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# Lecture 2-4

## Linear Time-Invariant System
## (LTI System)
## 線性非時變系統

# 解微分方程式基本複習

一階微分方程式： $$a_1 \frac{dy(t)}{dt} + a_0 \, y(t) = 0$$

解： $y(t) = ce^{rt}$

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系統特徵方程式： $a_1 r + a_0 = 0$

解系統特徵方程式： $$r = -\frac{a_0}{a_1}$$

解： $y(t) = ce^{-\frac{a_0}{a_1}t}$

二階微分方程式： $$\frac{d^2y(t)}{dt^2}+\frac{dy(t)}{dt}+y(t)=0$$

解： $$y(t)=c_1e^{r_1t}+c_2e^{r_2t}$$

---

系統特徵方程式： $$r^2+r+1=0$$

解系統特徵方程式： $$r_1,r_2=\frac{-1\pm j\sqrt{3}}{2}$$

解： $$y(t)=c_1e^{\left(\frac{-1+j\sqrt{3}}{2}\right)t}+c_2e^{\left(\frac{-1-j\sqrt{3}}{2}\right)t}$$

# Solving Differential Equation

略過本節

齊次方程式 (Homogeneous Equation) :

所有輸入相關項設為零 (無輸入激發情況下)

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y^{(h)}(t) = 0$$

齊次解 (Homogeneous Solution):

$$y^{(h)}(t) = \sum_{i=1}^{N} c_i \, e^{r_i t}$$

$r_i$： 系統特徵方程式(Characteristic Equation) 的根:

$$\sum_{k=0}^{N} a_k\, r^k = 0$$

只要上式成立，則前頁所示齊次解即成立。

# Solving Difference Equation

略過本節

齊次方程式 (Homogeneous Equation) :

所有輸入相關項設為零（無激發情況）

$$\sum_{k=0}^{N} a_k \; y^{(h)}[n-k] = 0$$

齊次解 (Homogeneous Solution):

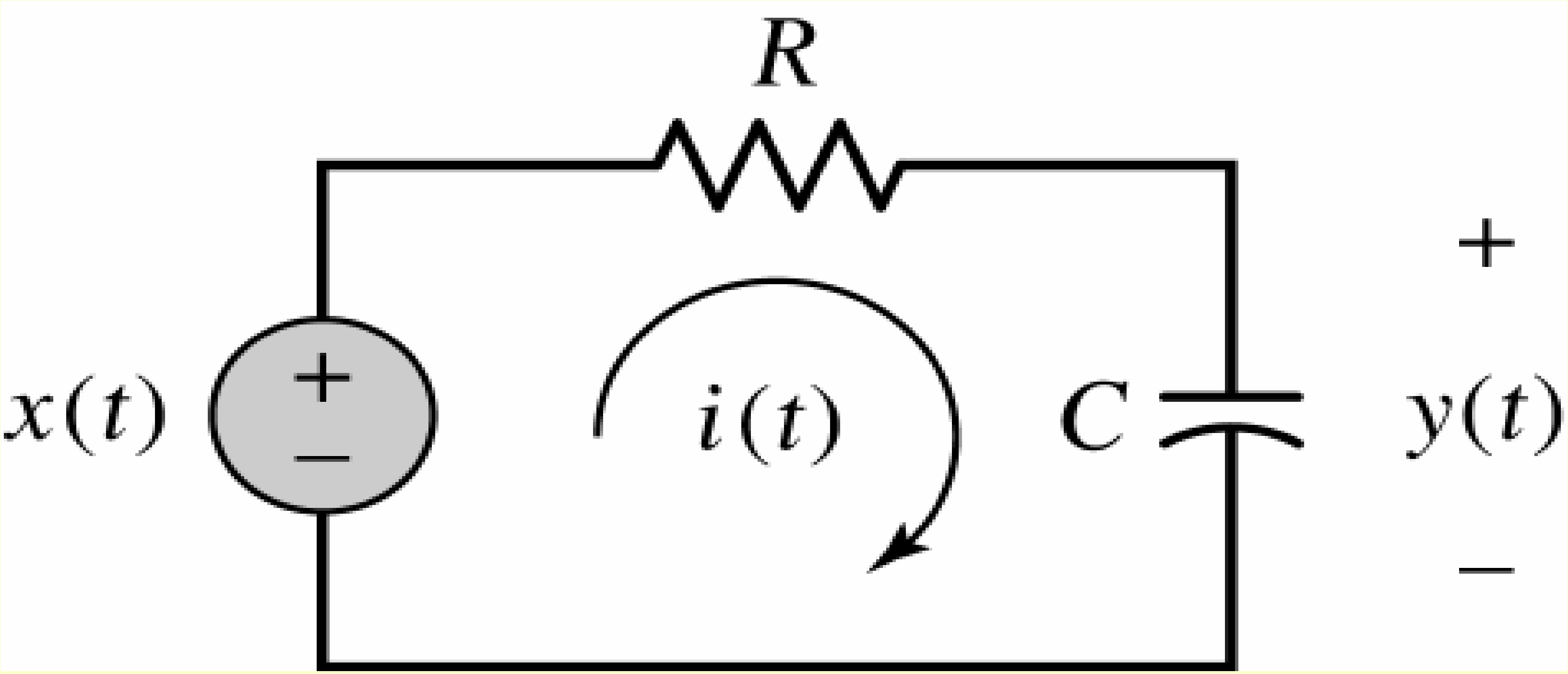
$$y^{(h)}[n] = \sum_{i=1}^{N} c_i \; r_i^k$$

$r_i$： 系統特徵方程式(Characteristic Equation) 的根:

$$\sum_{k=0}^{N} a_k \, r^{N-k} = 0$$

只要上式成立，則前頁所示齊次解即成立。

EX2.17 請用微分方程式描述下述RC circuit，其x(t)表輸入電壓、y(t)表輸出電壓，並求出此方程式的齊次解。



$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y^{(h)}(t) = 0$$

*Solution* :

$$\because\ y(t) + R\left(C\frac{d}{dt}y(t)\right) = x(t),$$

*Homogeneous Equation* :

$$y(t) + R\left(C\frac{d}{dt}y(t)\right) = 0, \quad \Rightarrow a_0 = 1, \quad a_1 = RC$$

*Characteristic Equation* :

$$\sum_{k=0}^{1} a_k r^k = a_0 + a_1 r = 1 + RCr = 0, \quad \therefore r = -\frac{1}{RC}$$

*Homogeneous Solution* :

$$y^{(h)}(t) = c_1 e^{rt} = c_1 e^{-\frac{t}{RC}}$$

# EX2.18 請用差分方程式描述一階遞迴系統，並求出其齊次解。

$$\sum_{k=0}^{N} a_k \, y^{(h)}[n-k] = 0$$

$$y[n] - \rho y[n-1] = x[n]$$

齊次方程式： $y[n] - \rho y[n-1] = 0, \quad a_0 = 1, a_1 = -\rho$

特徵方程式： $a_0 r + a_1 = r - \rho = 0,$

$r = \rho.$

一階齊次解： $y^{(h)}[n] = c\, r^n = c\rho^n$

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# 如何獲得特殊解? Particular Solution

- 特殊解 $y^{(p)}$ 代表微分或差分方程式對某已知輸入的任何一個解。
- 特殊解 $y^{(p)}$ 不是唯一的。

# EX2.19 請用差分方程式描述一階遞迴系統，並求出其特殊解。

$$y[n]-\rho y[n-1]=x[n], \quad if \quad x[n]=(1/2)^n$$

$Solution:$

假設 $y^{(p)}[n]=c_p\left(\frac{1}{2}\right)^n$，代入原差分方程式中:

$$c_p\left(\frac{1}{2}\right)^n-\rho\, c_p\left(\frac{1}{2}\right)^{n-1}=\left(\frac{1}{2}\right)^n, \quad multiplying \left(\frac{1}{2}\right)^{-n} to\ all\ terms,$$

$$c_p-\rho\, c_p\left(\frac{1}{2}\right)^{-1}=1, \quad \Rightarrow c_p(1-2\rho)=1, \quad \therefore c_p=\frac{1}{1-2\rho}$$

$$\therefore y^{(p)}[n]=\frac{1}{1-2\rho}\left(\frac{1}{2}\right)^n,$$

$$if\ \ \rho = 1/2\ case, \quad y^{(p)}[n] = c_p\left(\frac{1}{2}\right)^n = \frac{1}{1-2\rho}\left(\frac{1}{2}\right)^n,$$

*we cannot find a* $c_p$ *to satisfy the above condition.*

必須要假設另一種形態：

假設 $\quad y^{(p)}[n] = c_p n\left(\frac{1}{2}\right)^n,\quad$ 代入原差分方程式中

$Solution:$

假設 $y^{(p)}[n]=c_p n\left(\frac{1}{2}\right)^n$， 代入原差分方程式中:

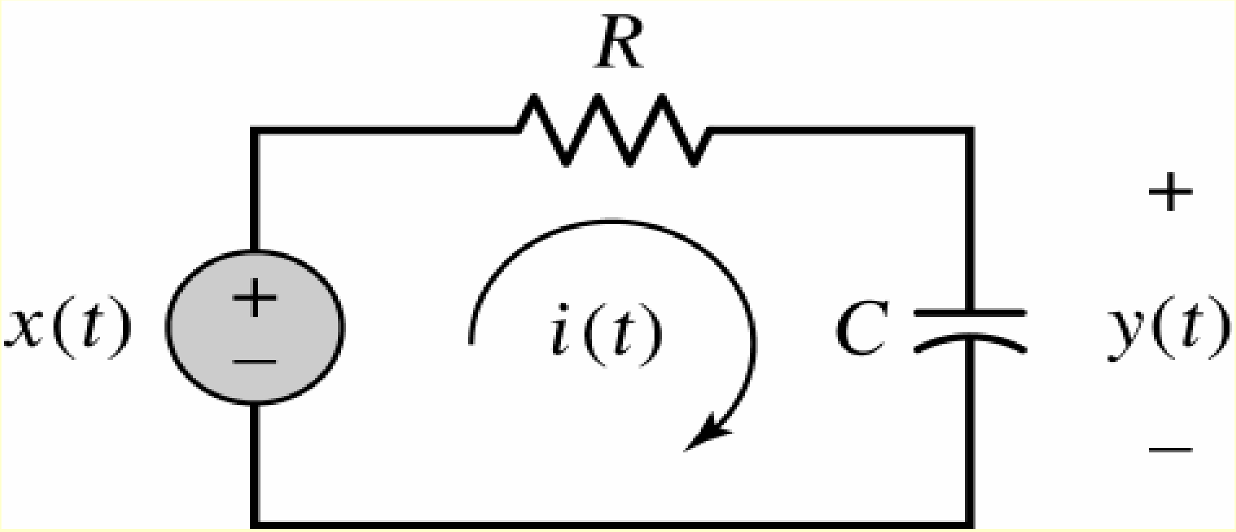
$$c_p n\left(\frac{1}{2}\right)^n-\rho\, c_p(n-1)\left(\frac{1}{2}\right)^{n-1}=\left(\frac{1}{2}\right)^n,$$

$$\therefore c_p n-2\rho\,(n-1)c_p=1, \quad \Rightarrow c_p(n-2n\rho+2\rho)=1,$$

$$\Rightarrow c_p[n(1-2\rho)+2\rho]=1, \quad \therefore c_p=\frac{1}{n(1-2\rho)+2\rho}$$

$$\therefore y^{(p)}[n]=\frac{1}{(1-2\rho)+2\rho}\left(\frac{1}{2}\right)^n$$

# EX2.20 請求出下列RC 電路的特殊解，已知輸入為 $x(t)=\cos(\omega_0 t)$ 伏特。



$$\because\ y(t) + R\,\mathrm{C}\frac{d}{dt}y(t) = x(t), \quad where \quad \underline{x(t) = \cos(\omega_0 t)}$$

$$Particular\ Solution:$$

$$y^{(p)}(t) = c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t),$$ 代入上式可得:

$$c_1\cos(\omega_0 t) + c_2\sin(\omega_0 t) + RC\left[-c_1\omega_0\sin(\omega_0 t) + c_2\omega_0\cos(\omega_0 t)\right]$$

$$= \cos(\omega_0 t)$$

$$\Rightarrow \cos(\omega_0 t)\left[c_1 + RCc_2\omega_0\right] + \sin(\omega_0 t)\left[c_2 - RCc_1\omega_0\right] = \cos(\omega_0 t),$$

$$\therefore \quad \begin{cases} c_1 + RCc_2\omega_0 = 1 \\ c_2 - RCc_1\omega_0 = 0 \end{cases}$$

$$\because \quad \begin{cases} c_1 + RCc_2\omega_0 = 1 \\ c_2 - RCc_1\omega_0 = 0 \end{cases}$$

$$\therefore \quad c_1 = \frac{1}{1+(RC\omega_0)^2}, \quad c_2 = \frac{RC\omega_0}{1+(RC\omega_0)^2}$$

$$\therefore \quad Particular\ Solution:$$

$$y^{(p)}(t) = \frac{1}{1+(RC\omega_0)^2}\cos(\omega_0 t) + \frac{RC\omega_0}{1+(RC\omega_0)^2}\sin(\omega_0 t)$$ 伏特

# 如何獲得完整解? Complete Solution

Procedure:

1. 從特徵方程式的根找出齊次解的形式 $y^{(h)}$。
2. 假設形式和輸入一樣，找出特殊解 $y^{(s)}$，而且和齊次解所有項目不同。
3. 決定齊次解的係數，使其完整解 $y=y^{(p)}+y^{(h)}$ 可滿足初始條件。

# EX2.21 請用差分方程式描述一階遞迴系統，並求出其完整解。

$$y[n]-\frac{1}{4}y[n-1]=x[n], \quad if \;\; x[n]=(1/2)^n u[n], \;\; and \;\; y[-1]=8$$

齊次方程式： $y[n]-\dfrac{1}{4}y[n-1]=0, \quad a_0=1, a_1=-\dfrac{1}{4}$

特徵方程式： $a_0 r + a_1 = r-\dfrac{1}{4}=0, \quad \therefore r=\dfrac{1}{4}.$

一階齊次解： $y^{(h)}[n]=c\,r^n=c\left(\dfrac{1}{4}\right)^n$

特殊解:

