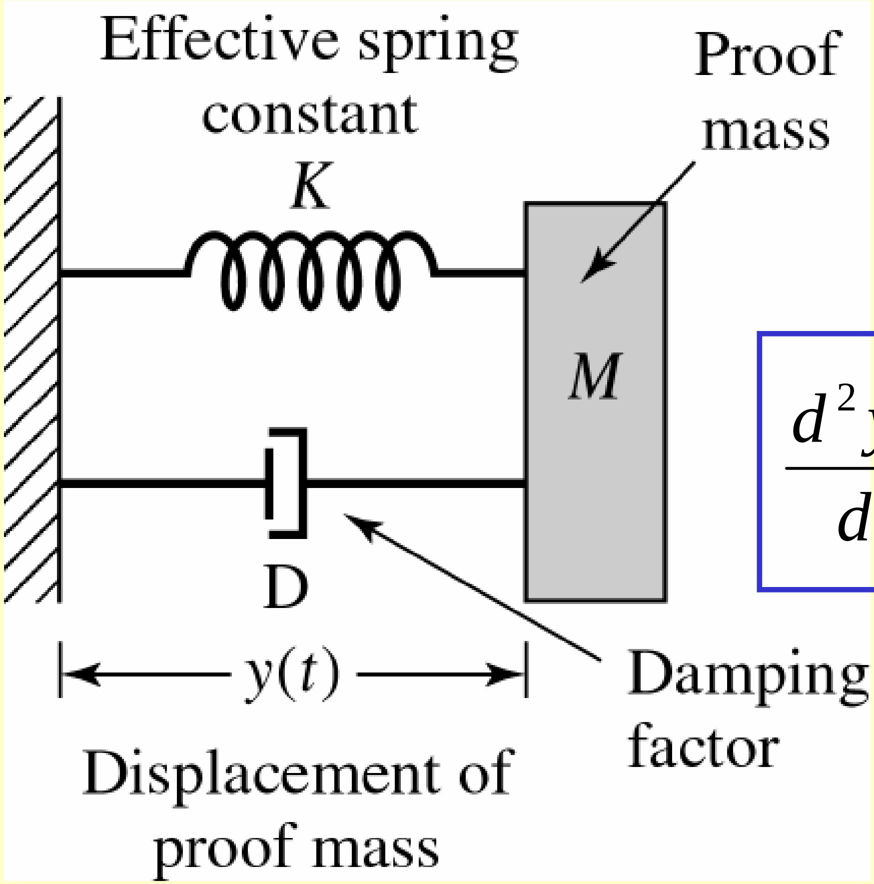
Simple *RC* circuit with small time constant, used as an approximator to a *differentiator*.



Simple *RC* circuit with large time constant used as an approximator to an *integrator*.



Mechanical lumped model of an accelerometer



$$\frac{d^2 y(t)}{dt^2} + \frac{D}{M}\frac{dy(t)}{dt} + \frac{K}{M} y(t) = x(t)$$

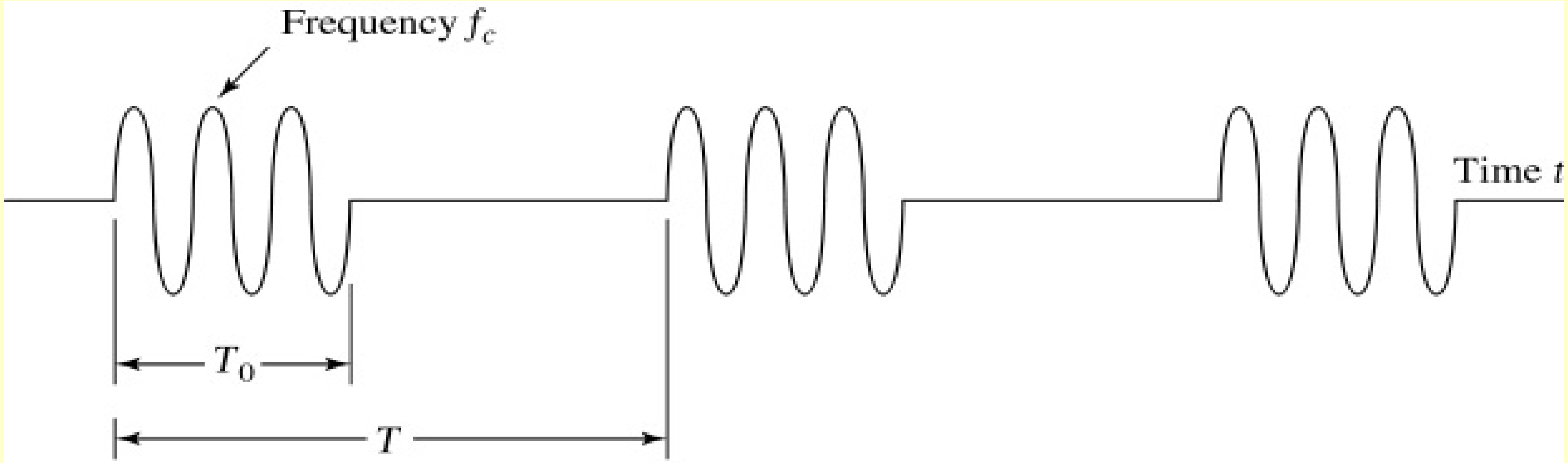
Radar Range Measurement

雷達與目標間距離：d

雷達脈波傳遞至目標並傳回所需時間 τ：

$$\tau = \frac{2d}{C}$$

C：雷達脈波傳遞速度



Radar Range Measurement (cont.)

距離解析度 (Range Resolution)

脈波持續時間 T_0 限制可測量最短目標距離:

$$d_{\min} = \frac{cT_0}{2} \quad meters$$

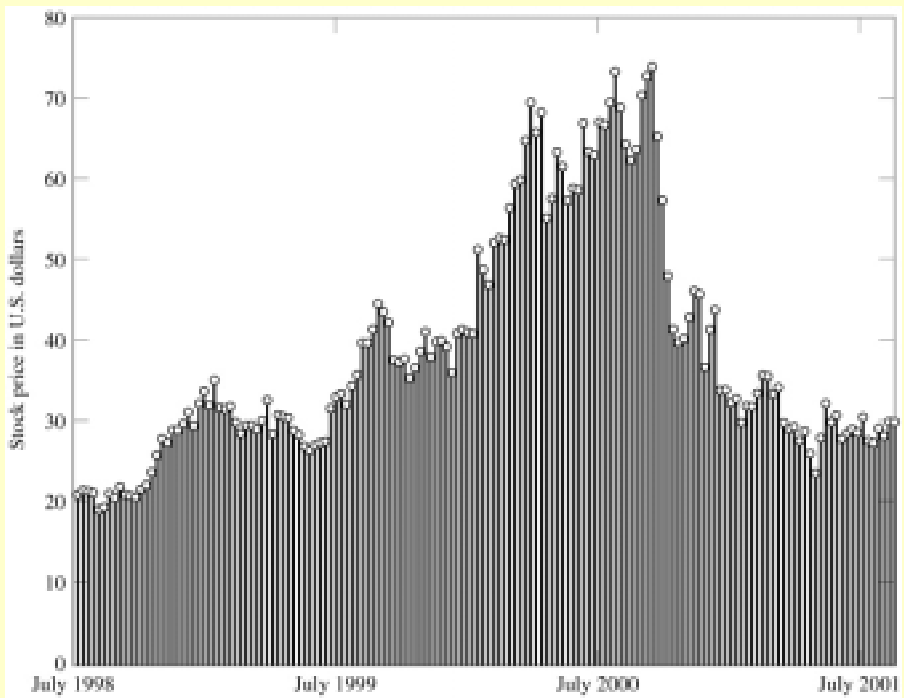
距離模糊度 (Range Ambiguity)

脈波間週期 T 限制可測量最遠目標距離:

$$d_{\max} = \frac{cT}{2} \quad meters$$

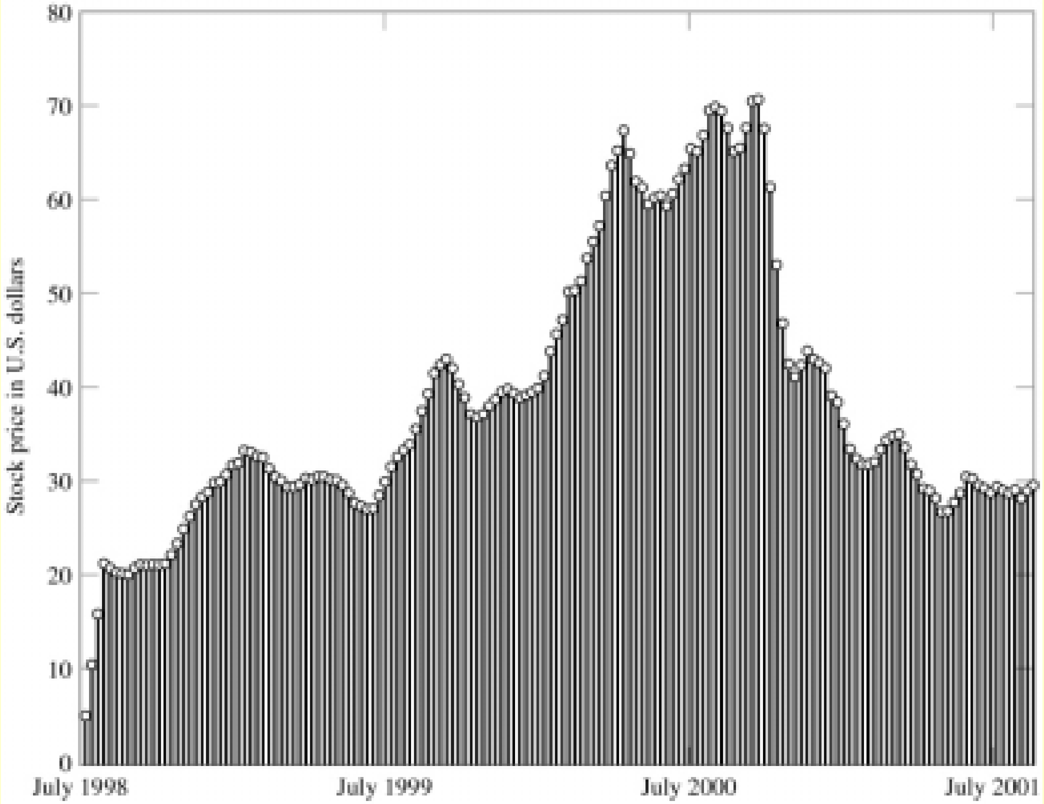
Moving Average System

Fluctuations in the closing stock price of Intel over a three-year period.