假設 $y^{(p)}[n]=c_p\left(\frac{1}{2}\right)^n$, 代入原差分方程式中:

$$c_p\left(\frac{1}{2}\right)^n-\frac{1}{4}c_p\left(\frac{1}{2}\right)^{n-1}=\left(\frac{1}{2}\right)^n,\quad \textit{multiplying}\left(\frac{1}{2}\right)^{-n}\textit{to all terms,}$$

$$c_p-\frac{1}{4}c_p\left(\frac{1}{2}\right)^{-1}=1,\quad \Rightarrow c_p(1-\frac{1}{2})=1,\quad \therefore c_p=\frac{1}{1/2}=2$$

$$\therefore y^{(p)}[n]=c_p\left(\frac{1}{2}\right)^n=2\left(\frac{1}{2}\right)^n,$$

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# 完整解(Complete Solution):

$$y[n] = y^{(h)}[n] + y^{(p)}[n] = c\left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n$$

加入初始值:

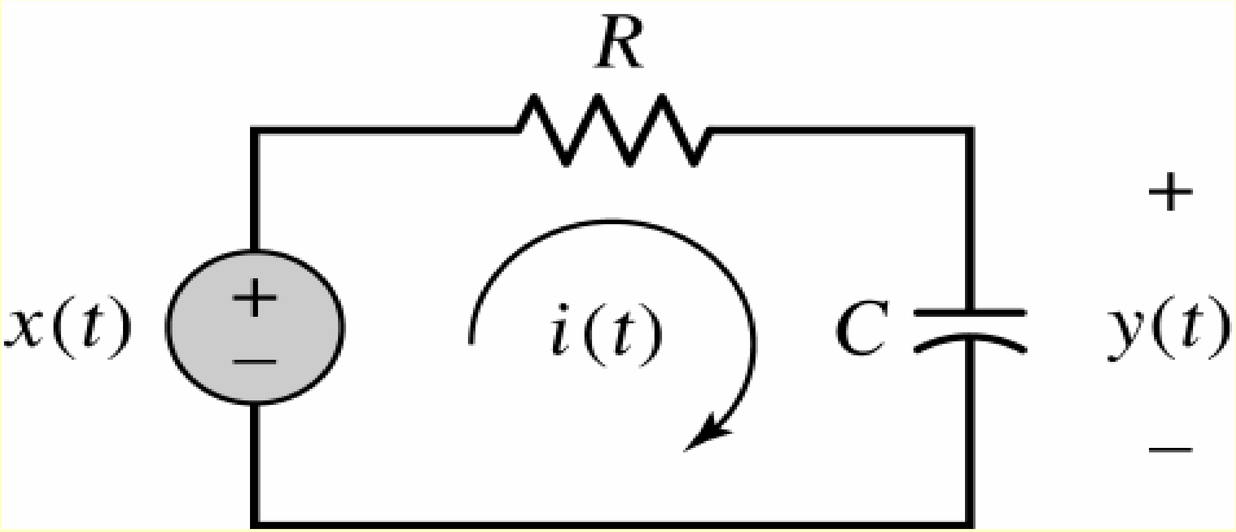
$$y[0] = x[0] + \frac{1}{4}y[-1] = (1/2)^0 + \frac{1}{4}(8) = 1 + 2 = 3,$$

$$= c\left(\frac{1}{4}\right)^0 + 2\left(\frac{1}{2}\right)^0 = c + 2$$

$$\therefore c = 1,$$

$$\therefore y[n] = \left(\frac{1}{4}\right)^n + 2\left(\frac{1}{2}\right)^n, \quad \forall n \geq 0$$

EX2.22 請求出下列RC 電路的完整解，已知輸入為 $x(t)=\cos(t)u(t)$ 伏特，假設 R=1 Ω、C=1F，且電容初始電壓為 $y(0^-)=2$ 伏特。



## *Homogeneous Solution:*

$$y^{(h)}(t) = c\, e^{rt} = c\, e^{-\frac{1}{RC}}$$

## *Particular Solution:*

$$y^{(p)}(t) = \frac{1}{1+(RC)^2}\cos(t) + \frac{RC}{1+(RC)^2}\sin(t)$$

## *Complete Solution:*

R = 1, C = 1, $\omega_0$=1

$$y(t) = c\, e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t)$$

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$$\because \text{ initial voltage}: y(0^-) = y(0^+) = 2,$$

$$2 = c^{-(0^+)} + \frac{1}{2}\cos(0^+) + \frac{1}{2}\sin(0^+) = c + \frac{1}{2},$$

$$\therefore \quad c = \frac{3}{2}$$

**Complete Solution:**

$$y(t) = \frac{3}{2}e^{-t} + \frac{1}{2}\cos(t) + \frac{1}{2}\sin(t)$$

# 如何用 Differential and Difference Equations 描述系統特性?

自然響應 (Natural Response):

- 零輸入時的系統輸出，描述任何儲存得能量並滿足初始條件。
- 類似 齊次解 ： $y^{(n)}$

強迫響應 (Forced Response):

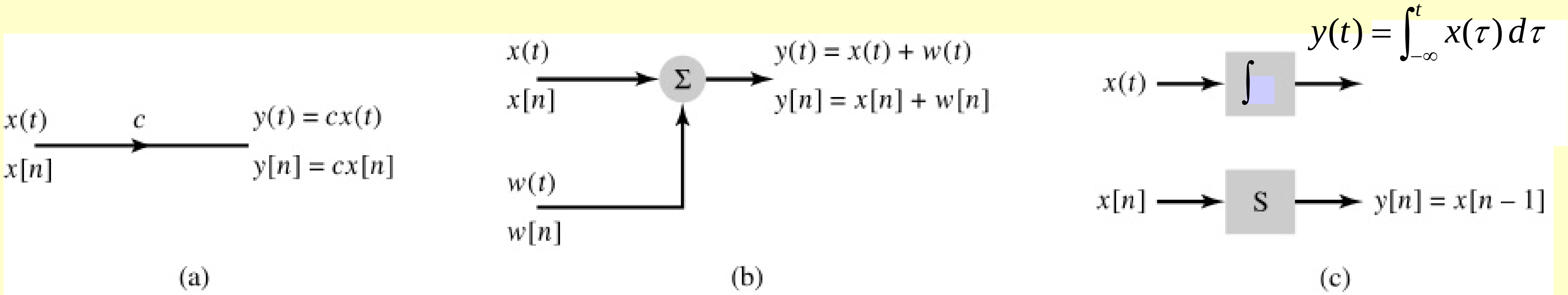
- 假設初始為零時，由輸入訊號造成的系統輸出。
- 類似 特殊解 ： $y^{(f)}$

完整輸出 : $y = y^{(n)} + y^{(f)}$

# 如何用方塊圖(Block Diagram）描述系統

本節新開始

- 方塊圖表示輸入訊號基本運算的互連狀態
- 方塊圖描述系統內部計算或操作如何排列
- 方塊圖表示運算
  - 純量乘法
  - 加法
  - 積分
  - 時間平移
  - …




Symbols for elementary operations in block diagram descriptions of systems.
(a) Scalar multiplication.
(b) Addition.
(c) Integration for continuous-time systems and time shifting for discrete-time systems.

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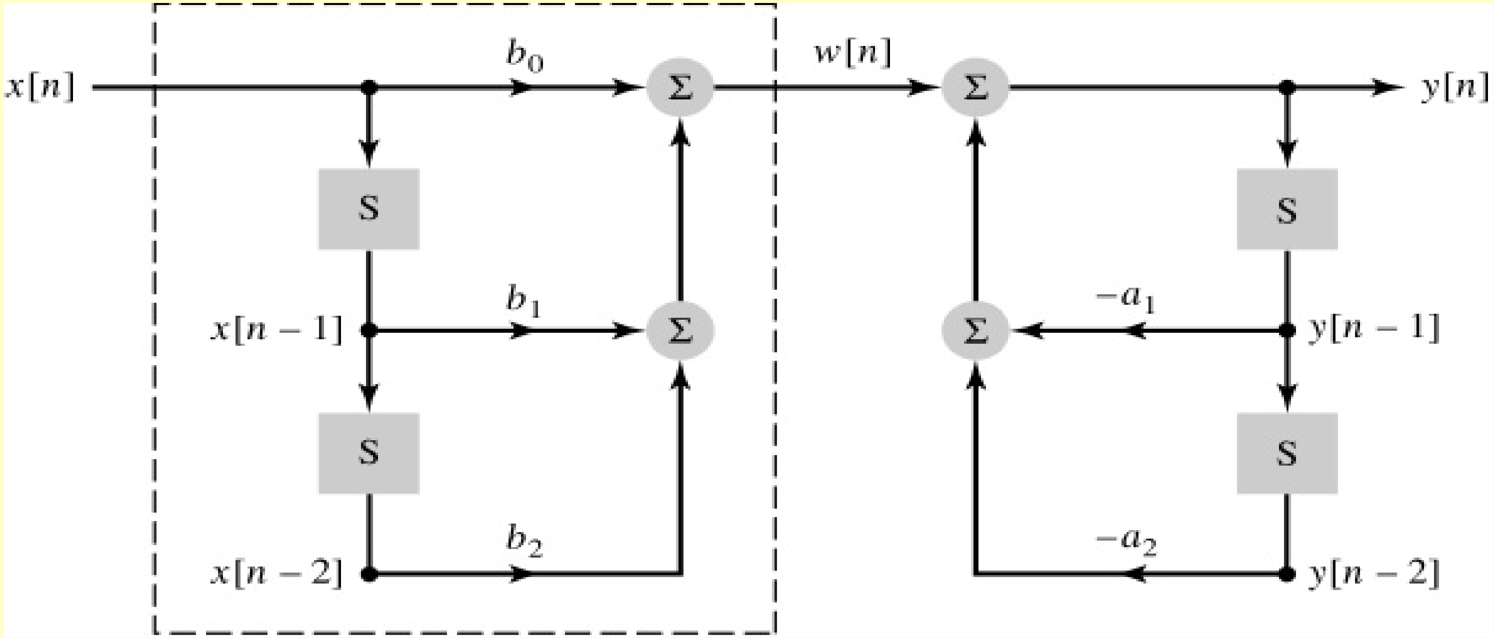
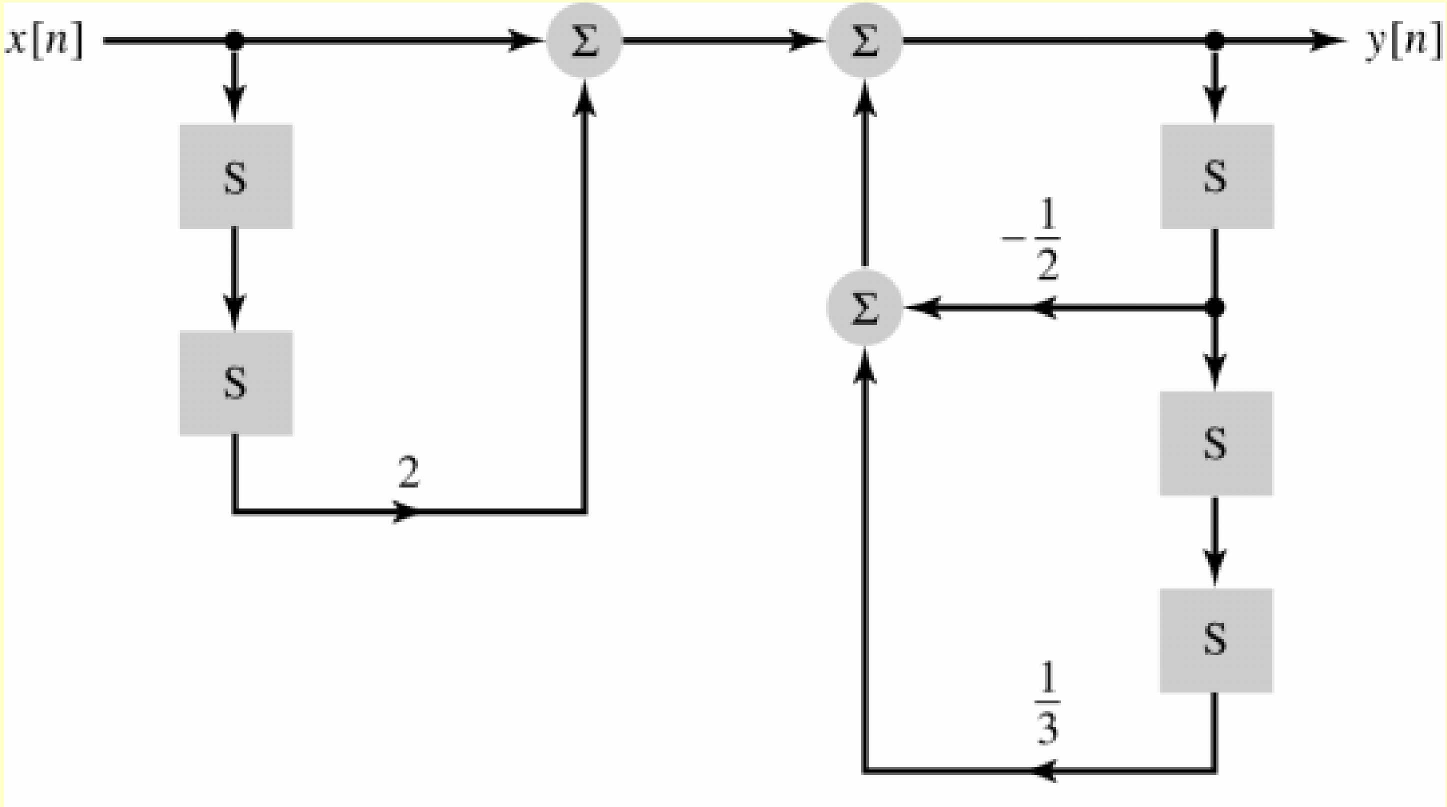



Block diagram of A second-order difference equation. (Direct form I : needs 4 memory locations:
x[n-1], x[n-2], y[n-1], y[n-2] )

# Block diagram representation for Problem 2.33 (2.34b in next slide)




(a)

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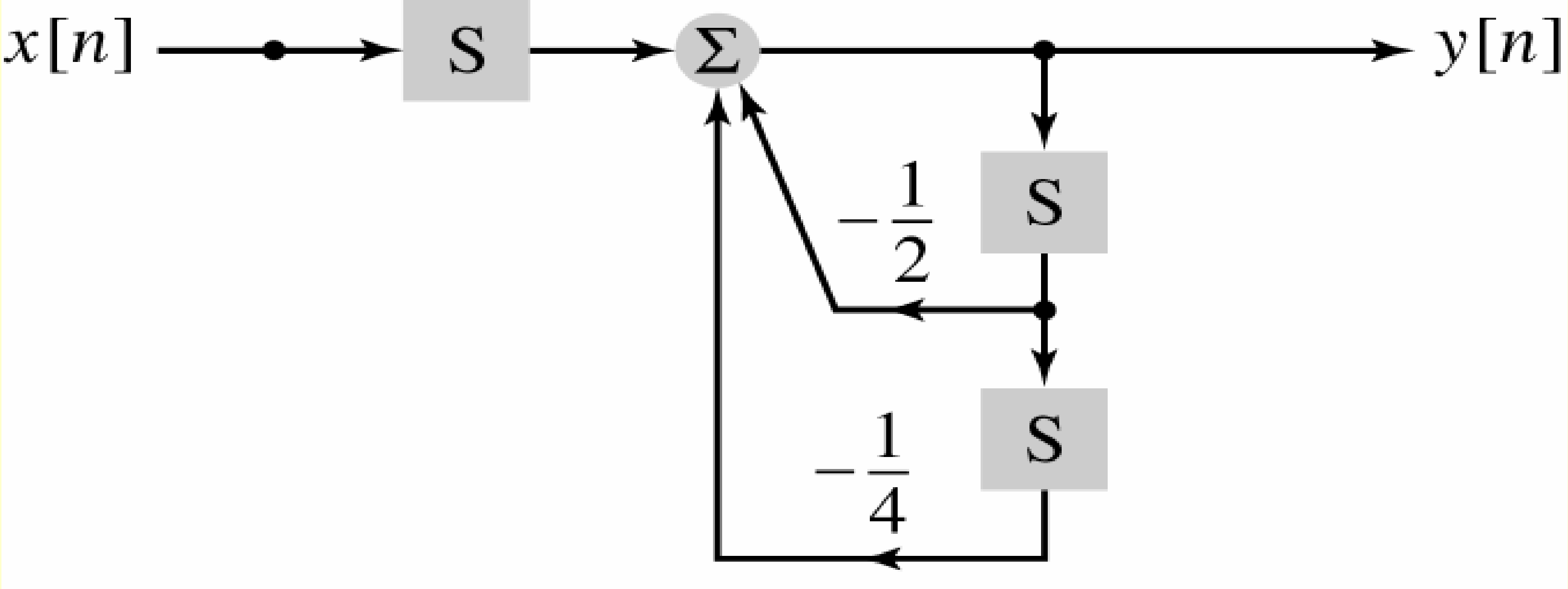
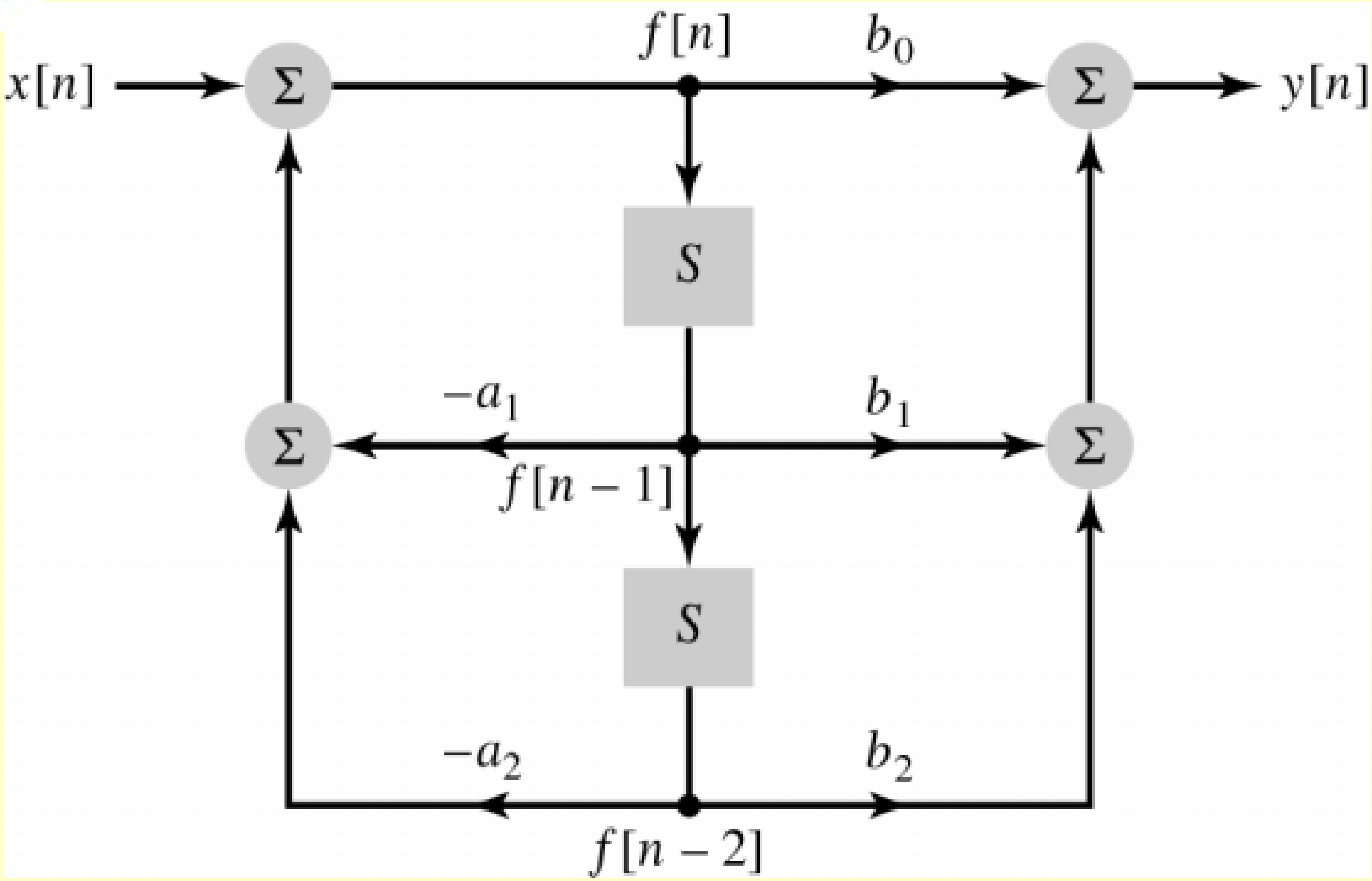



(b)




Block diagram of A second-order difference equation.
(Direct form II : needs 2 memory locations:
*f*[n-1], *f*[n-2] )

*Direct form I :*

$$a_0 y[n] + a_1 y[n-1] + a_2 y[n-2] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2]$$

$$\therefore \sum_{k=0}^{2} a_k y[n-k] = \sum_{p=0}^{2} b_p x[n-p]$$

$$define \sum_{k=0}^{2} a_k f[n-k] = x[n], \quad \therefore x[n-p] = \sum_{k=0}^{2} a_k f[n-k-p]$$

$$\therefore \sum_{k=0}^{2} a_k y[n-k] = \sum_{p=0}^{2} b_p \sum_{k=0}^{2} a_k f[n-k-p] = \sum_{k=0}^{2} a_k \sum_{p=0}^{2} b_p f[n-k-p]$$

$$\therefore y[(n-k)] = \sum_{p=0}^{2} b_p f[(n-k)-p]$$

$$\therefore y[n] = \sum_{p=0}^{2} b_p f[n-p]$$

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$$Direct\ \ form\ II:$$

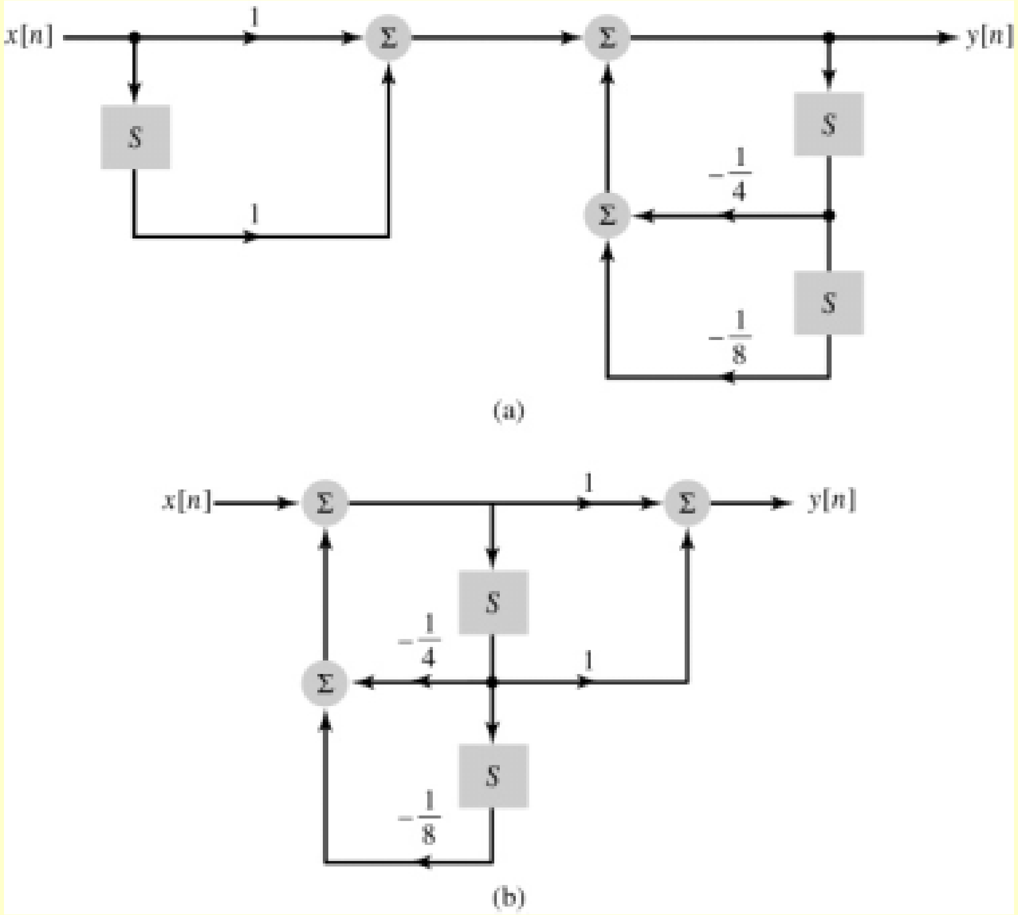
$$a_0 = 1$$

$$\sum_{k=0}^{2} a_k f[n-k] = x[n]$$

$$f[n] = x[n] - a_1 f[n-1] - a_2 f[n-2]$$

$$y[n] = \sum_{p=0}^{2} b_p f[n-p] = b_0 f[n] + b_1 f[n-1] + b_2 f[n-2]$$

# Solution to Problem 2.24. (a) Direct form 1, (b) Direct form 2.



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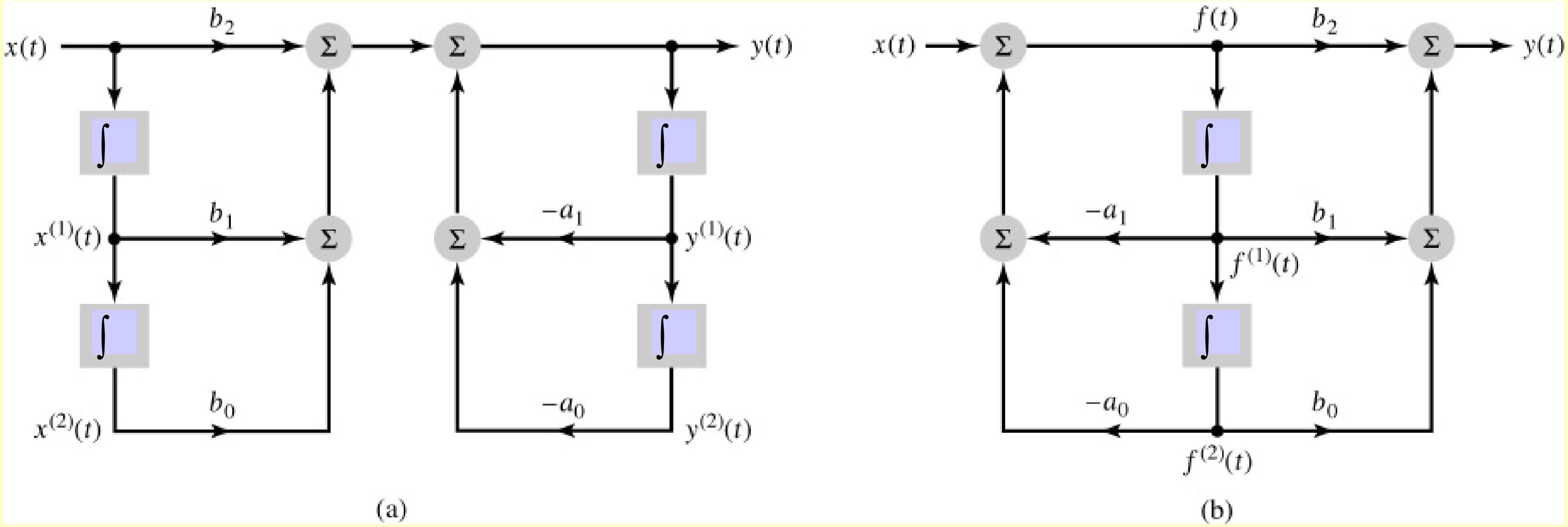
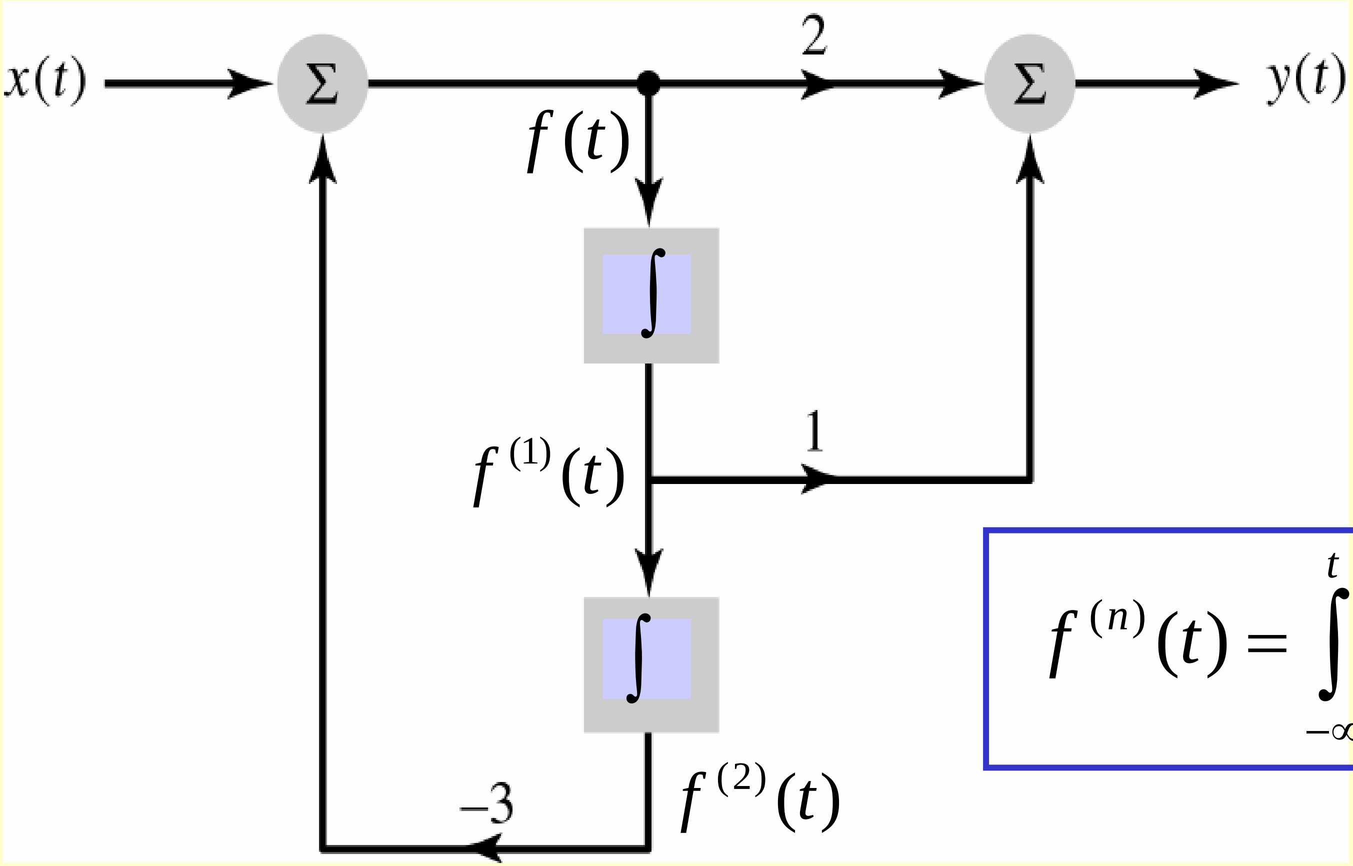
# Continuous-time LTI system described by a second-order integral equation. (a) Direct form I. (b) Direct form II.