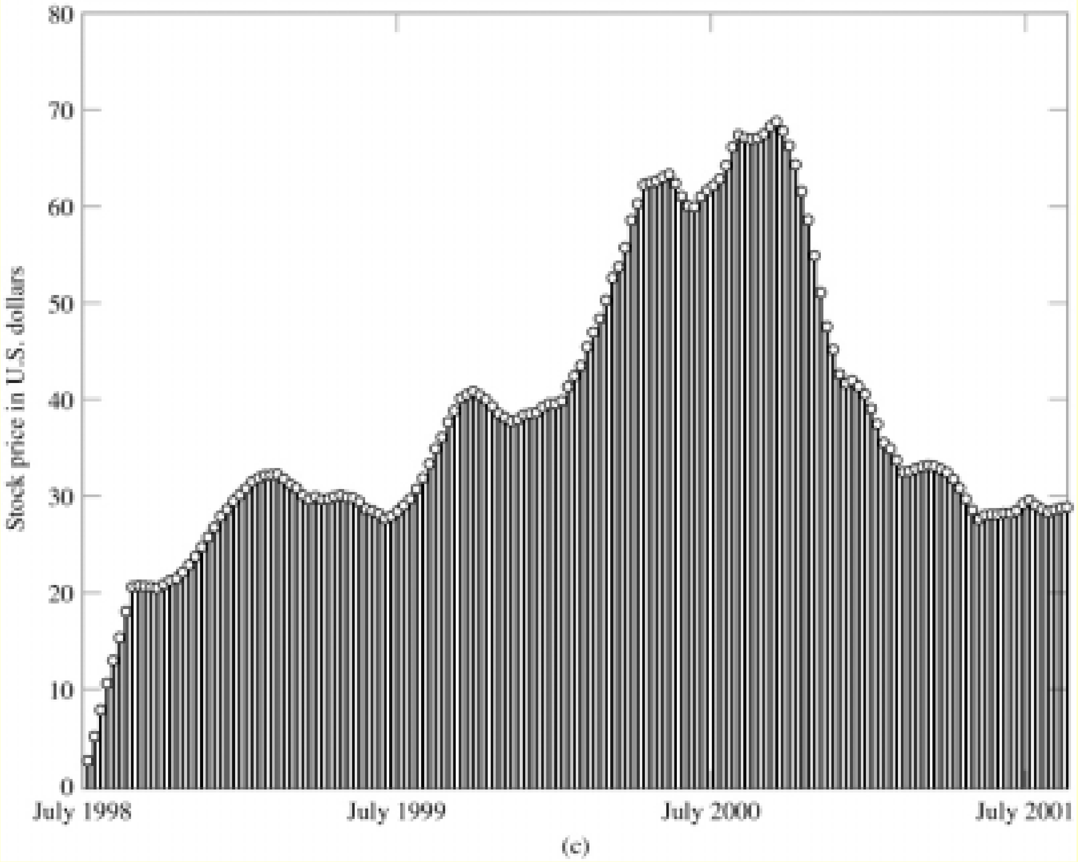
(a)

Output of a *4-point* moving-average system

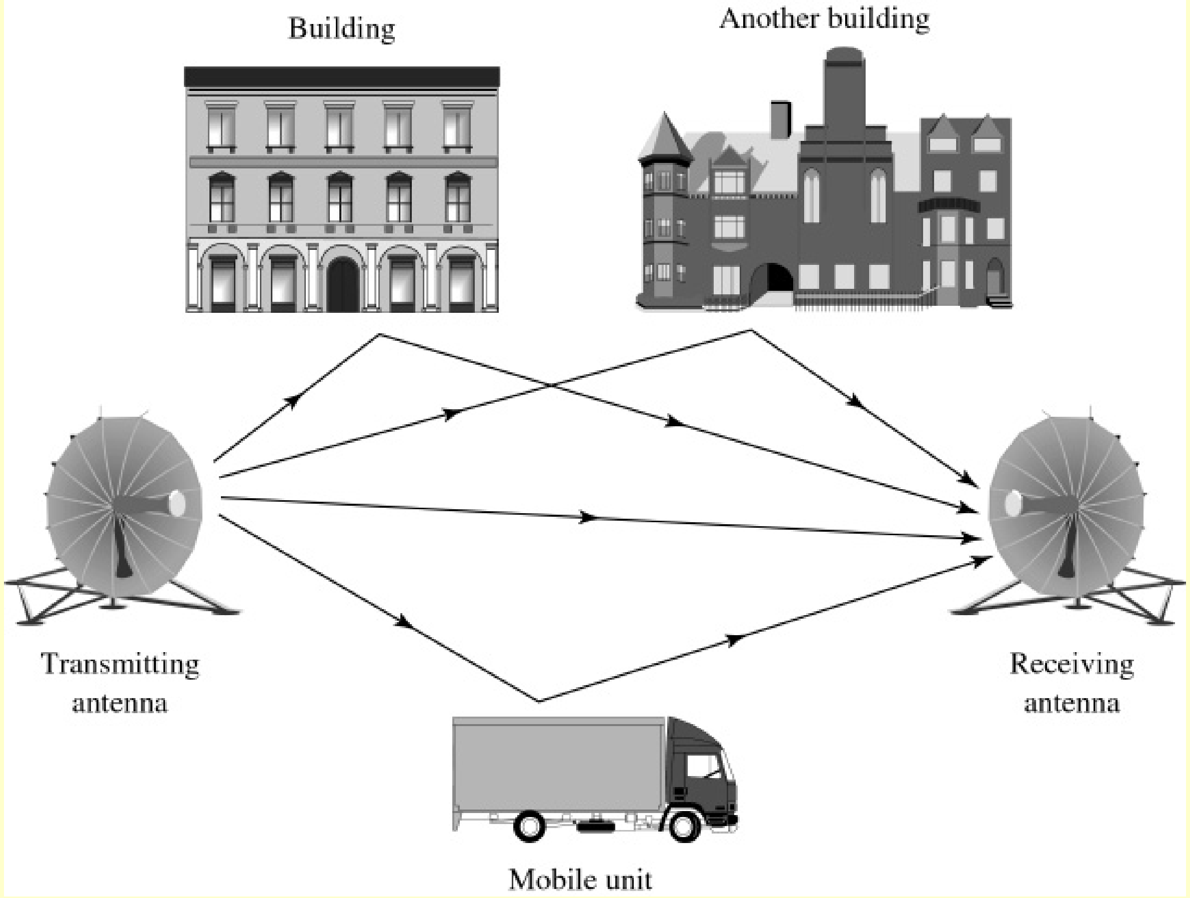


(b)

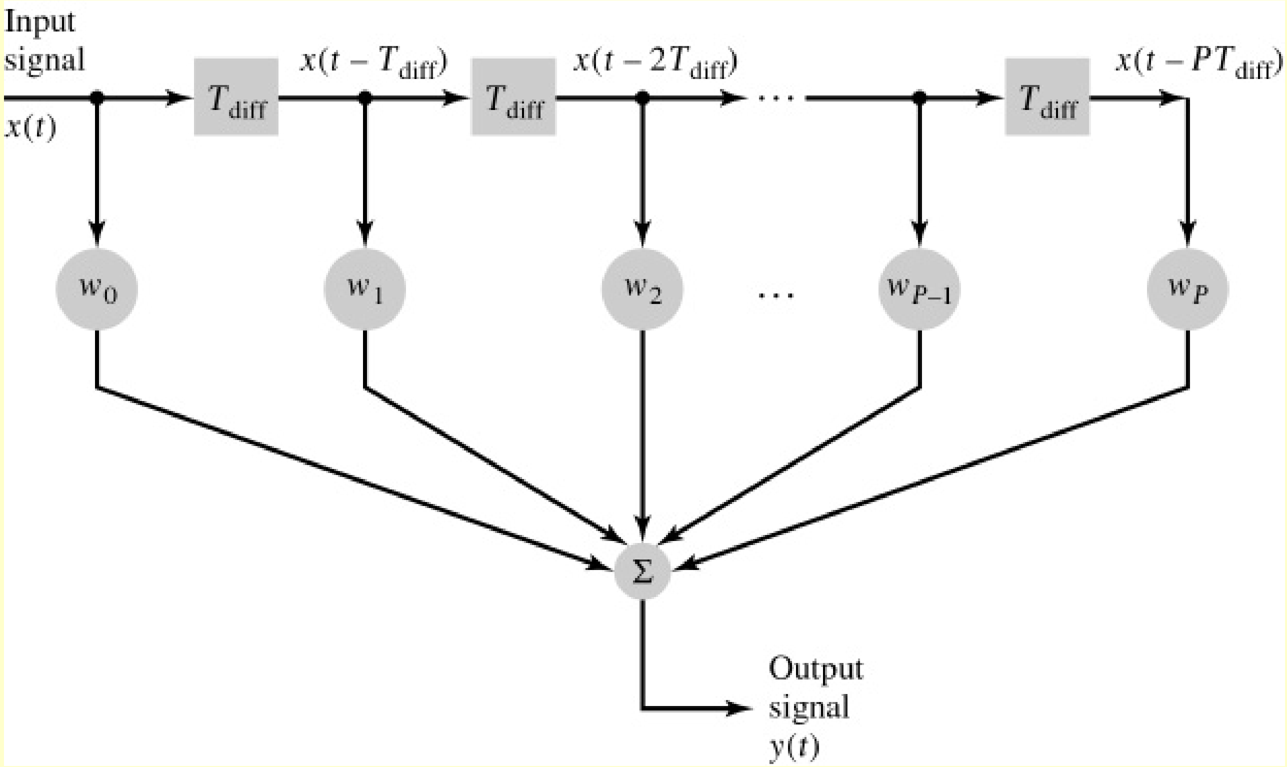
Output of an ***8-point*** moving-average system



Multiple propagation paths in a wireless communication environment

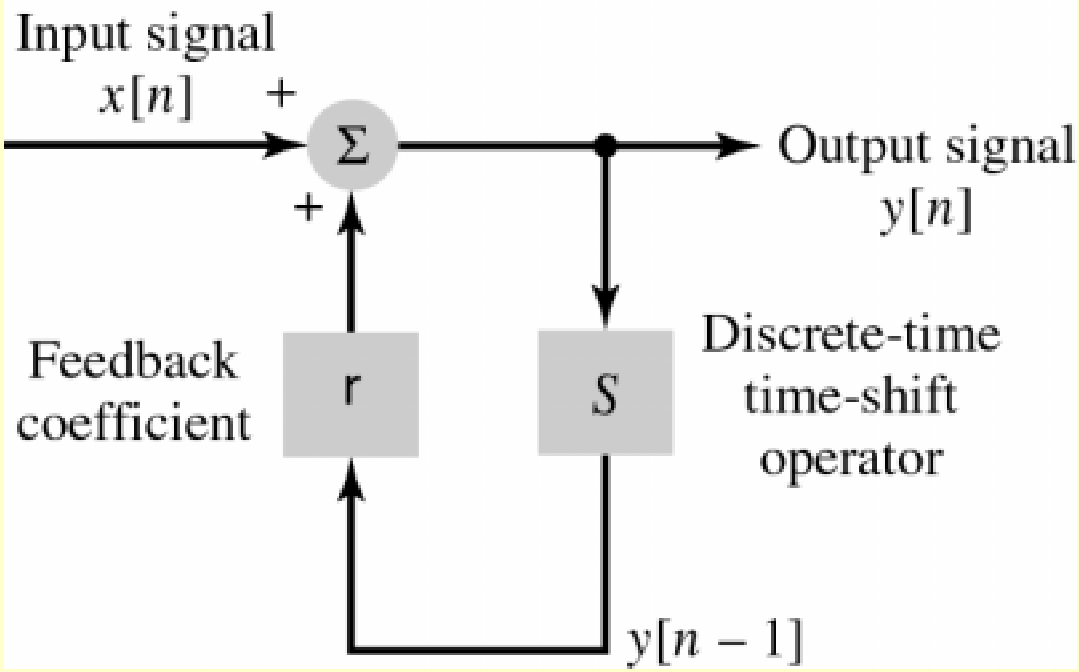


Tapped-delay-line model of a linear communication channel, assumed to be time-invariant