# Block diagram representation for Problem 2.25.




$$f^{(n)}(t) = \int_{-\infty}^{t} f^{(n-1)}(\tau)\, d\tau$$

$$y(t) = 2f(t) + f^{(1)}(t)$$

$$f(t) = x(t) - 3f^{(2)}(t)$$

$\therefore$

$$y(t) = 2\left(x(t) - 3f^{(2)}(t)\right) + f^{(1)}(t) = 2x(t) - 6f^{(2)}(t) + f^{(1)}(t)$$

$$\frac{dy(t)}{dt} = 2\frac{dx(t)}{dt} - 6f^{(1)}(t) + f(t) = 2\frac{dx(t)}{dt} - 6f^{(1)}(t) + x(t) - 3f^{(2)}(t)$$

$$\frac{d^2y(t)}{dt^2} = 2\frac{d^2x(t)}{dt^2} - 6f(t) + \frac{dx(t)}{dt} - 3f^{(1)}(t)$$

$$= 2\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 3\left(2f(t) + f^{(1)}(t)\right) = 2\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 3y(t)$$

$\therefore$

$$\boxed{\frac{d^2y(t)}{dt^2} + 3y(t) = 2\frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt}}$$

# State-Variable Description of LTI System 以狀態變數描述 LTI 系統

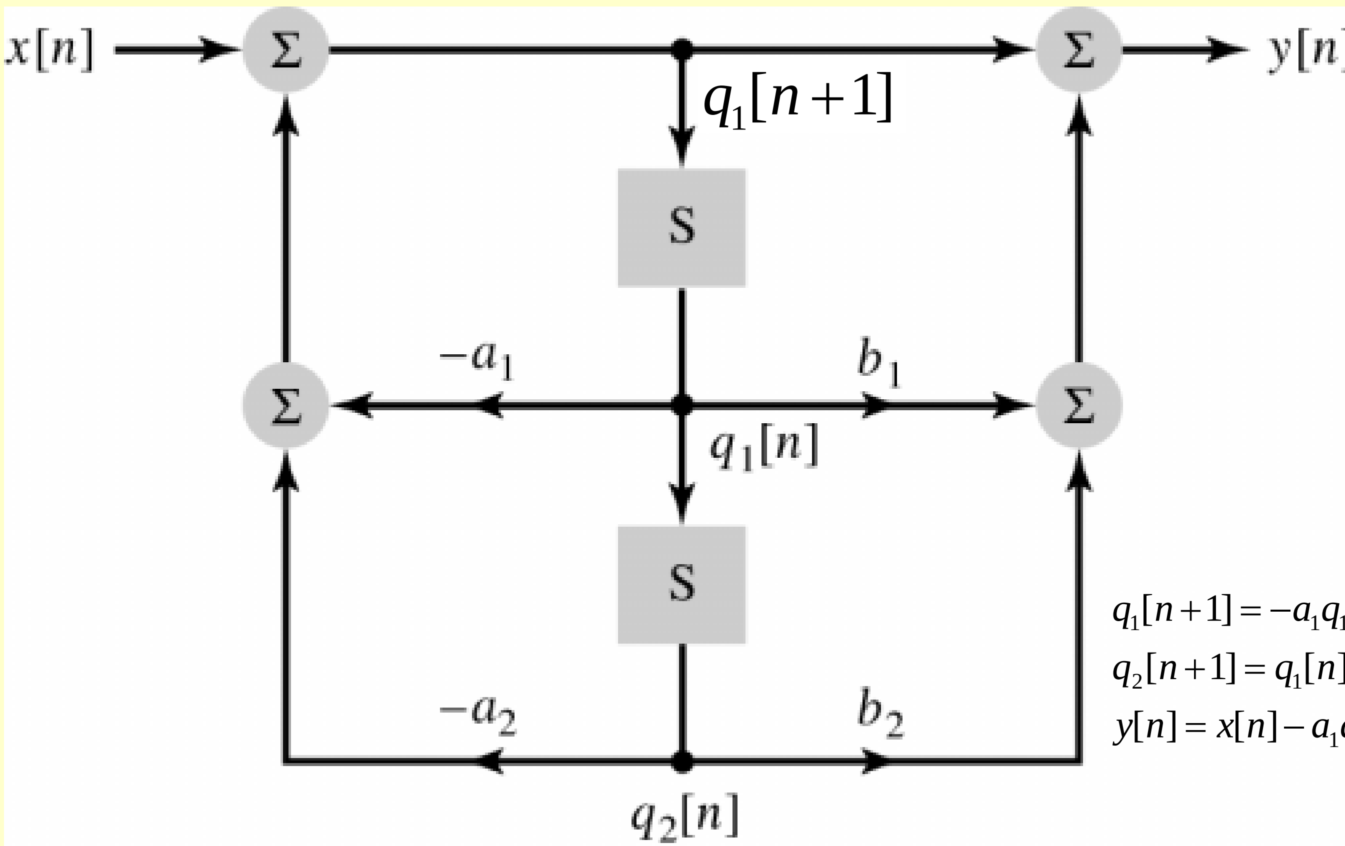
- 描述系統輸出與目前系統狀態和輸入之間的關係。
- 狀態變數(state variables) 描述以矩陣的形式表示。
- 多種輸入選擇暗示系統狀態不是唯一。
  - 狀態可能會改變 $q_1[n] \rightarrow q_2[n]$
- 範例：

$$q_1[n+1] = -a_1 q_1[n] - a_2 q_2[n] + x[n]$$

$$q_2[n+1] = q_1[n]$$

$$y[n] = x[n] - a_1 q_1[n] - a_2 q_2[n] + b_1 q_1[n] + b_2 q_2[n]$$

# Direct form II representation of a second-order discrete-time LTI system depicting state variables $q_1[n]$ and $q_2[n]$.




$$q_1[n+1] = -a_1 q_1[n] - a_2 q_2[n] + x[n]$$
$$q_2[n+1] = q_1[n]$$
$$y[n] = x[n] - a_1 q_1[n] - a_2 q_2[n] + b_1 q_1[n] + b_2 q_2[n]$$

- $q_1[n+1] = -a_1 q_1[n] - a_2 q_2[n] + x[n]$
  $q_2[n+1] = q_1[n]$
- $y[n] = x[n] - a_1 q_1[n] - a_2 q_2[n] + b_1 q_1[n] + b_2 q_2[n]$

$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} b_1 - a_1 & b_2 - a_2 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + [1] x[n]$$

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$$\begin{bmatrix} q_1[n+1] \\ q_2[n+1] \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} x[n]$$

$$y[n] = \begin{bmatrix} b_1 - a_1 & b_2 - a_2 \end{bmatrix} \begin{bmatrix} q_1[n] \\ q_2[n] \end{bmatrix} + [1] x[n]$$

$$\Rightarrow \quad \underline{q}[n+1] = A\underline{q}[n] + \underline{b}\, x[n]$$

$$y[n] = C\underline{q}[n] + Dx[n]$$

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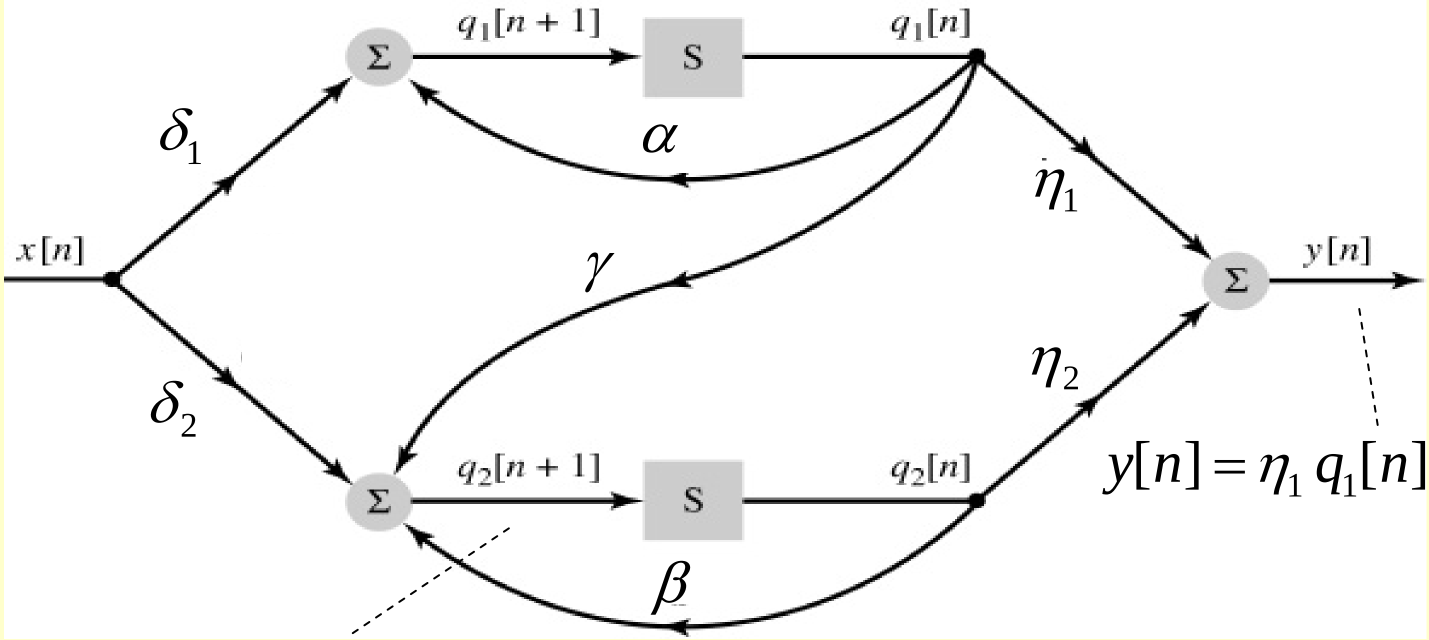
# Block diagram of LTI system for Example 2.28.

請用狀態變數描述下圖： $q_1[n+1] = \alpha\, q_1[n] + \delta_1 x[n]$




$y[n] = \eta_1\, q_1[n] + \eta_2 q_2[n]$

$q_2[n+1] = \gamma\, q_1[n] + \beta\, q_2[n] + \delta_2 x[n]$

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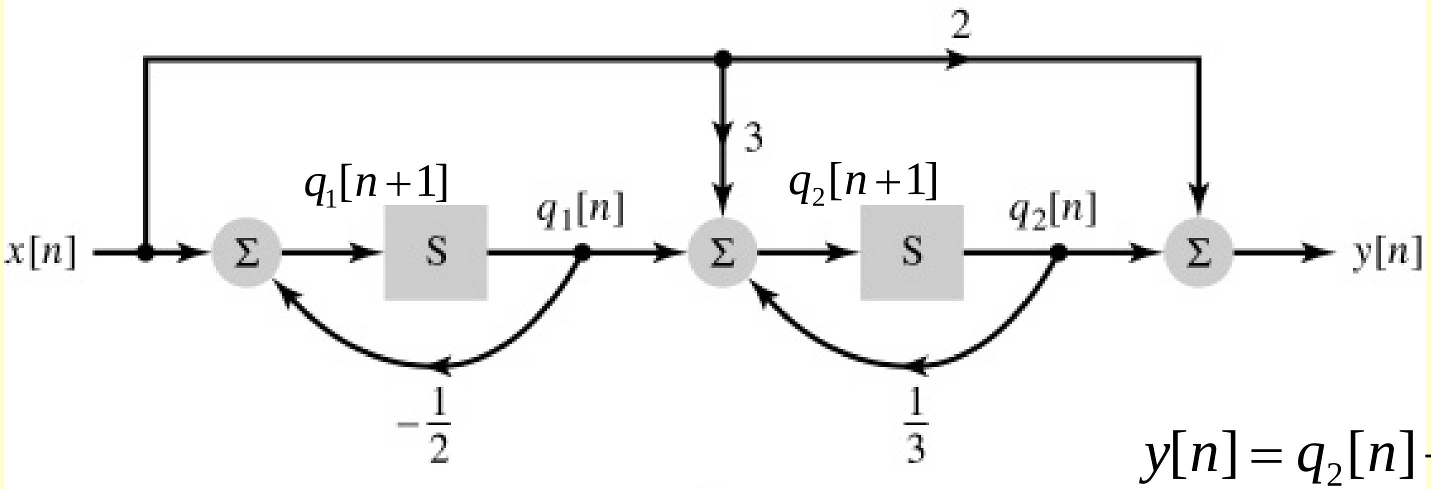
# Block diagram of LTI system for Problem 2.26 (2.41b on next slide).

請用狀態變數描述下圖：

$$q_1[n+1] = -\frac{1}{2}q_1[n] + x[n]$$

$$q_2[n+1] = q_1[n] + \frac{1}{3}q_2[n] + 3x[n]$$




$$y[n] = q_2[n] + 2x[n]$$

(a)

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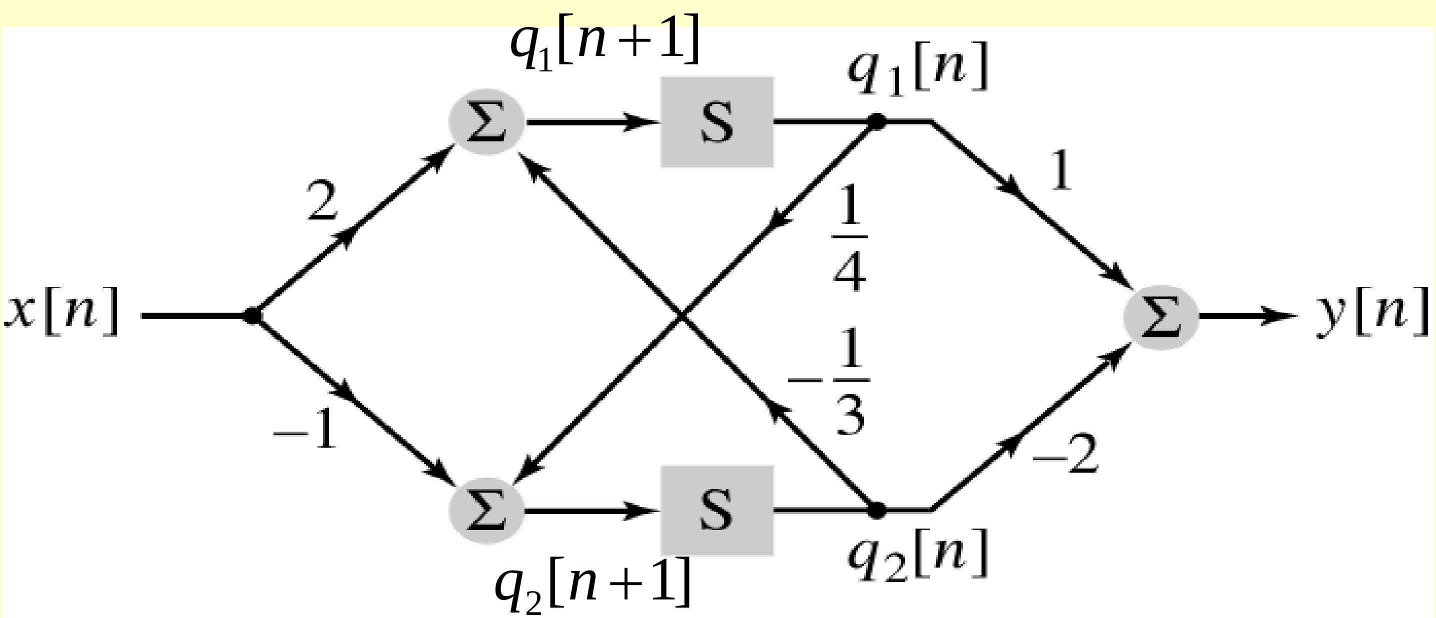
請用狀態變數描述下圖：

$$q_1[n+1] = a_{11}\, q_1[n] + a_{12}\, q_2[n] + b_1\, x[n]$$

$$q_2[n+1] = a_{21}\, q_1[n] + a_{22}\, q_2[n] + b_2\, x[n]$$

找出 這些參數值？

$$y[n] = c_1\, q_1[n] + c_2\, q_2[n] + d\, x[n]$$



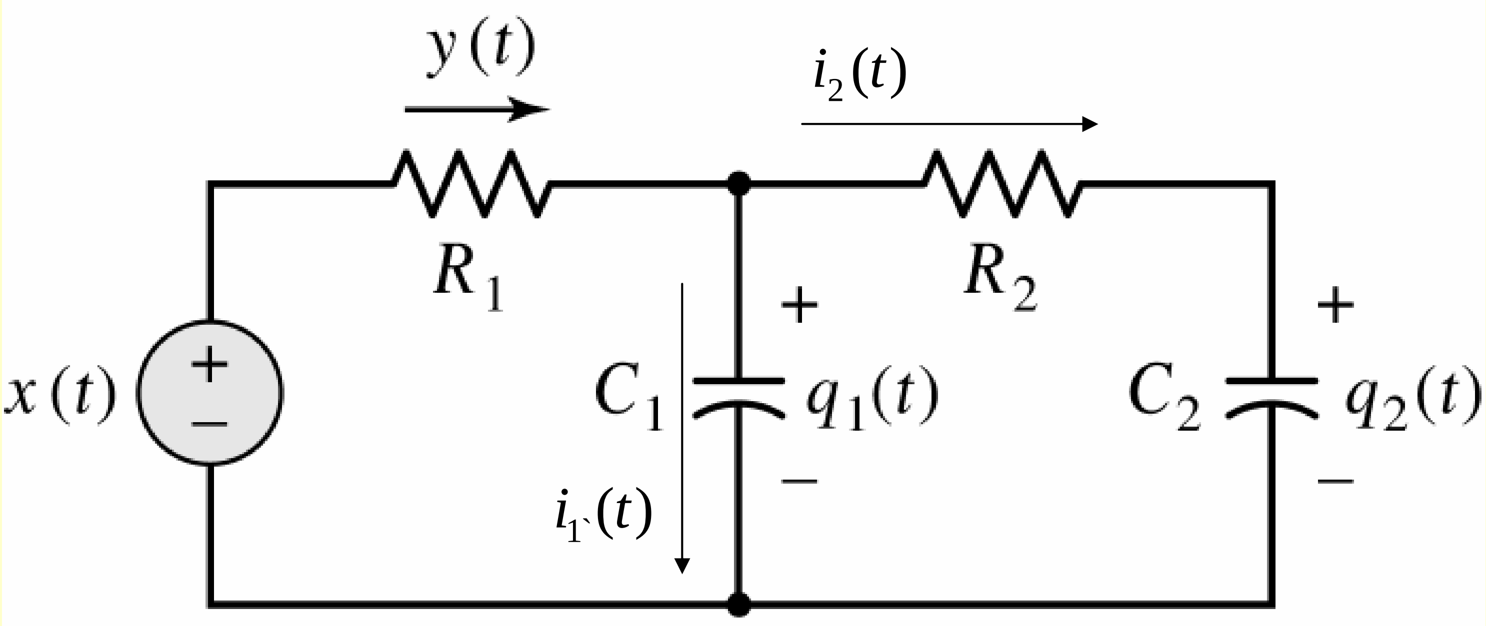
(b)

# Circuit diagram of LTI system for Example 2.29.

請用狀態變數描述下圖：

$$x(t) = y(t)R_1 + q_1(t)$$

$$q_1(t) = i_2(t)R_2 + q_2(t)$$



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# Solution:

$$x(t) = y(t)R_1 + q_1(t) \quad \rightarrow \quad y(t) = \frac{1}{R_1}x(t) - \frac{1}{R_1}q_1(t)$$

$$q_1(t) = i_2(t)R_2 + q_2(t) \quad \rightarrow \quad i_2(t) = \frac{1}{R_2}q_1(t) - \frac{1}{R_2}q_2(t)$$

$$\because q_1 = \frac{1}{C_1}\int_{-\infty}^{t} i_1(\tau)\,d\tau \quad \rightarrow i_1(t) = C_1\frac{dq_1(t)}{dt}$$

$$\because q_2 = \frac{1}{C_2}\int_{-\infty}^{t} i_2(\tau)\,d\tau \quad \rightarrow i_2(t) = C_2\frac{dq_2(t)}{dt}$$

$$\because \quad i_2(t) = \frac{1}{R_2} q_1(t) - \frac{1}{R_2} q_2(t) = C_2 \frac{dq_2(t)}{dt}$$

$$\therefore \quad \boxed{\frac{dq_2(t)}{dt} = \frac{1}{R_2 C_2} q_1(t) - \frac{1}{R_2 C_2} q_2(t)}$$

$$\because \quad i_1(t) = y(t) - i_2(t) = \left( \frac{1}{R_1} x(t) - \frac{1}{R_1} q_1(t) \right) - \left( \frac{1}{R_2} q_1(t) - \frac{1}{R_2} q_2(t) \right)$$

$$= C_1 \frac{dq_1(t)}{dt}$$

$$\therefore \quad \frac{dq_1(t)}{dt} = \left( \frac{1}{C_1 R_1} x(t) - \frac{1}{C_1 R_1} q_1(t) \right) - \left( \frac{1}{C_1 R_2} q_1(t) - \frac{1}{C_1 R_2} q_2(t) \right)$$

$$\boxed{= \frac{1}{C_1 R_1} x(t) - \left( \frac{1}{C_1 R_1} + \frac{1}{C_1 R_2} \right) q_1(t) + \frac{1}{C_1 R_2} q_2(t)}$$

$$\frac{dq_2(t)}{dt} = \frac{1}{R_2C_2} q_1(t) - \frac{1}{R_2C_2} q_2(t)$$

$$\frac{dq_1(t)}{dt} = \frac{1}{C_1R_1} x(t) - \left( \frac{1}{C_1R_1} + \frac{1}{C_1R_2} \right) q_1(t) + \frac{1}{C_1R_2} q_2(t)$$

$$\begin{bmatrix} \frac{dq_1}{dt} \\ \frac{dq_2}{dt} \end{bmatrix} = \begin{bmatrix} -\left( \frac{1}{R_1C_1} + \frac{1}{R_2C_1} \right) & \frac{1}{R_2C_1} \\ \frac{1}{R_2C_2} & -\frac{1}{R_2C_2} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1C_1} \\ 0 \end{bmatrix} x(t)$$

$$= A\,\underline{q} + bx$$

$$y(t) = \left(x(t) - q_1(t)\right) / R_1 = -\frac{1}{R_1} q_1(t) + \frac{1}{R_1} x(t)$$

$$= \begin{bmatrix} -\frac{1}{R_1} & 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1} \end{bmatrix} x(t)$$

$$= c\underline{q} + dx$$

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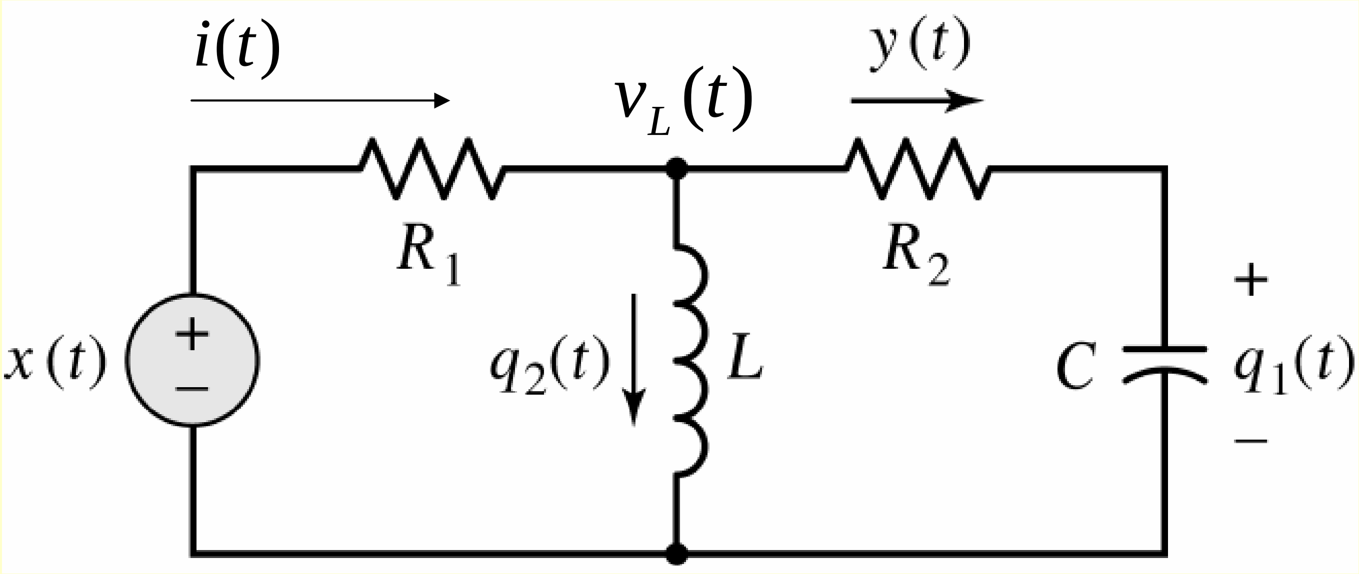
# Circuit diagram of LTI system for Problem 2.27.