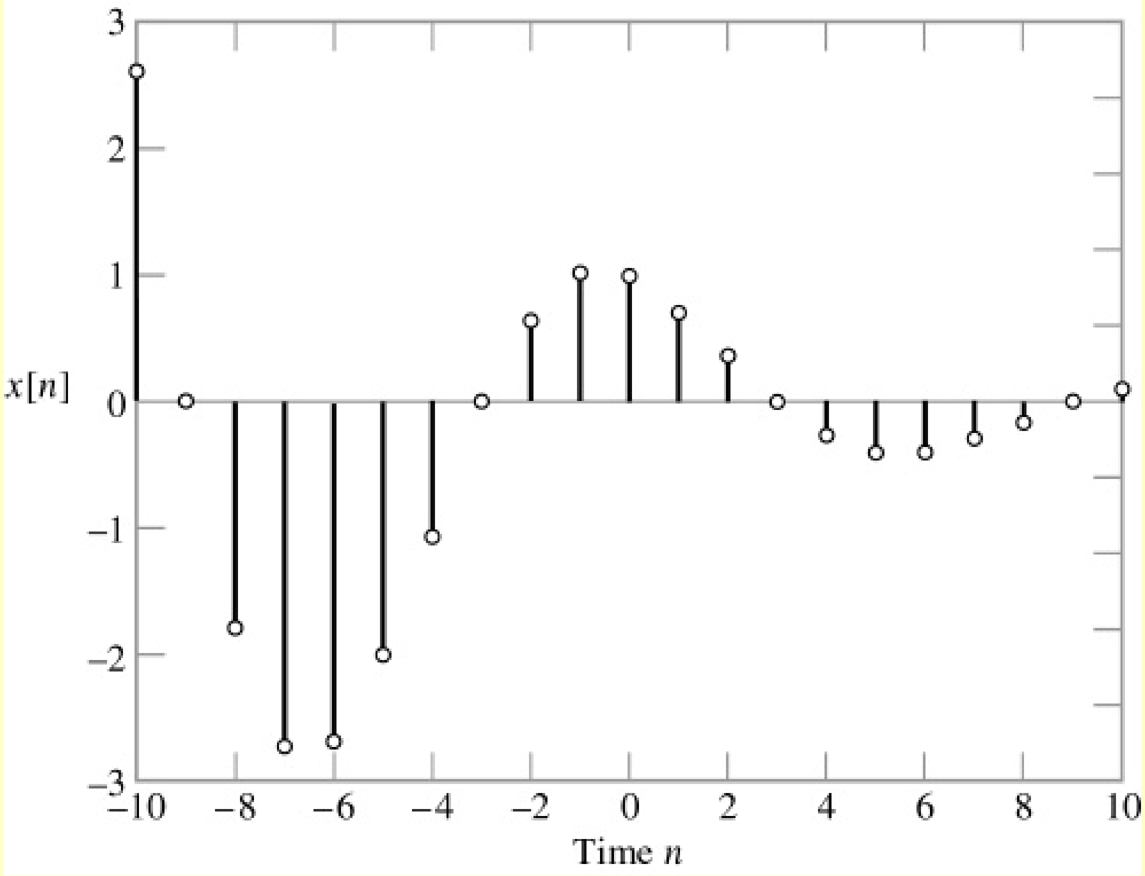


Block diagram of first-order recursive discrete-time filter

The operator *S* shifts the output signal *y*[*n*] by one sampling interval, producing *y*[*n* – 1]. The feedback coefficient *p* determines the stability of the filter.



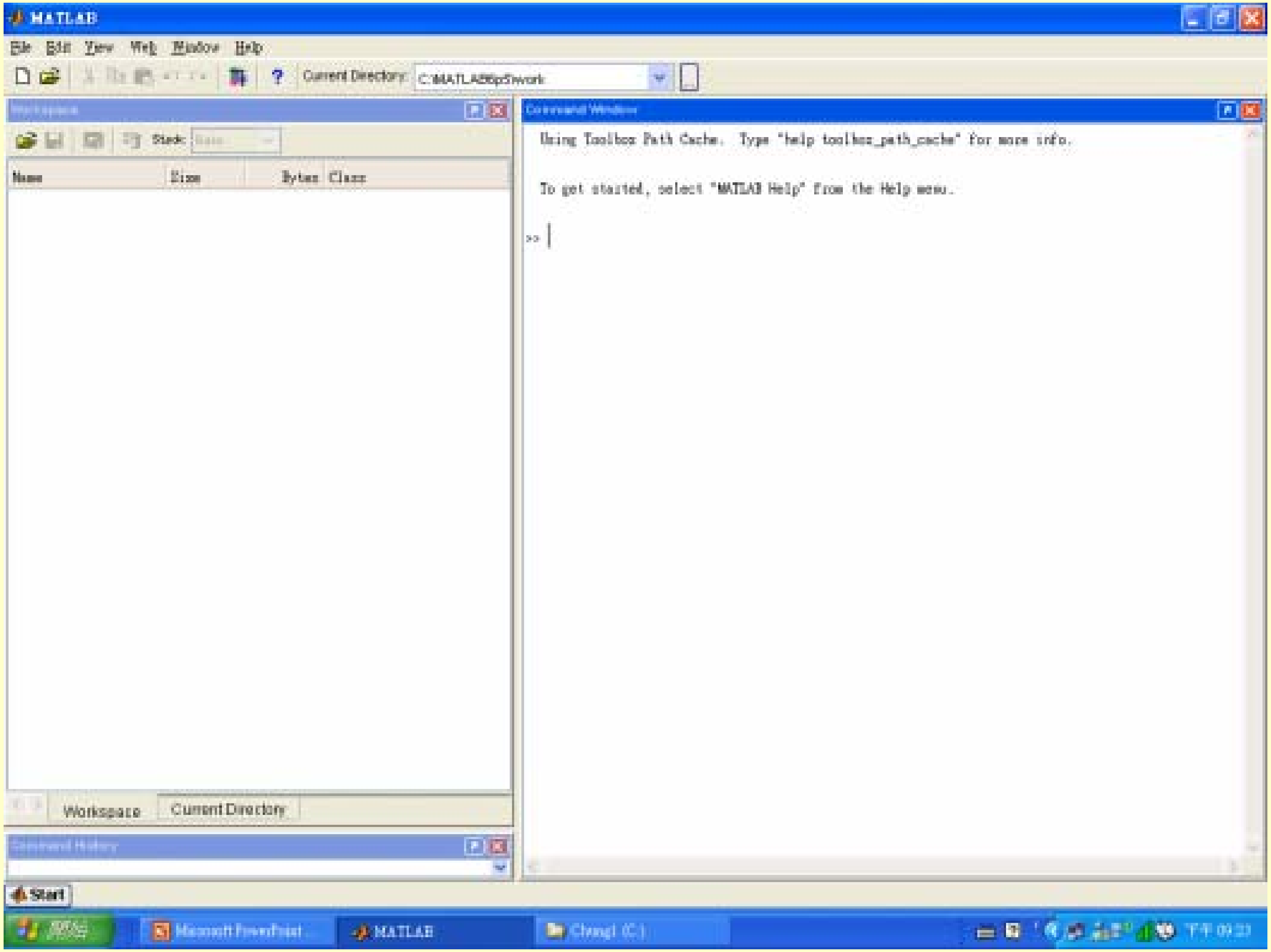
Exponentially damped sinusoidal sequence



Exploring Concepts with MATLAB

- Periodic Signals
- Exponential Signals
- Sinusoidal Signals
- Exponentially Damped Sinusoidal Signals
- Step, Impulse and Ramp Functions
- User Defined Functions

MATLAB Window 畫面



Periodic Signal

- Generate Square Wave:

$$A = 1;$$

$$w0 = 10 * pi;$$

$$rho = 0.5;$$

$$t = 0 : 0.001 : 1;$$

$$sq = A * square(w0 * t, rho);$$

$$plot(t, sq)$$

$$axis([0 \quad 1 \quad -1.1 \quad 1.1])$$

Exponential Signal

- Generate Function: x = b exp(-at)