請用狀態變數描述下圖：

$$\frac{dq_1(t)}{dt} = ?$$

$$\frac{dq_2(t)}{dt} = ?$$



$$\because x(t) = R_1\left(y(t) + q_2(t)\right) + y(t)R_2 + q_1(t)$$

$$= y(t)\left(R_1 + R_2\right) + q_1(t) + R_1 q_2(t)$$

$$\therefore y(t) = \frac{x(t)}{R_1 + R_2} - \frac{1}{R_1 + R_2} q_1(t) - \frac{R_1}{R_1 + R_2} q_2(t)$$

$$\because q_1(t) = \frac{1}{C}\int_{-\infty}^{t} y(\tau)\, d\tau,$$

$$\therefore \frac{dq_1(t)}{dt} = \frac{1}{C} y(t)$$

$$= \frac{x(t)}{C(R_1 + R_2)} - \frac{1}{C(R_1 + R_2)} q_1(t) - \frac{R_1}{C(R_1 + R_2)} q_2(t)$$

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$$\because\ x(t) = R_1\left(y(t) + q_2(t)\right) + L\frac{dq_2(t)}{dt}$$

$$\therefore\ \frac{dq_2(t)}{dt} = \frac{x(t)}{L} - \frac{R_1}{L}y(t) - \frac{R_1}{L}q_2(t)$$

$$= \frac{x(t)}{L} - \frac{R_1}{L}\left(\frac{x(t)}{R_1+R_2} - \frac{1}{R_1+R_2}q_1(t) - \frac{R_1}{R_1+R_2}q_2(t)\right) - \frac{R_1}{L}q_2(t)$$

$$= \left(\frac{1}{L} - \frac{R_1}{L(R_1+R_2)}\right)x(t) + \frac{R_1}{L(R_1+R_2)}q_1(t) + \left(\frac{R_1^2}{L(R_1+R_2)} - \frac{R_1}{L}\right)q_2(t)$$

$$= \frac{R_2}{L(R_1+R_2)}x(t) + \frac{R_1}{L(R_1+R_2)}q_1(t) - \frac{R_1R_2}{L(R_1+R_2)}q_2(t)$$

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$$y(t) = \frac{x(t)}{R_1 + R_2} - \frac{1}{R_1 + R_2} q_1(t) - \frac{R_1}{R_1 + R_2} q_2(t)$$

$$\frac{dq_1(t)}{dt} = \frac{x(t)}{C(R_1 + R_2)} - \frac{1}{C(R_1 + R_2)} q_1(t) - \frac{R_1}{C(R_1 + R_2)} q_2(t)$$

$$\frac{dq_2(t)}{dt} = \frac{R_2}{L(R_1 + R_2)} x(t) + \frac{R_1}{L(R_1 + R_2)} q_1(t) - \frac{R_1 R_2}{L(R_1 + R_2)} q_2(t)$$

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$$\begin{bmatrix} \dfrac{dq_1(t)}{dt} \\ \dfrac{dq_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{C(R_1+R_2)} & \dfrac{-R_1}{C(R_1+R_2)} \\ \dfrac{R_1}{L(R_1+R_2)} & -\dfrac{R_1R_2}{L(R_1+R_2)} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{C(R_1+R_2)} \\ \dfrac{R_2}{L(R_1+R_2)} \end{bmatrix} x(t)$$

$$= A \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + b\, x(t)$$

$$y(t) = \begin{bmatrix} -\dfrac{1}{R_1+R_2} & -\dfrac{R_1}{R_1+R_2} \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + \begin{bmatrix} \dfrac{1}{R_1+R_2} \end{bmatrix} x(t)$$

$$C \begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix} + D\, x(t)$$

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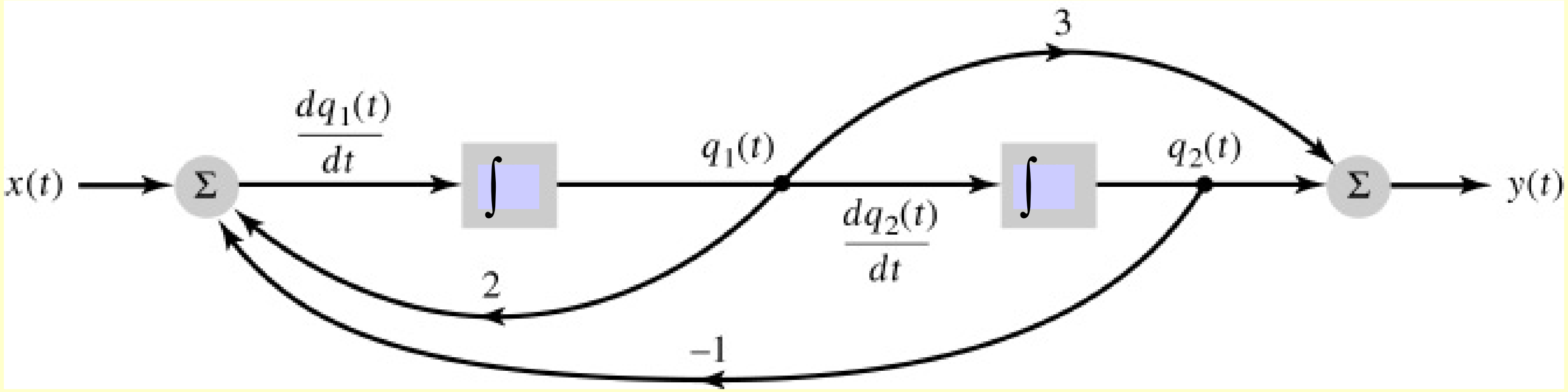
# Block diagram of LTI system for Example 2.30.

請用狀態變數描述下圖：



# Convolution Sum using MATLAB.

$$>> h = [-1, \quad 0.5];$$

$$>> x = [2, \quad 4, \quad -2];$$

$$>> y = conv(x, h)$$

$$y =$$

$$2 \quad 5 \quad 0 \quad -1$$

# Convolution sum computed using MATLAB.

$$>> h = 0.25 * ones(1, 4);$$

$$>> x = ones(1, 10);$$

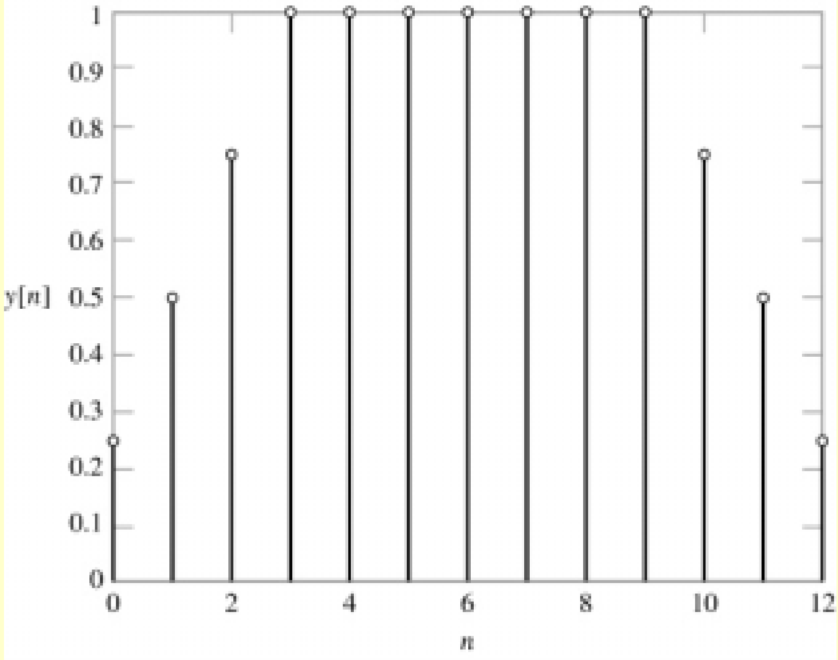
$$>> n = 0:12;$$

$$>> y = conv(x, h);$$

$$>> stem(n, y);$$

$$>> xlabel('n');$$

$$>> ylabel('y[n]')$$



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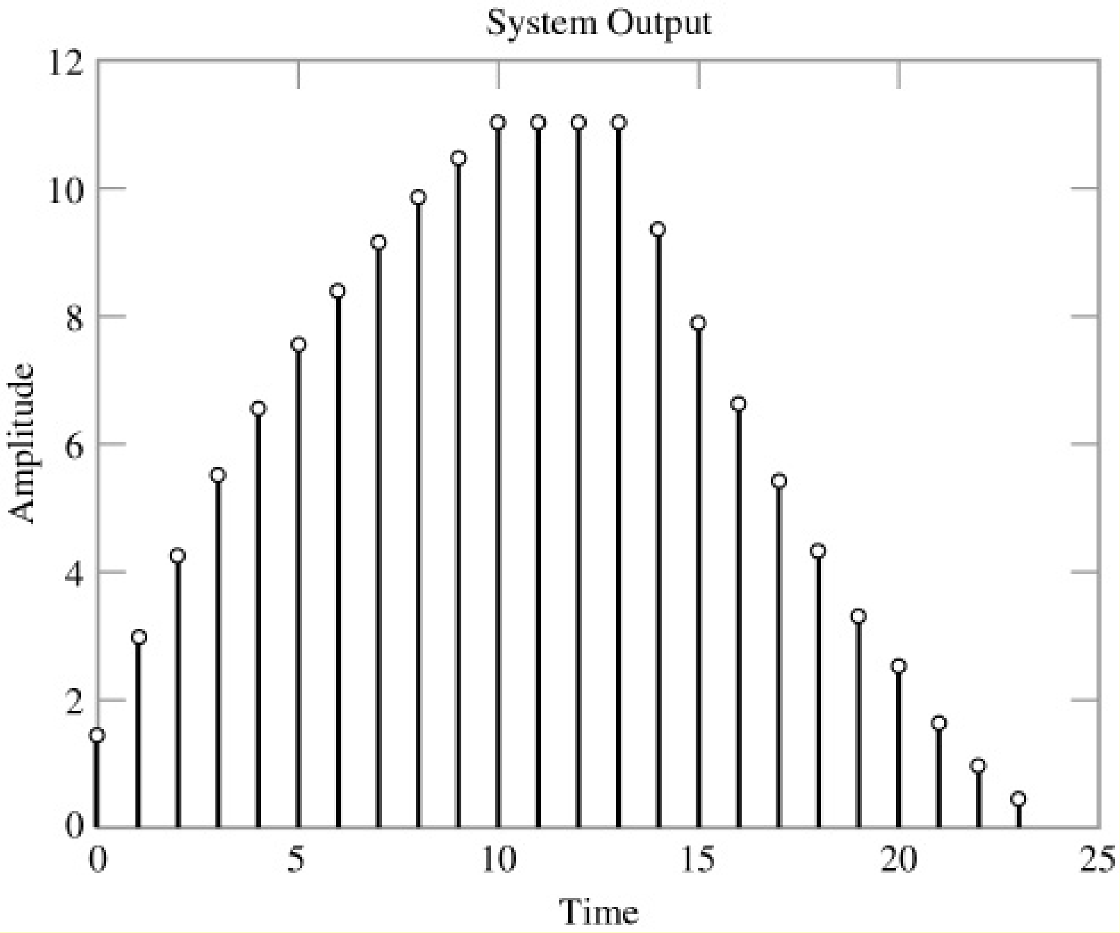
# Solution to Problem 2.29.



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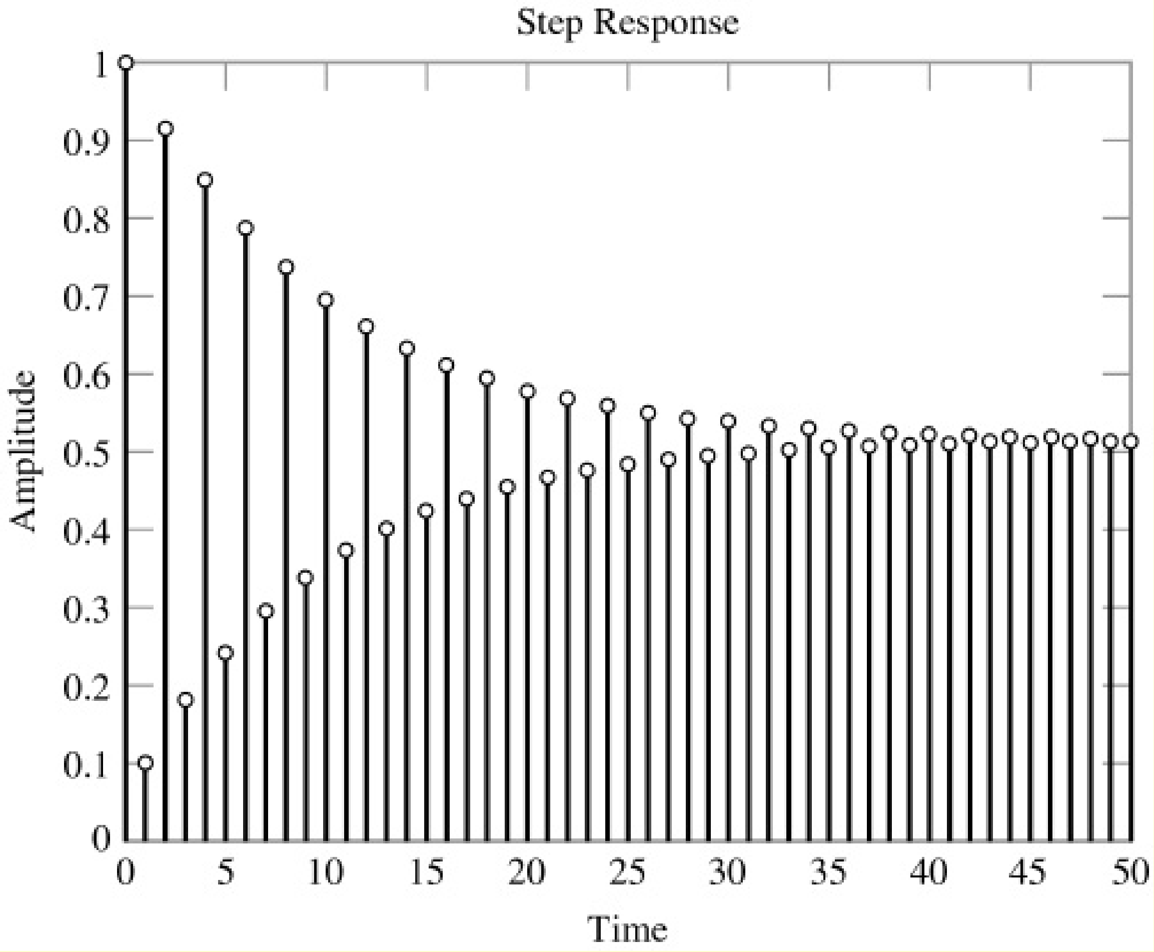
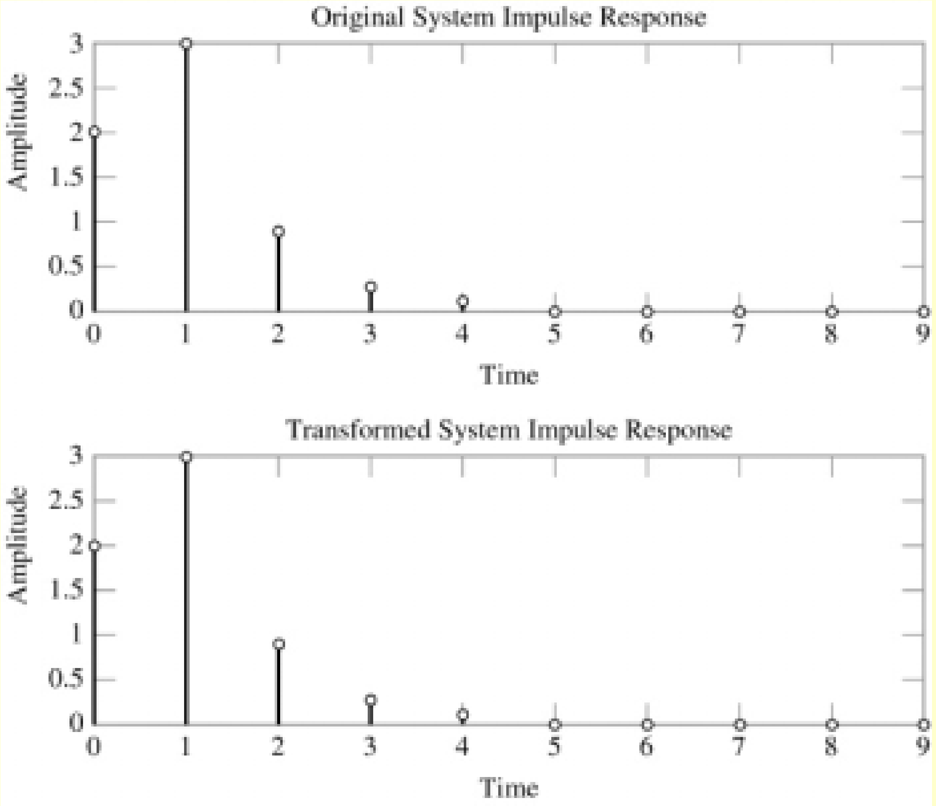
# Step response computed using MATLAB.



# Impulse responses associated with the original and transformed state-variable descriptions computer using MATLAB.



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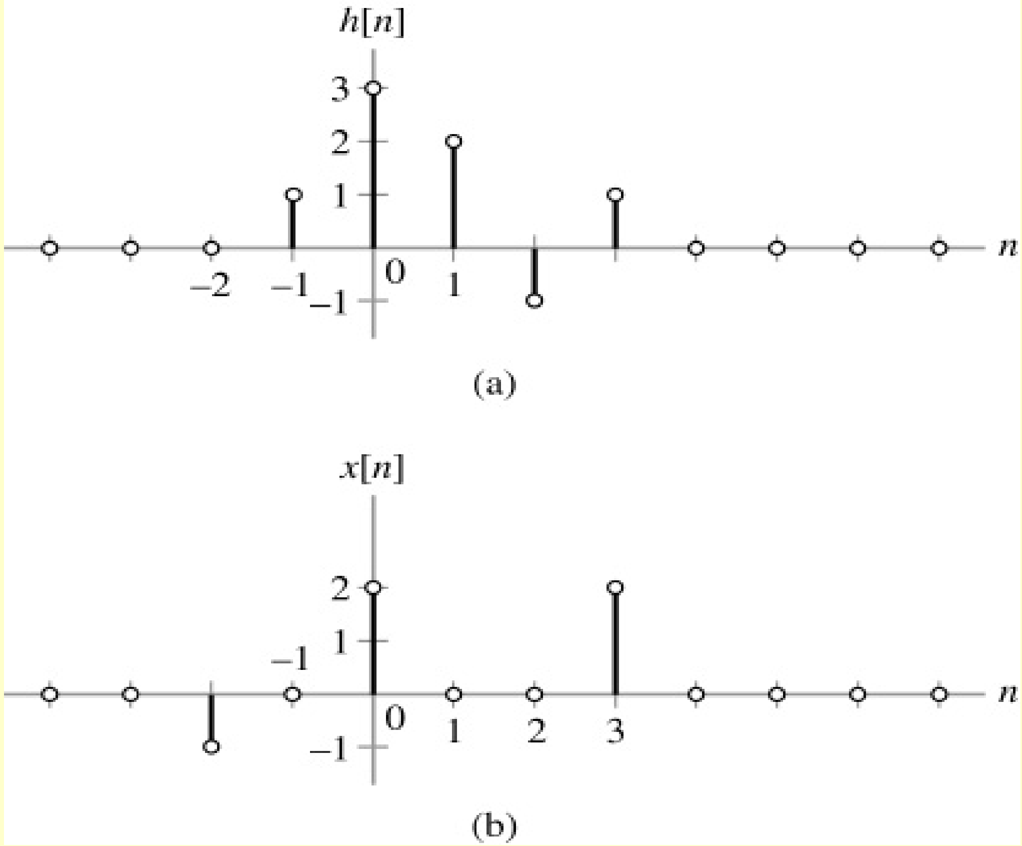
# P2.32: y[n] = x[n] ∗ h[n] = ?



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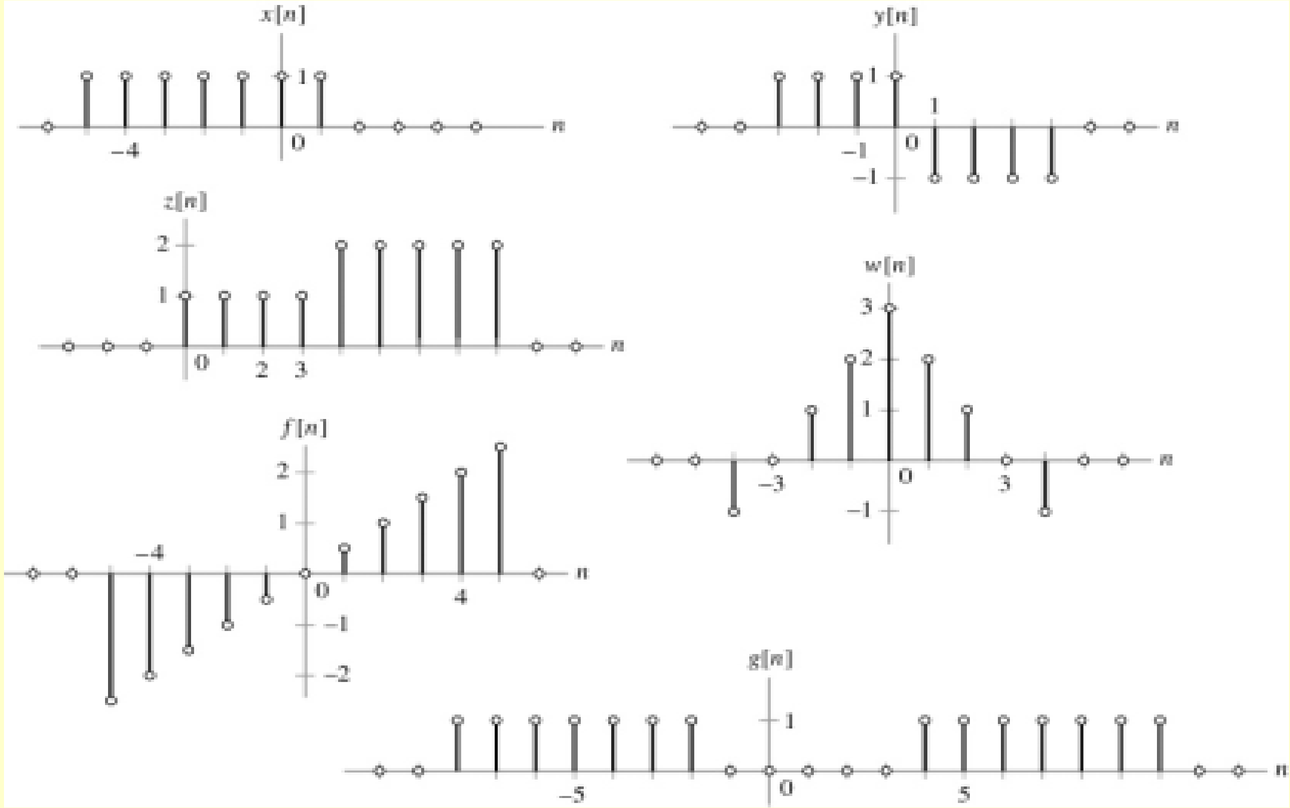
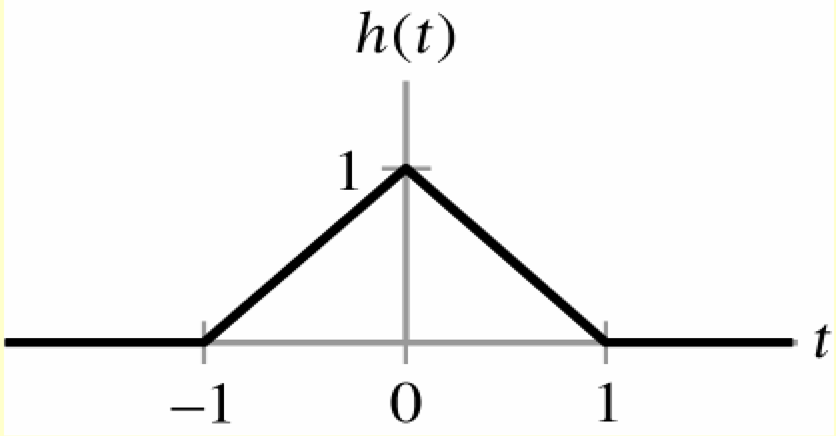
# P2.33: x[n] ∗ y[n] = ? z[n] ∗w[n] = ? f[n] ∗g[n] = ?



P2.38: LTI 系統脈衝響應如下，若輸入為：

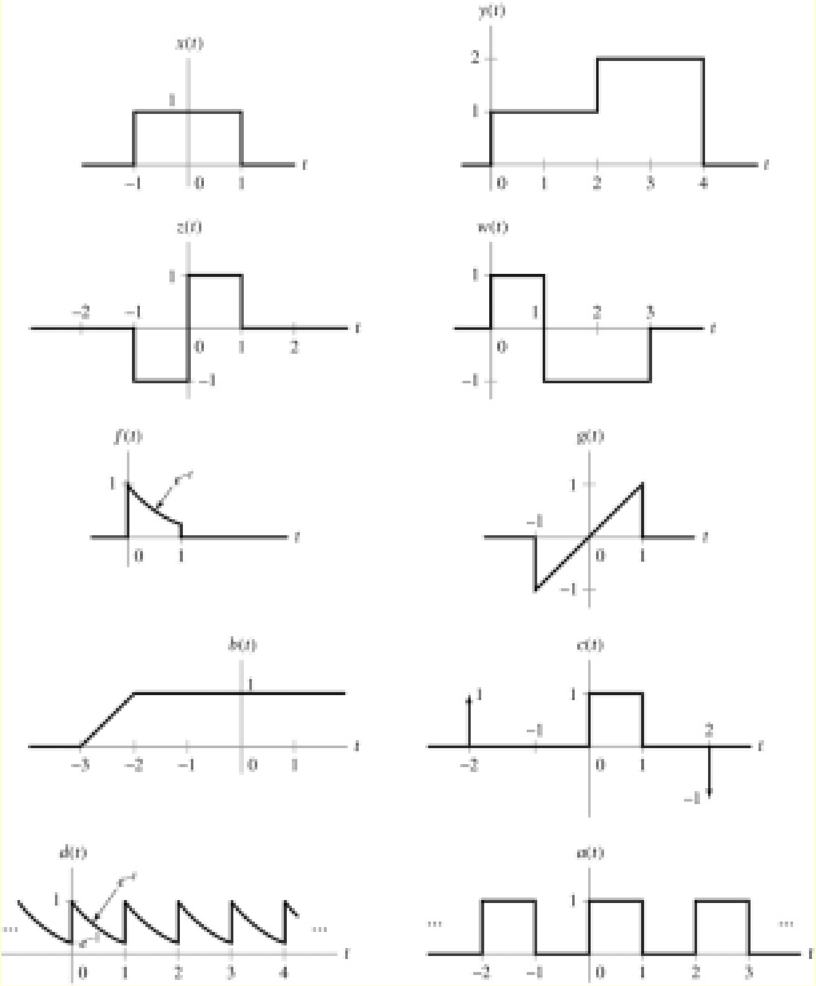
(a) $x(t) = 2\delta(t+2) + \delta(t-2)$

(b) $x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$



P2.40: $x(t) * y(t)$ = ? $z(t) * w(t)$ = ? $f(t) * g(t)$ = ?

$b(t) * c(t)$ = ? $d(t) * a(t)$ = ?



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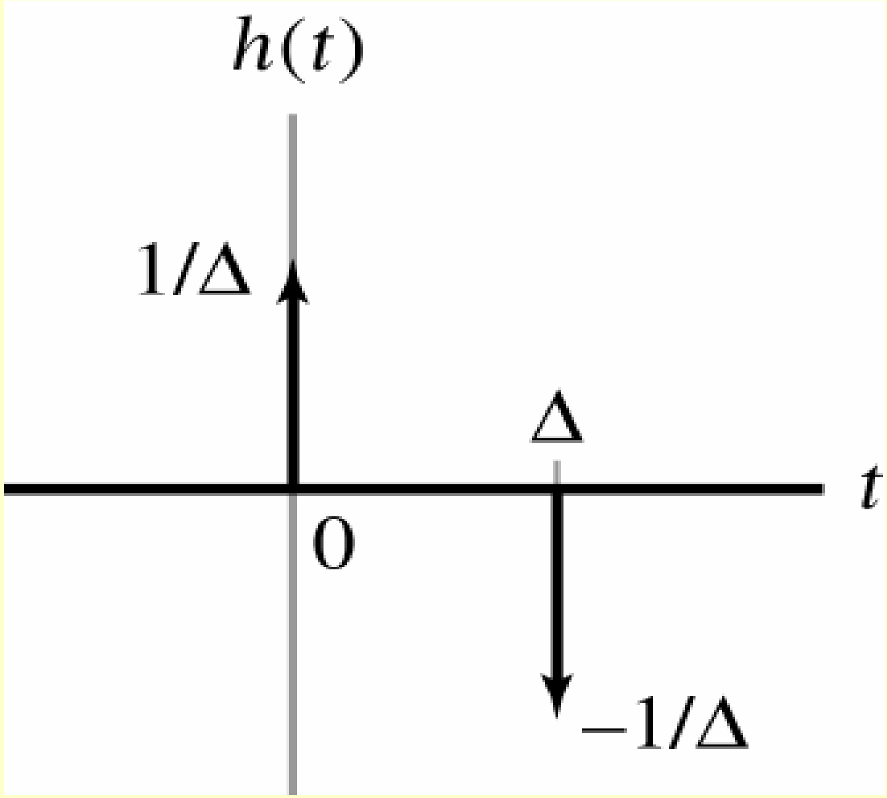
P2.43: 利用下圖系統脈衝響應，寫出系統輸出與輸入關係式=？