$$b = 5;$$

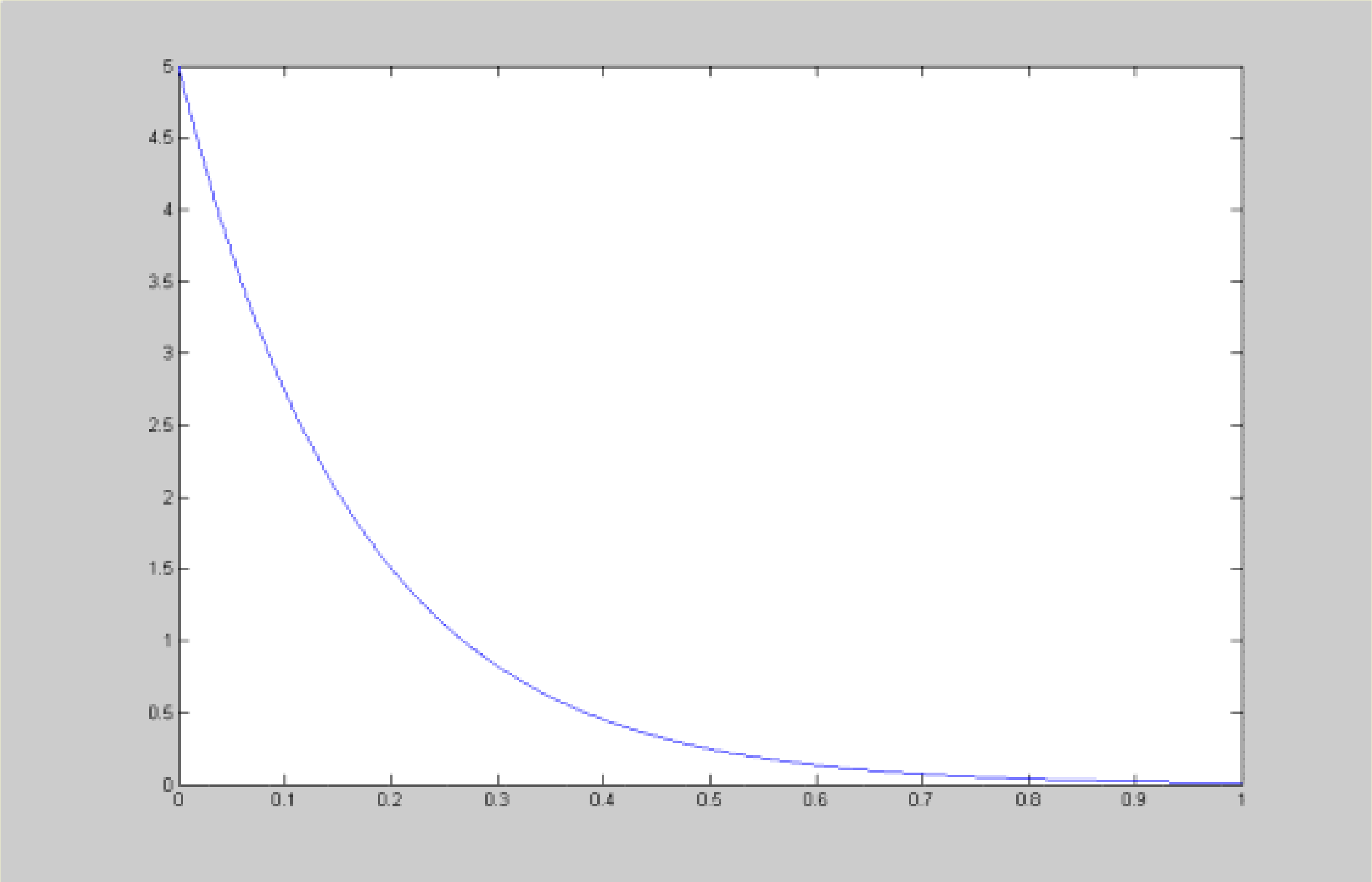
$$a = 6;$$

$$t = 0 : 0.001 : 1;$$

$$x = b * \exp(-a * t);$$

$$plot(t, x)$$

Exp Signal Plot



Sinusoidal Signal

- Generate Function: $x = A\cos(\omega_o t + \phi)$

$$A = 4;$$

$$w0 = 20 * pi;$$

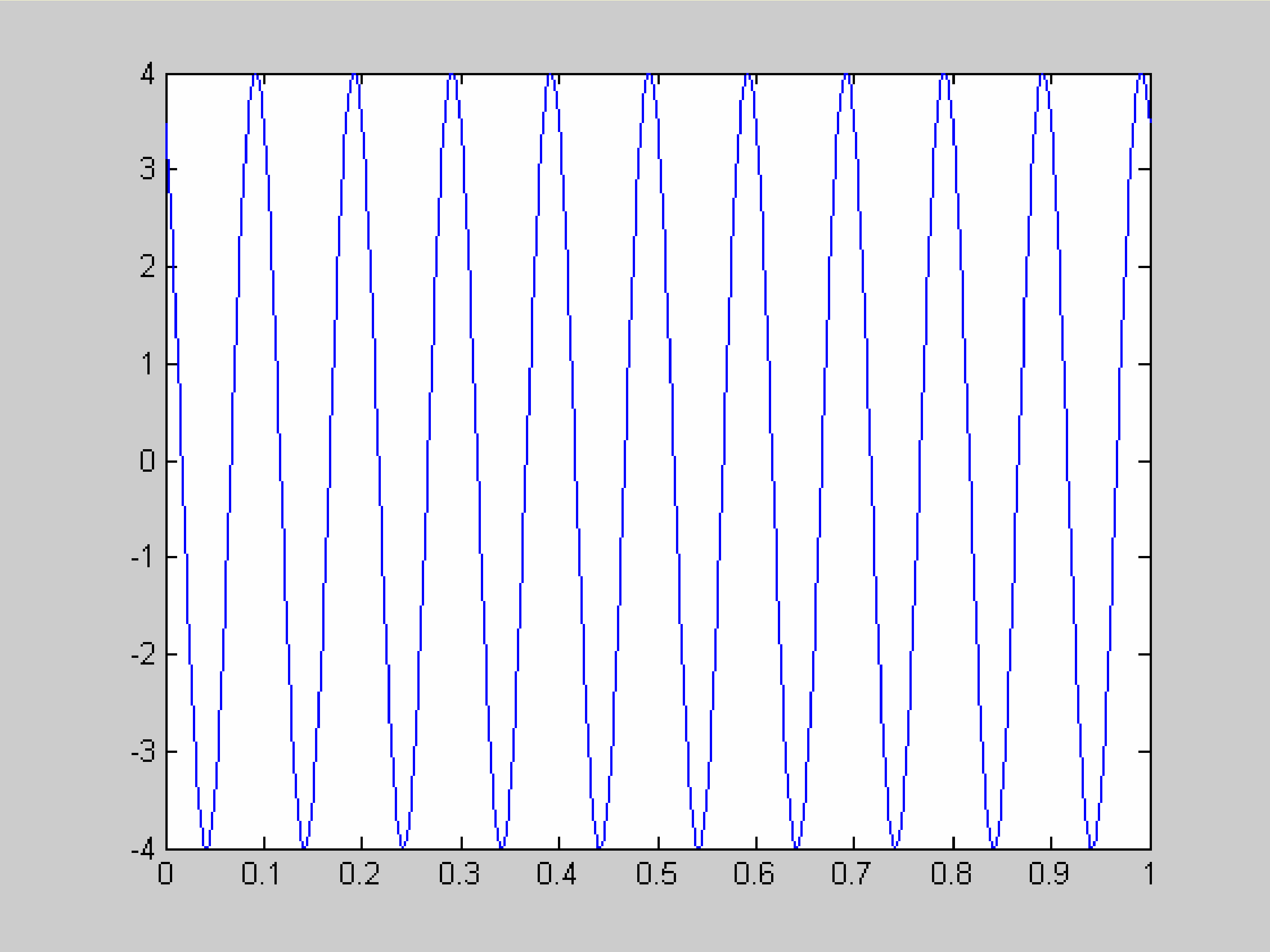
$$phi = pi/6;$$

$$t = 0:0.001:1;$$

$$x = A*\cos(w0*t + phi);$$

$$plot(t, x)$$

Sine Signal Plot



Exponentially Damped Sinusoidal Signal

- Generate Function: x = A e^{-at} $\sin(\omega_o t + \phi)$

$$A = 60;$$

$$w0 = 20 * pi;$$

$$phi = 0;$$

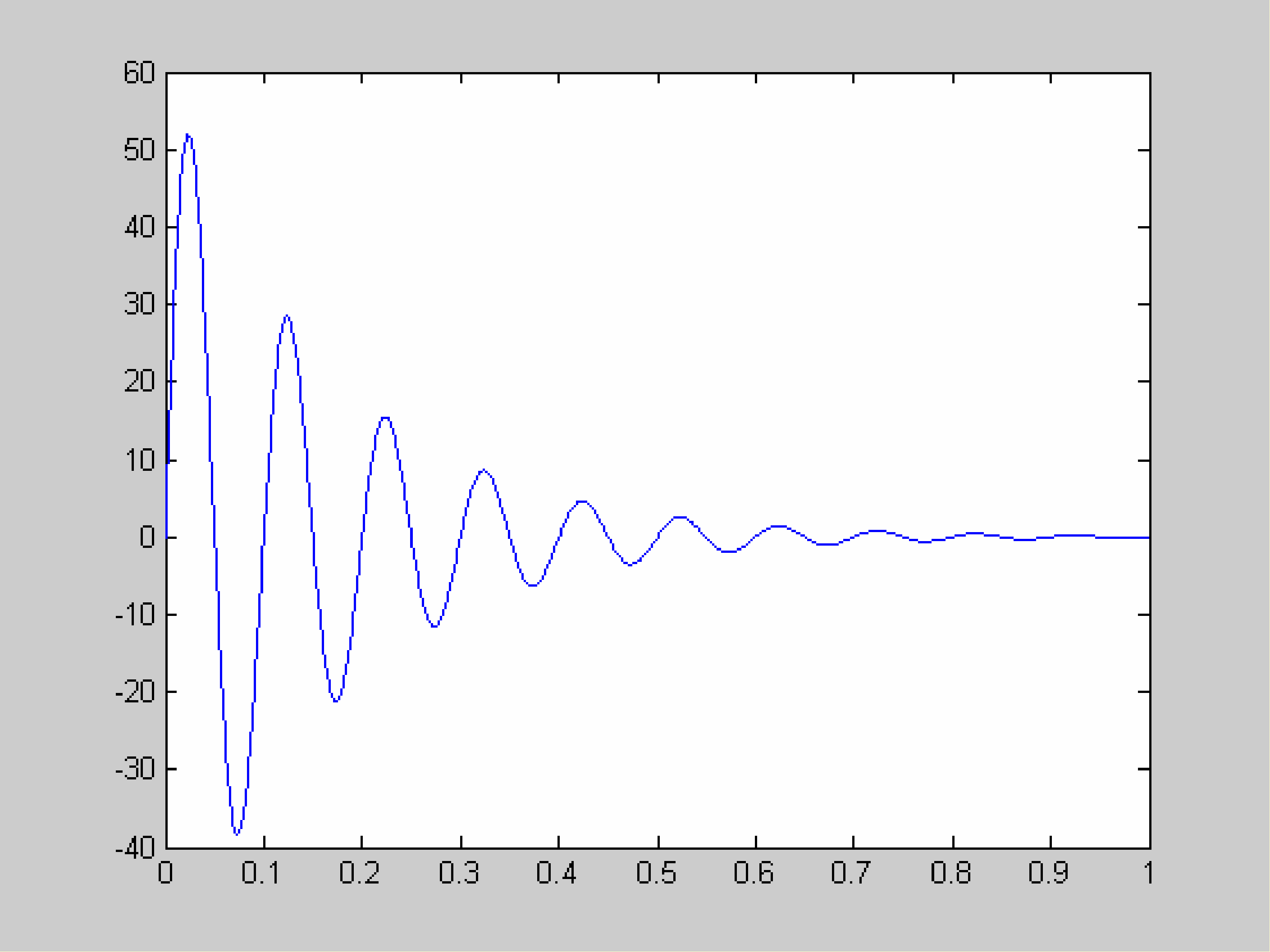
$$a = 6;$$

$$t = 0 : 0.001 : 1;$$

$$x = A * \sin(w0 * t + phi) .* \exp(-a * t);$$

$$plot(t, x)$$

Exp. Damped Sine Signal Plot



Step, Impulse and Ramp Function

- Generate Step Function:

$$u = [zeros(1,50), ones(1,50)];$$

- Generate Impulse Function:

$$delta = [zeros(1,49), 1, zeros(1,49)];$$

- Generate Ramp Function:

$$ramp = 0:0.1:10;$$

User Defined Function

- Generate ".m" file to define function:

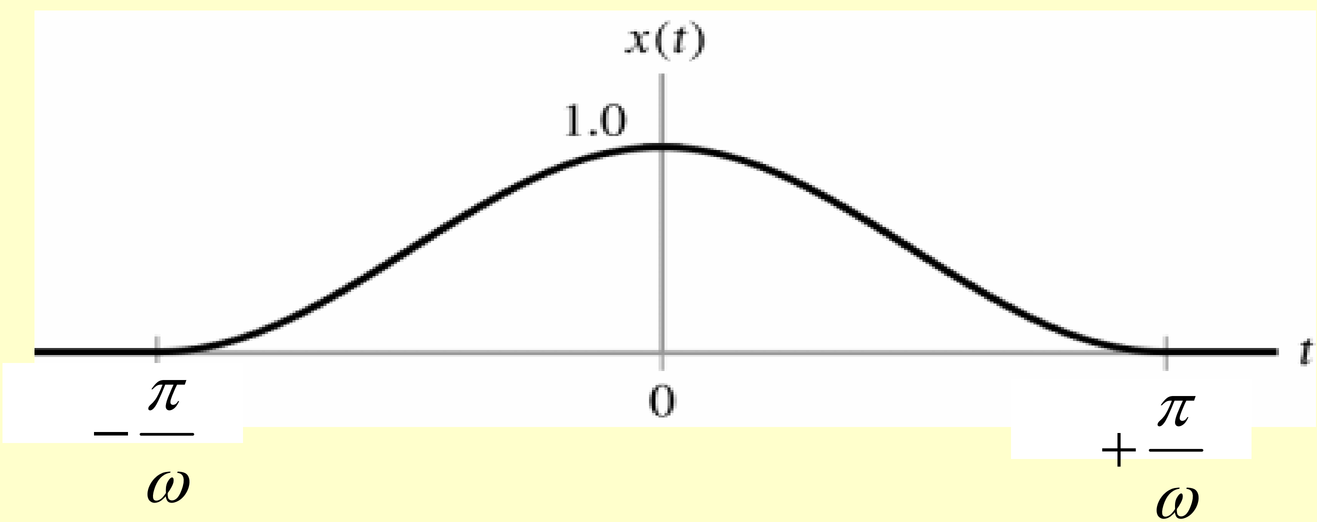
$$function\ g = rect(x)$$

$$g = zeros(size(x));$$
$$set = find(abs(x) <= 0.5);$$
$$g(set) = ones(size(set));$$

P1.46 試求下圖訊號能量總和 ？

$$E=\int_{-\pi/\omega}^{+\pi/\omega} x^2(t)\,dt$$

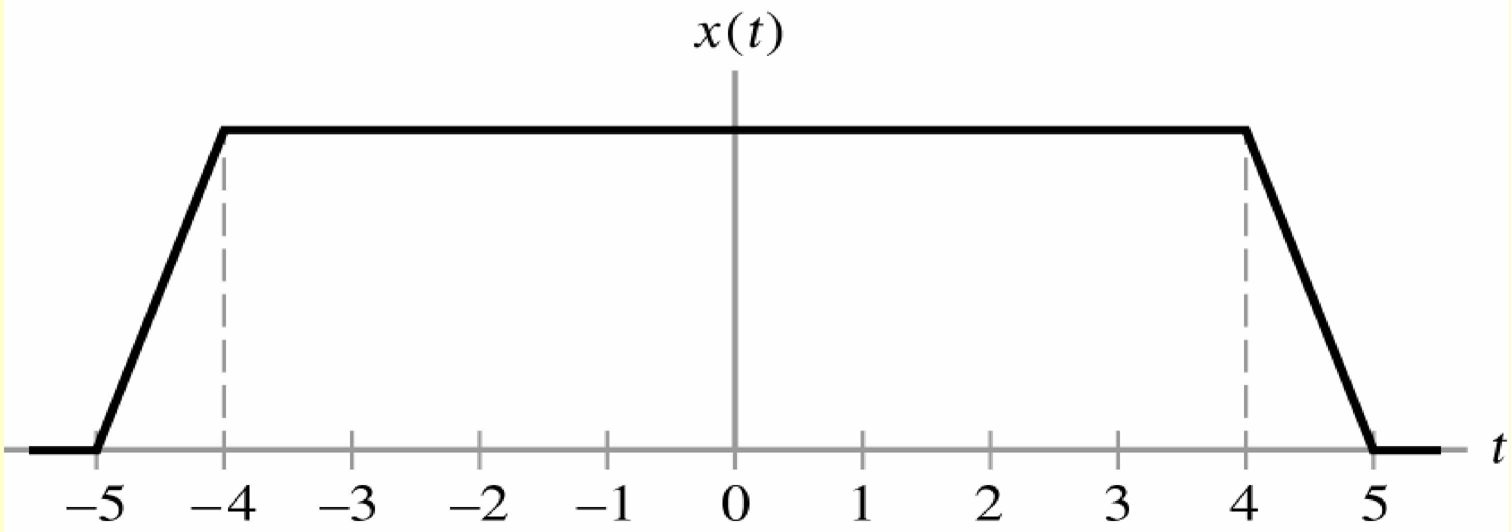
$$x(t)=\begin{cases}\frac{1}{2}\left[1+\cos(\omega t)\right], & -\pi/\omega \le t \le \pi/\omega \\ 0, & others\end{cases}$$



P1.47 試求下圖訊號能量總和 ？

$$E = \int_{-5}^{+5} x^2(t)\, dt$$

$$x(t) = \begin{cases} 5-t, & 4 \le t \le 5 \\ 1, & -4 \le t \le 4 \\ t+5, & -5 \le t \le -4 \\ 0, & others \end{cases}$$

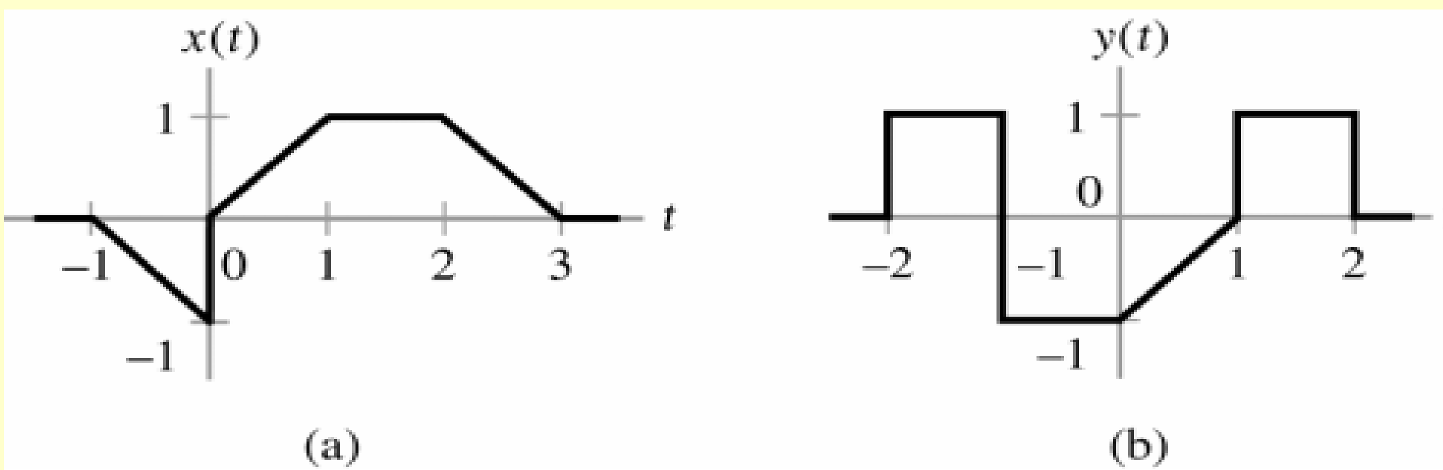


P1.52 請繪出下列訊號圖？

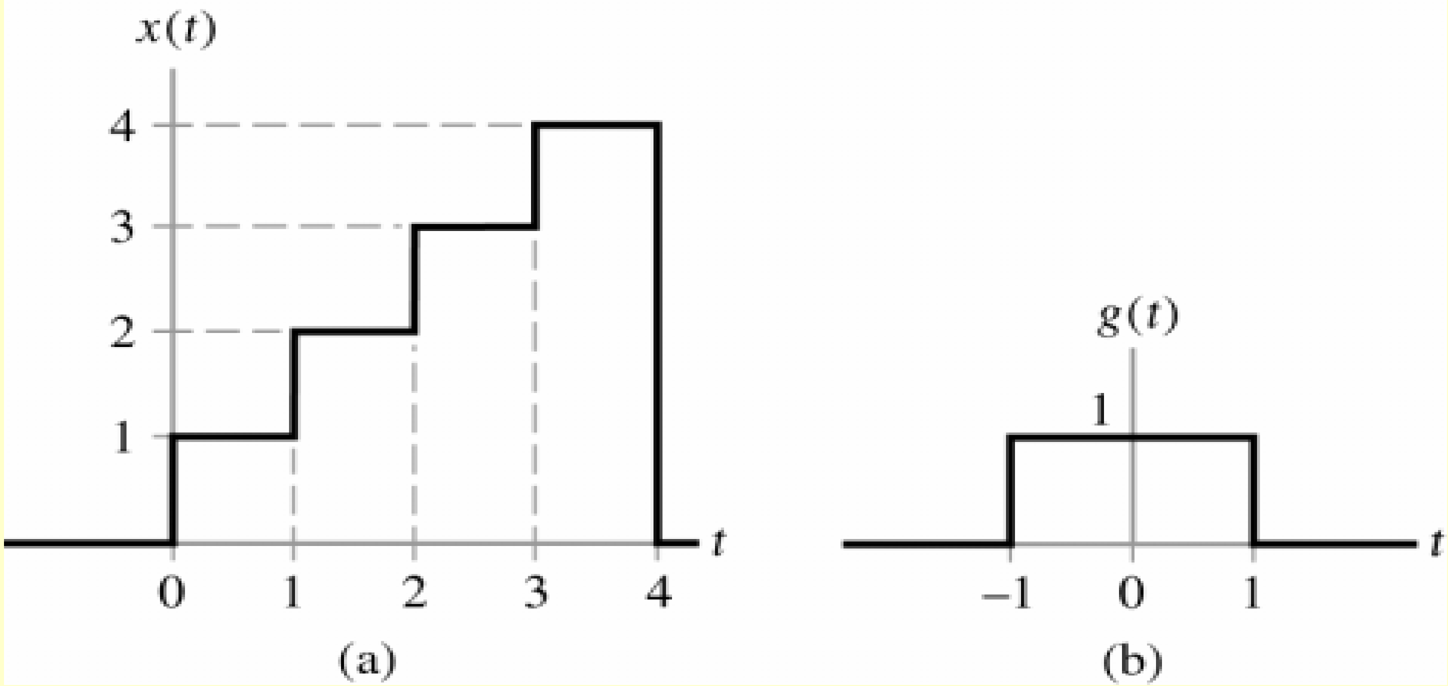
$$(a)\quad x(t)y(t-1)$$

$$(b)\quad x(t-1)y(-t)$$

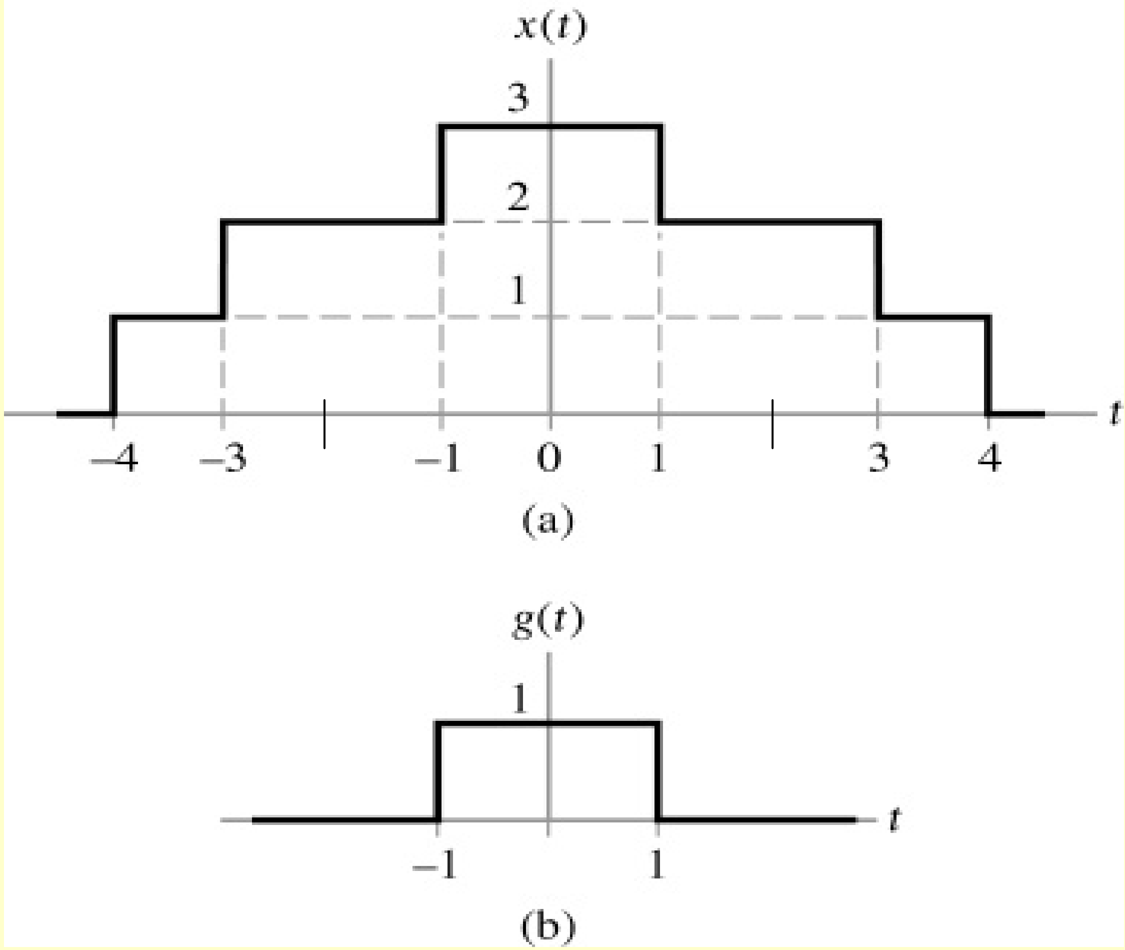
$$(c)\quad x(t+1)y(t-2)$$



P1.53 試以 g(t) 表示 x(t) ？



P1.55 試以 **g(t)** 表示 **x(t)** ？



(a)