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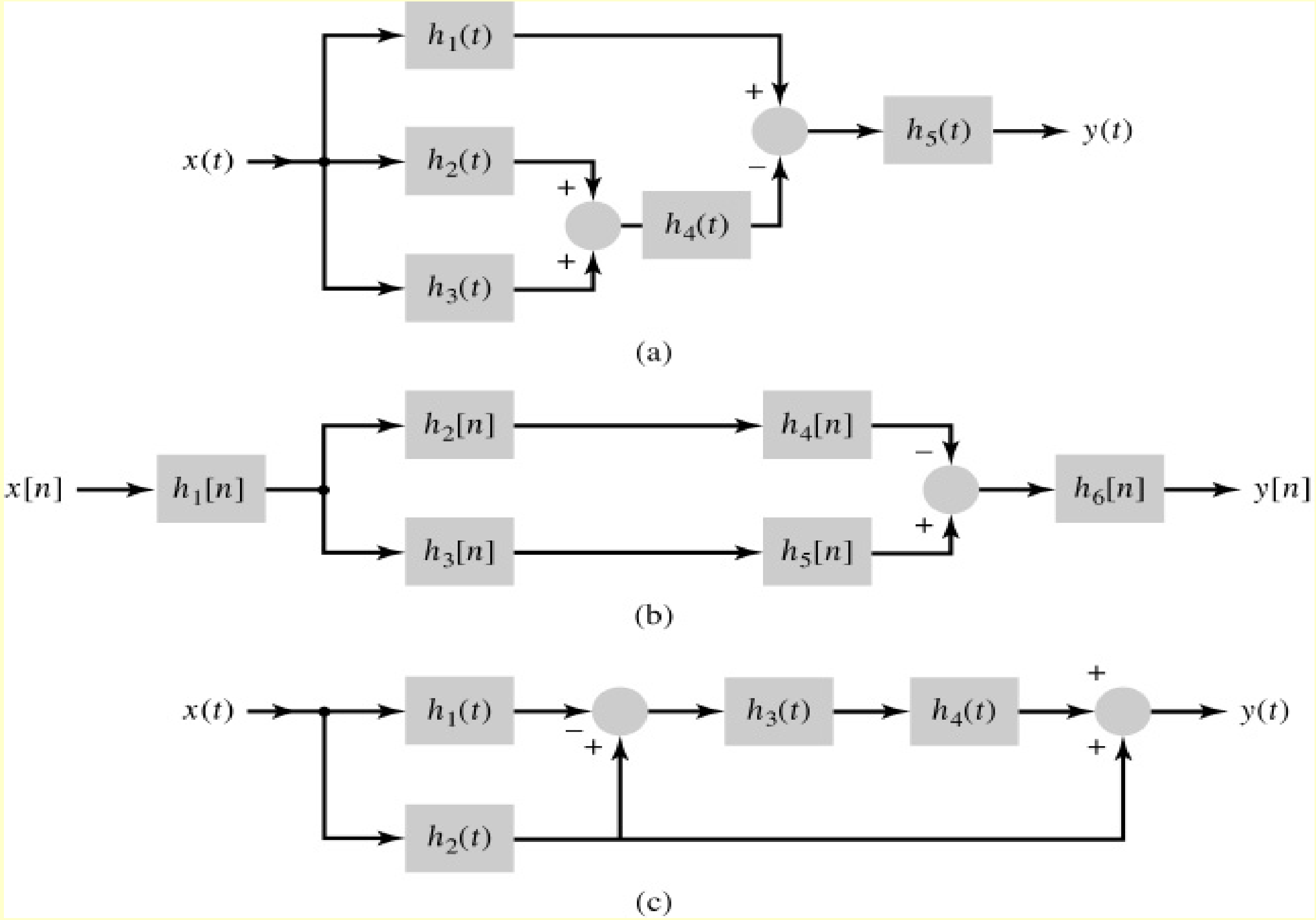
# P2.46: 找出下列各圖總系統脈衝響應 = ?



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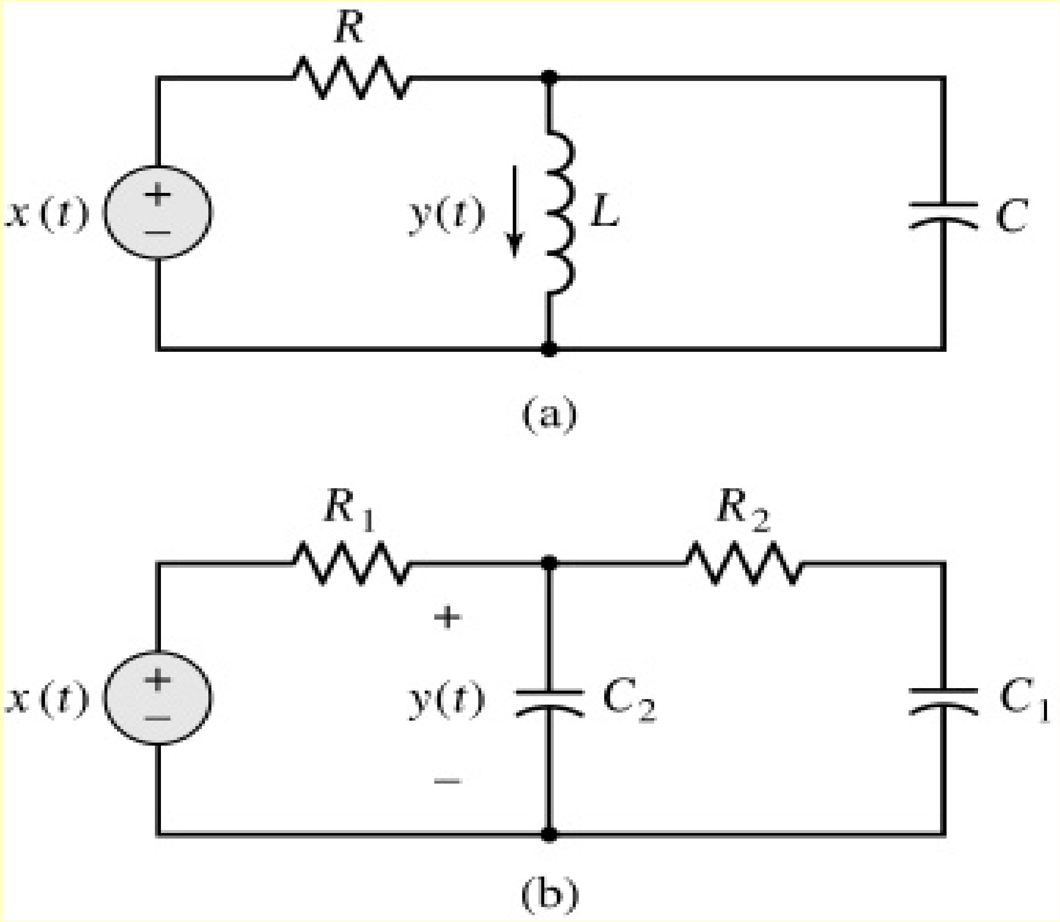
# P2.52: 以微分方程式描述下列電路 = ?



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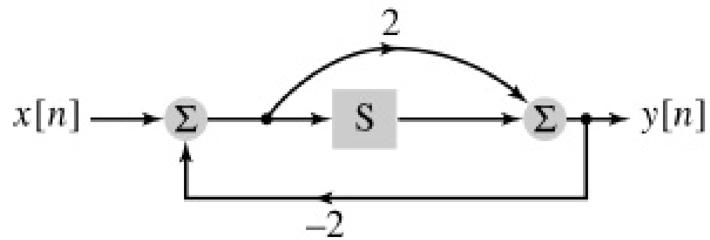
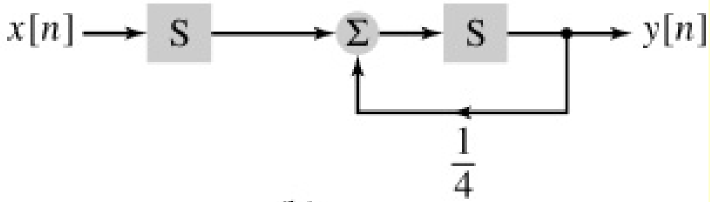
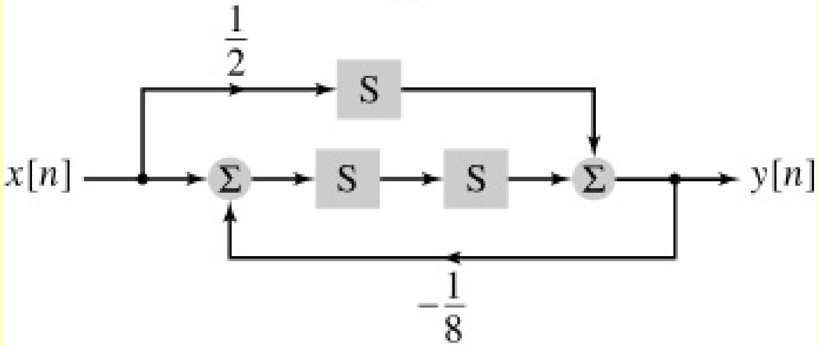
# P2.65: 以差分方程式描述下列電路 = ?



(a)



(b)



(c)

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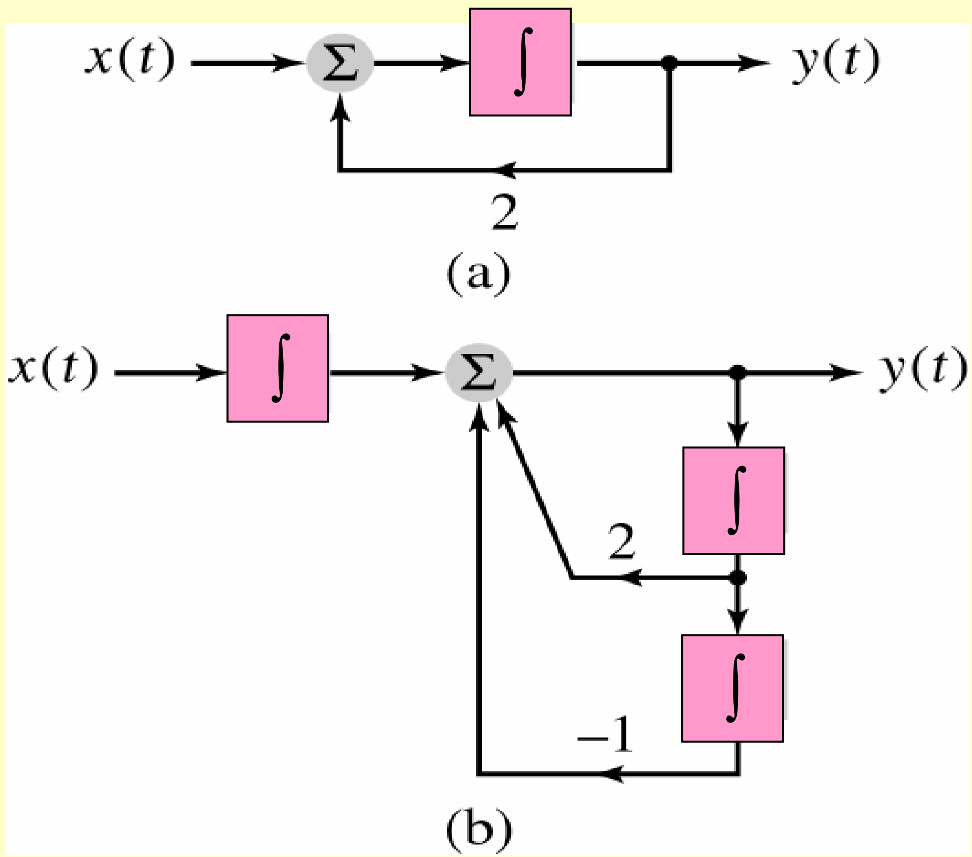
# P2.68: 以微分方程式描述下列電路 = ?




(a)

(b)

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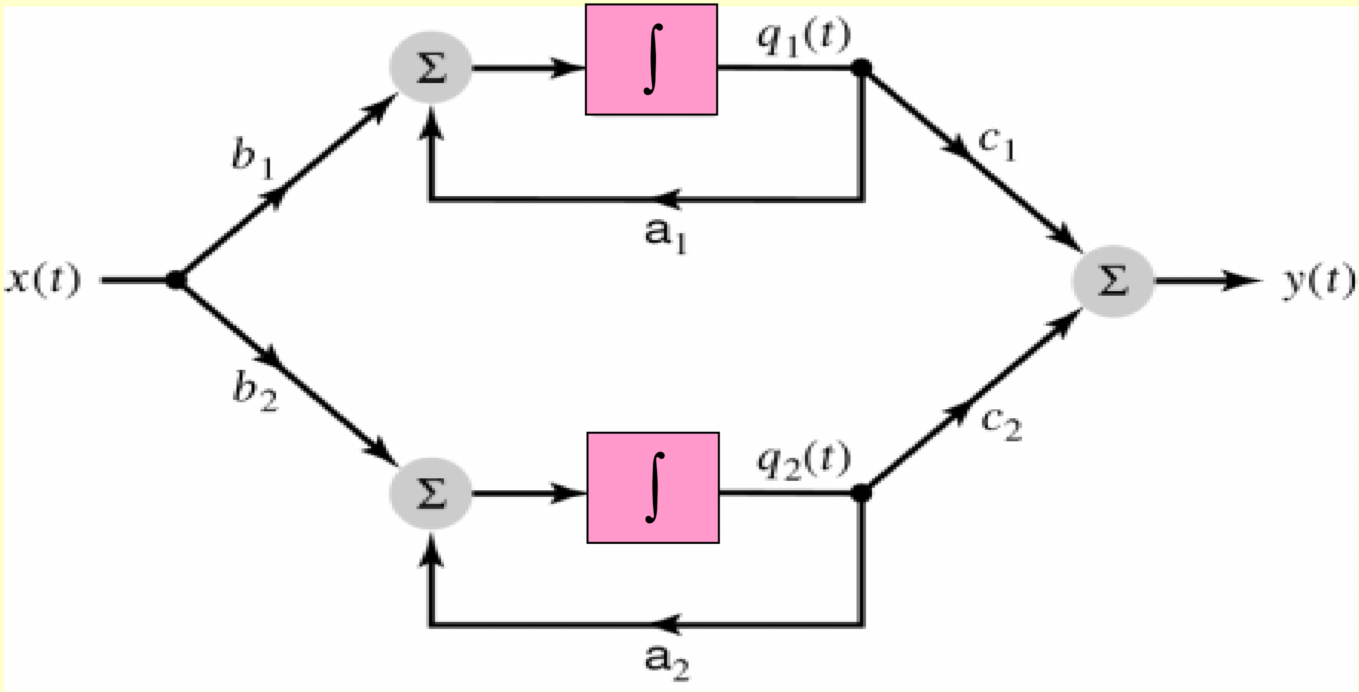
# P2.74: 以狀態變數方式描述下列電路 = ?



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# P2.75: 進階習題