(b)

P1.56 請繪出下列訊號圖？

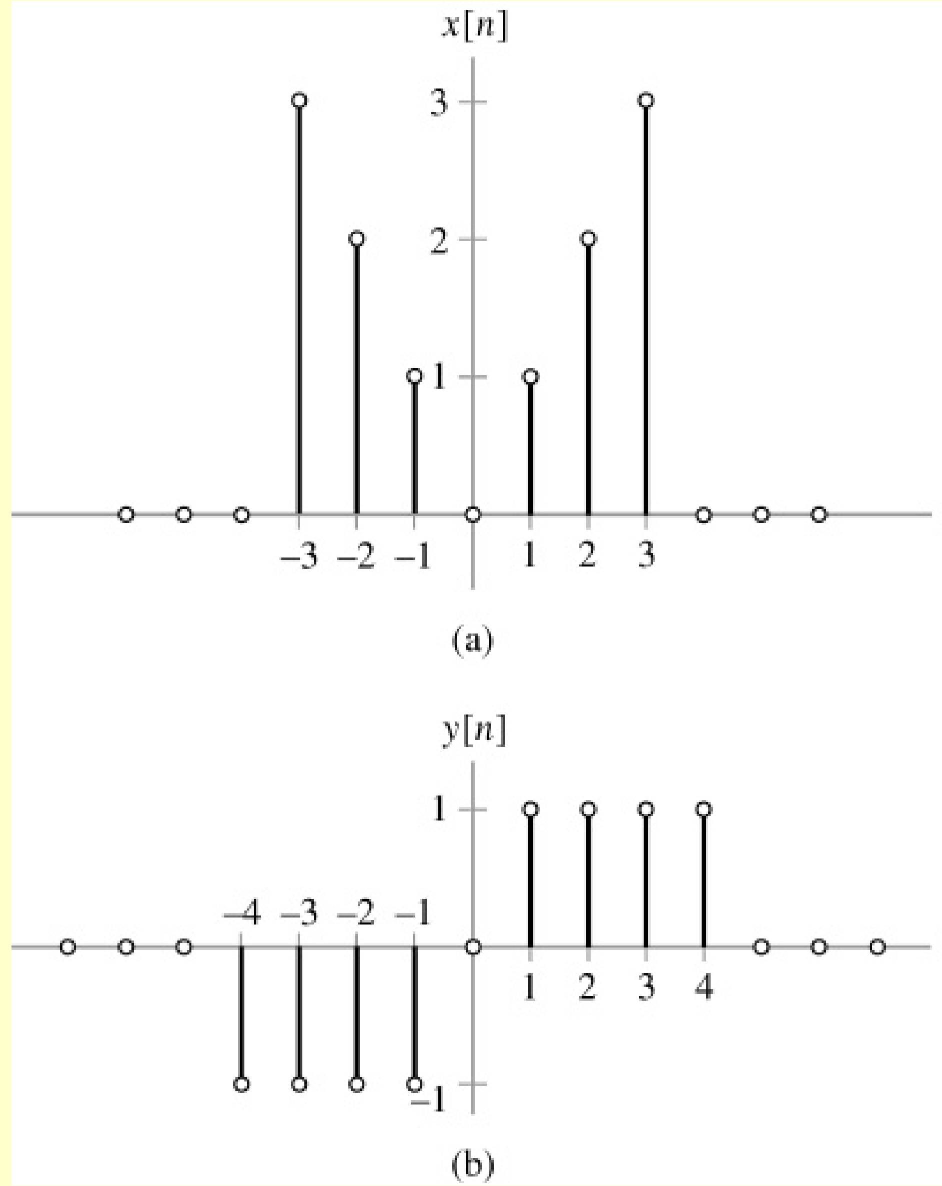
(1) $x[2n]$

(2) $x[3n-1]$

(3) $y[1-n]$

(4) $y[2-2n]$

(5) $x[n-2]+y[n+2]$



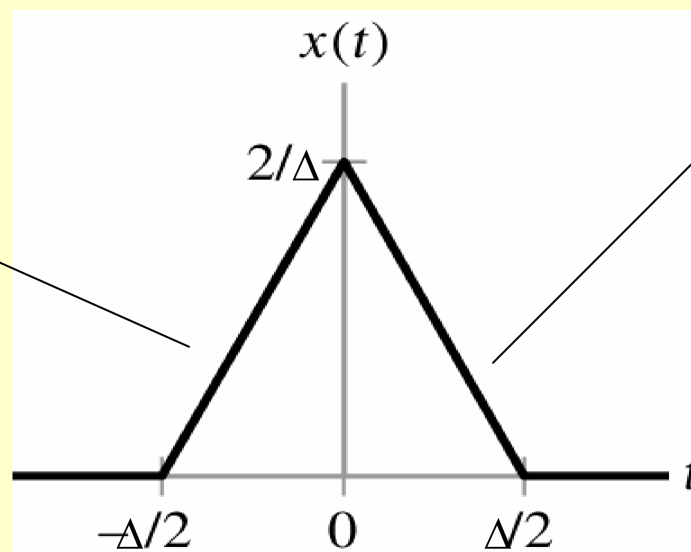
P1.62 將下面三角訊號送入一微分器，回答下列問題

(a) 微分器輸出 **y(t)** 為何？

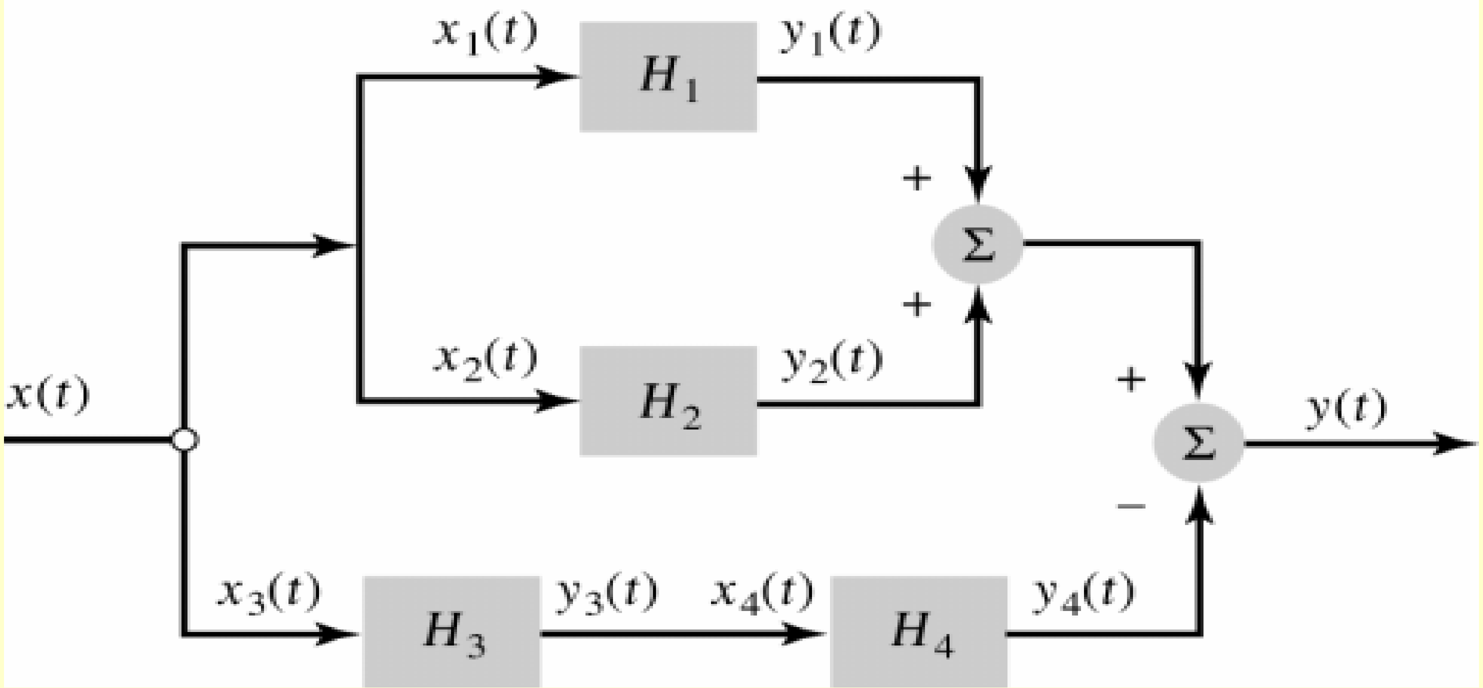
(b) 當Δ 趨近於零時，輸出 **y(t)** 變化為何？以 δ**(t)** 說明

$$\frac{4}{\Delta^2}t+\frac{2}{\Delta}$$

$$-\frac{4}{\Delta^2}t+\frac{2}{\Delta}$$



P1.63 試求出總系統？



P1.75

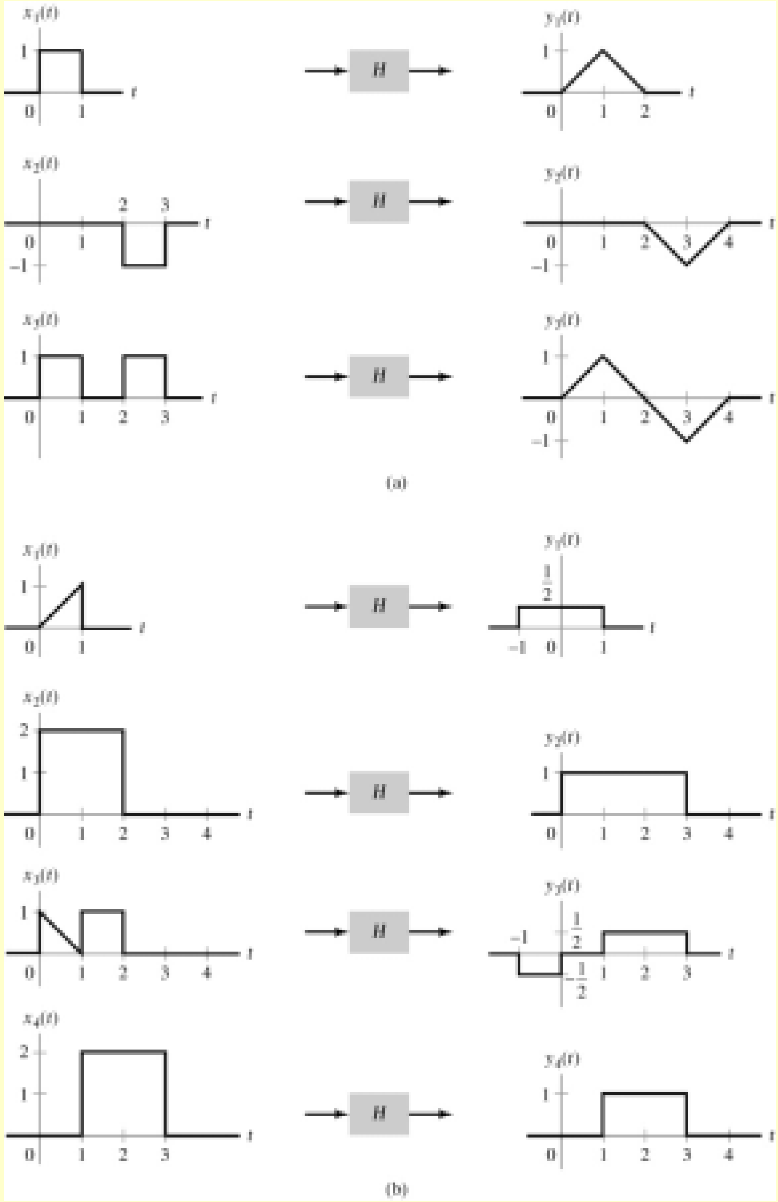
說明右圖各系統特性？

記憶性？

因果性？

線性？

(非)時變性？



P1.76

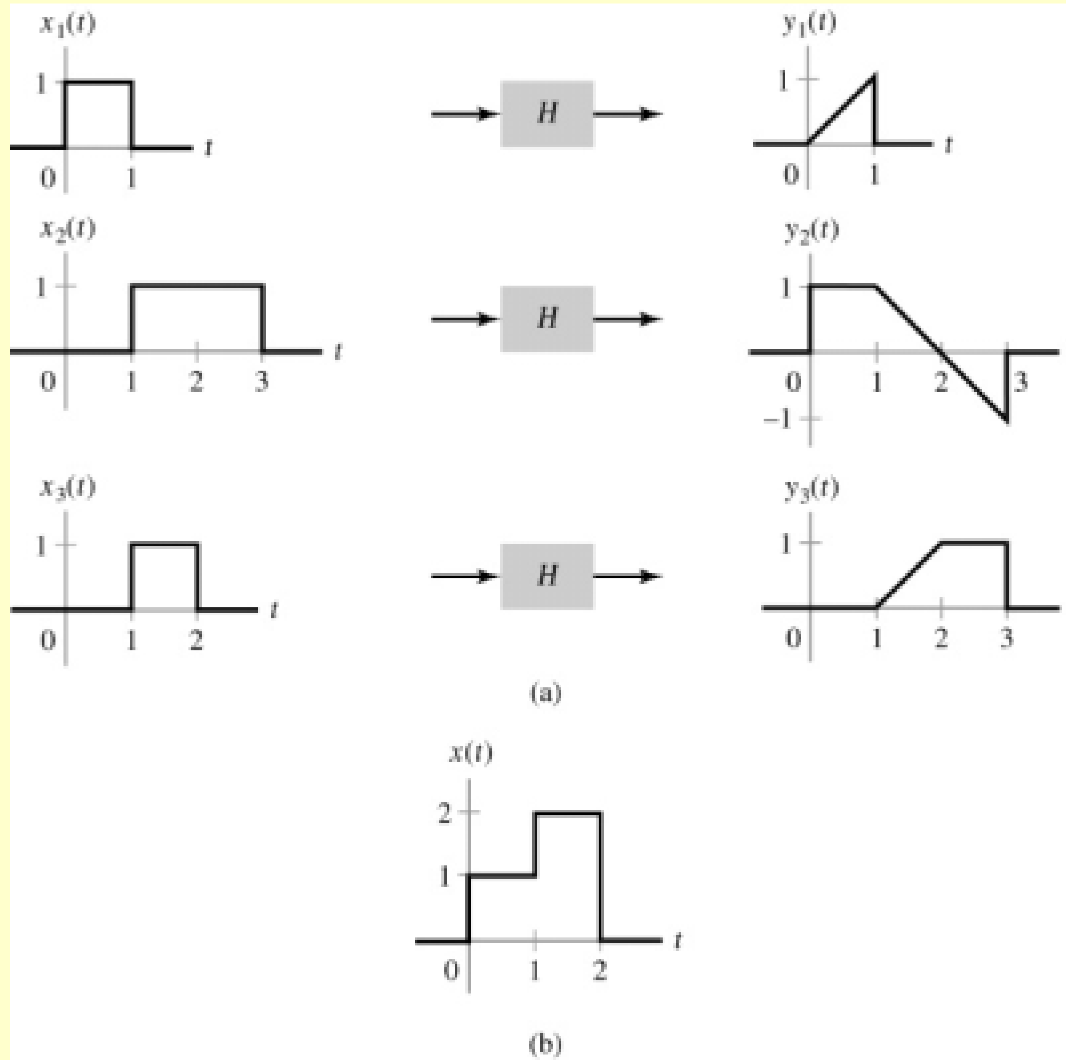
說明右圖各系統特性？

記憶性？

因果性？

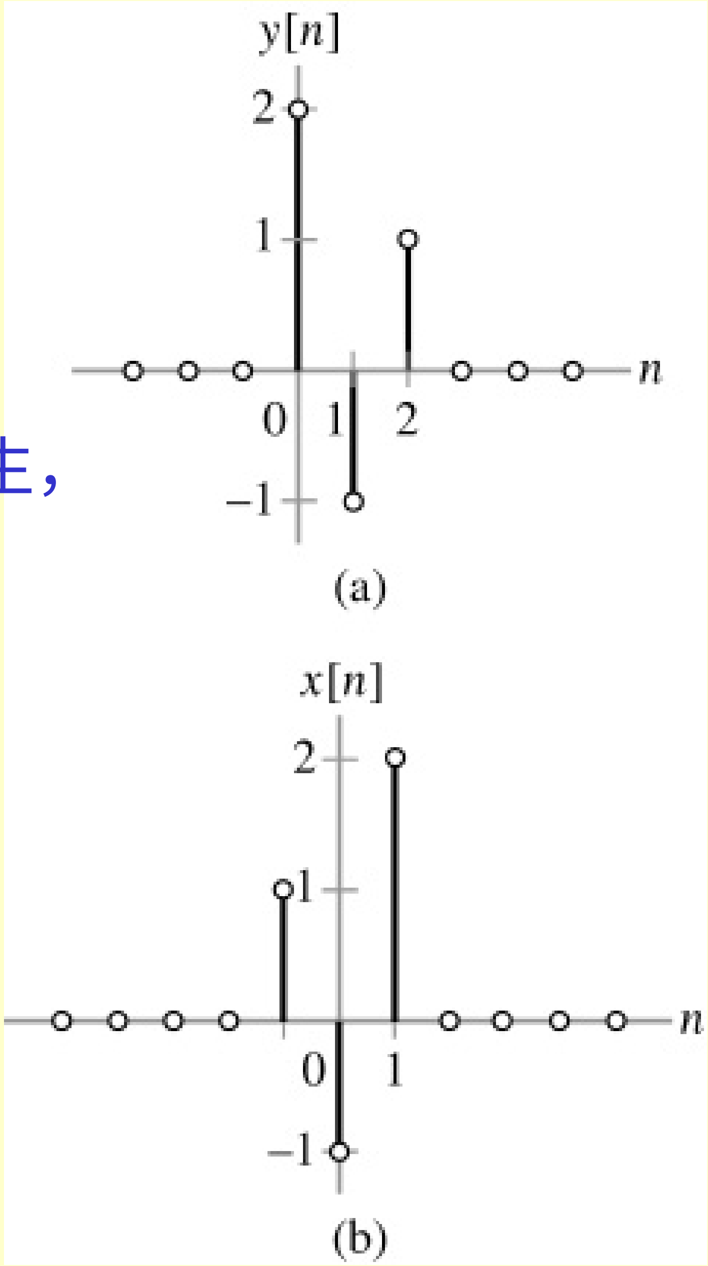
線性？

(非)時變性？

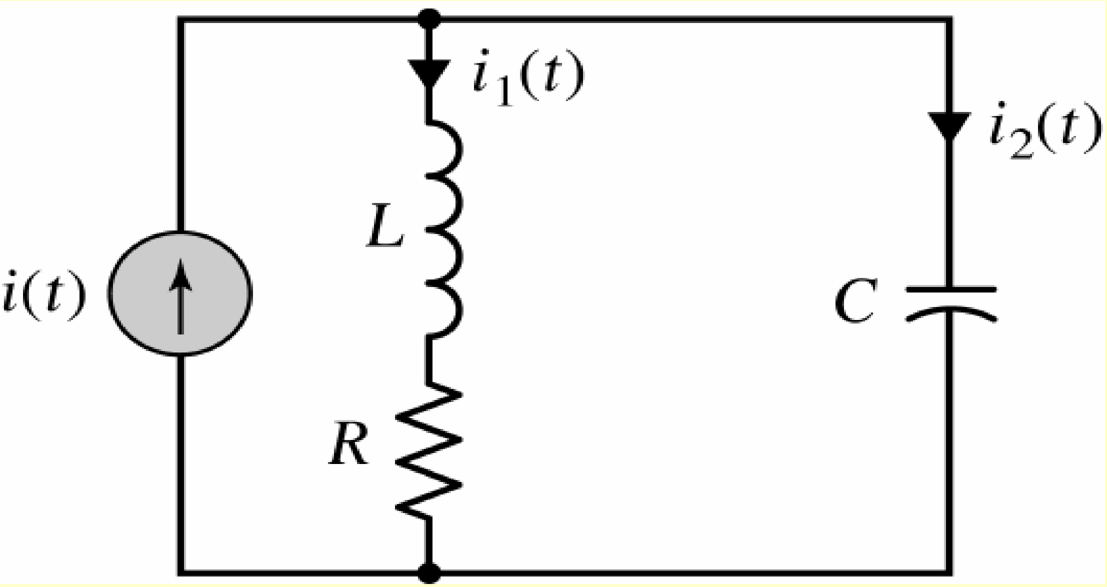


P1.77

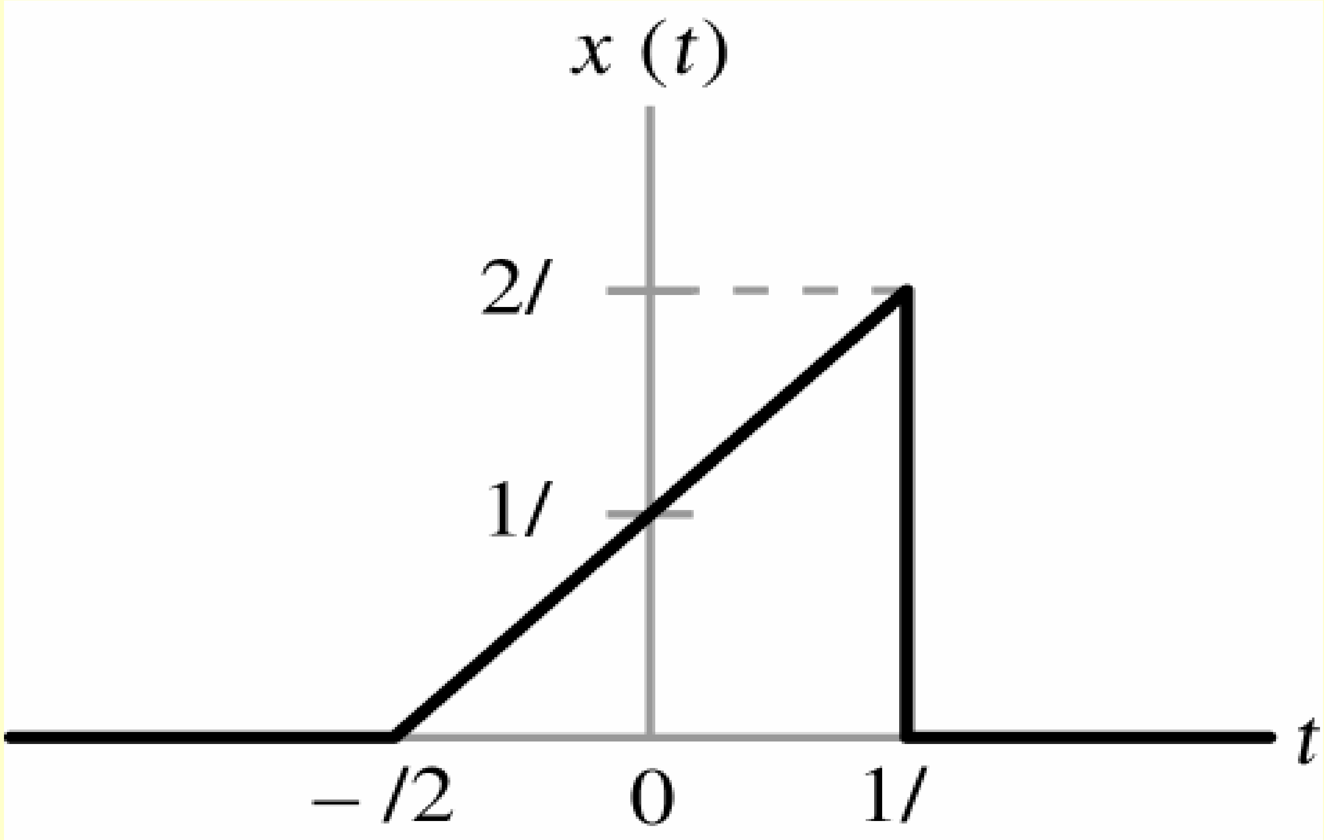
若 (a) 輸出 y[n] 是 x[n] = δ[n] 所產生，現以 (b) x[n] 輸入 則 y[n] 為何？



P1.79 試寫出微分方程式來描述下列電路？



P1.80

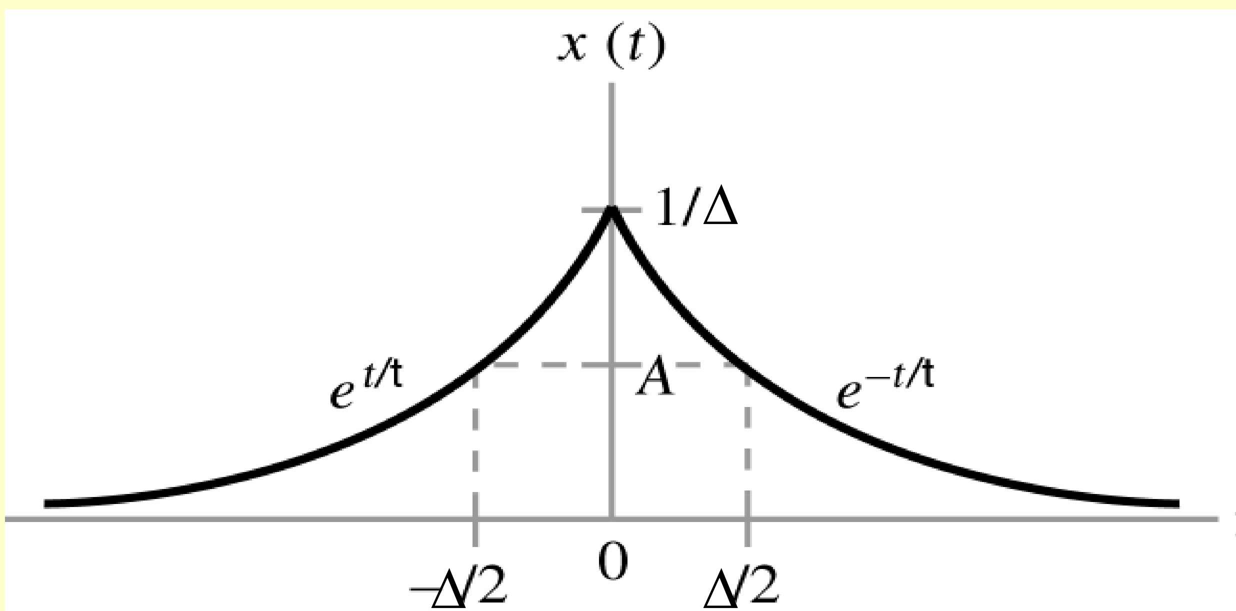


P1.81 若下圖 H 為線性非時變系統，請舉例說明？

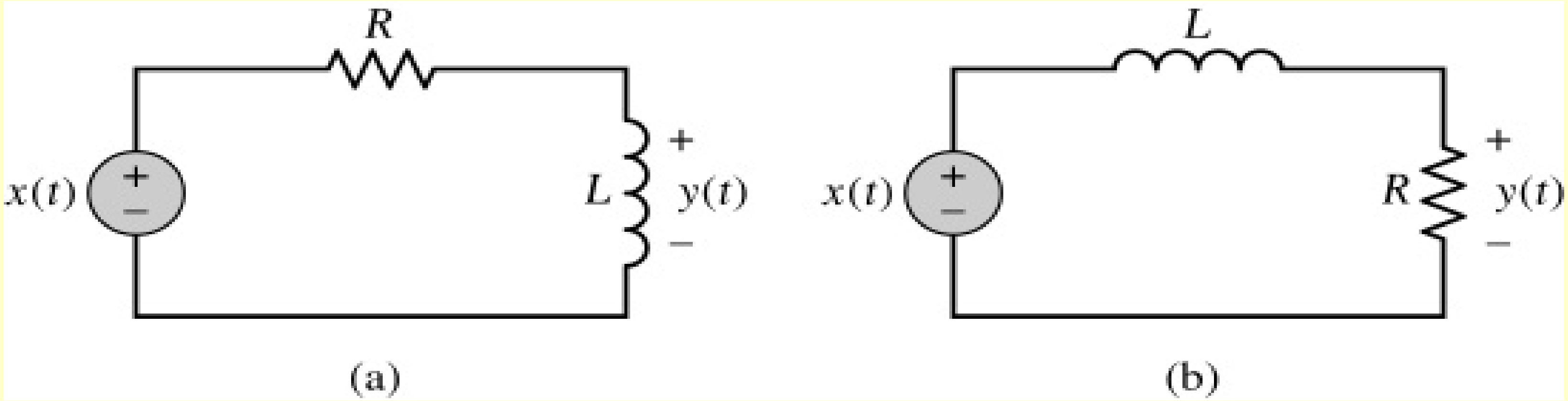
P1.82

試求 A = ?

$$x_\Delta(t) = \frac{1}{\Delta}\left(e^{+t/\tau}u(-t) + e^{-t/\tau}u(t)\right)$$



P1.83 電路 (a) 和 (b) 是否互為反運算 ？

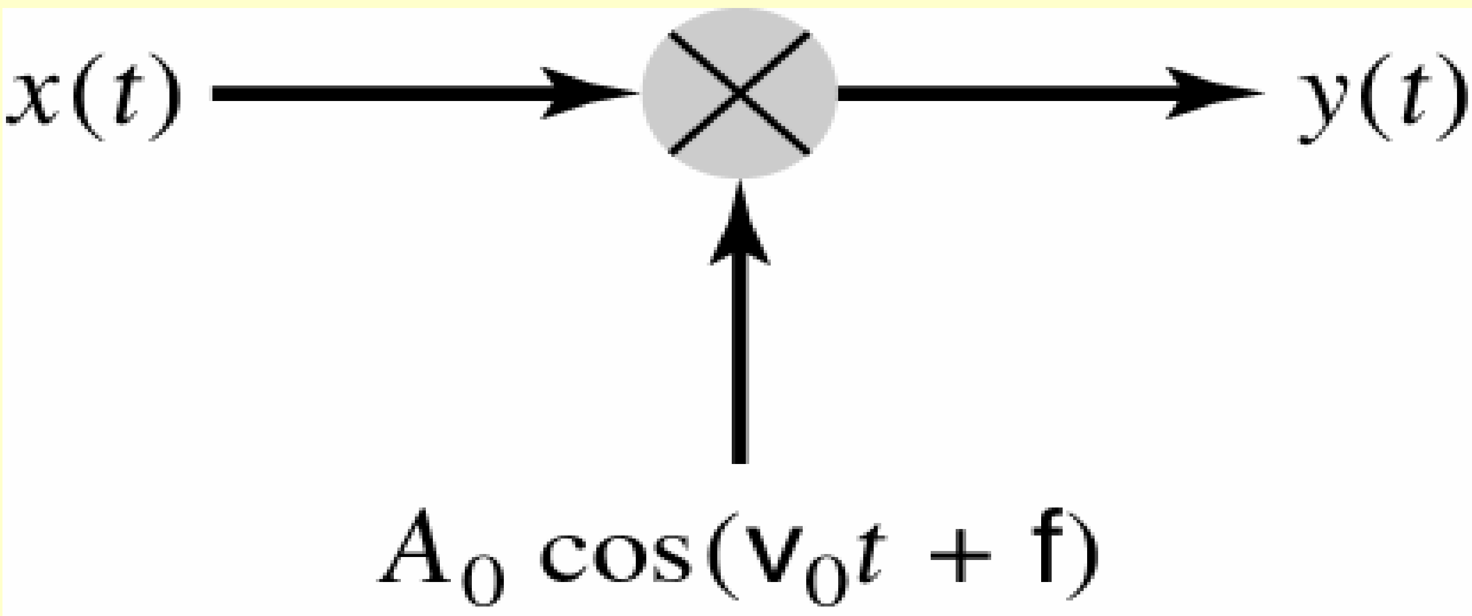


$$y(t) = A_0 \cos\left(\omega_0 t + \phi\right) x(t)$$

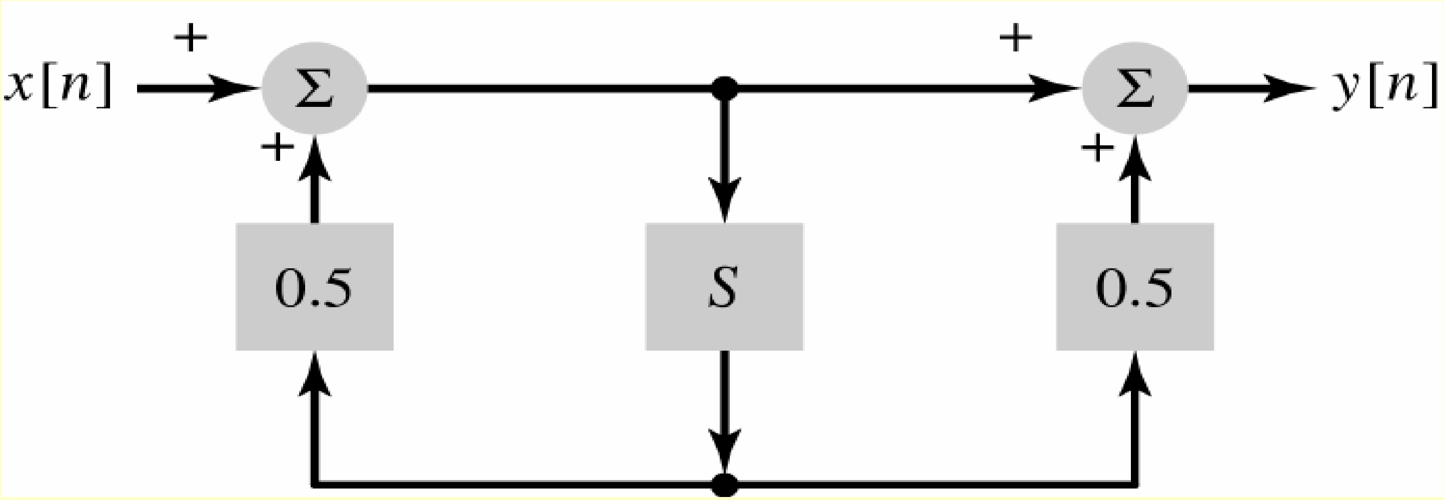
P1.84 系統輸出:

(a) 證明此系統為線性

(b) 證明此系統為時變性



P1.89 以輸入 x[n] 寫出輸出 y[n] 表示式？



P1.92 試繪出下圖中方塊圖？