大地測量課程

幾何大地測量
(Geometric Geodesy)

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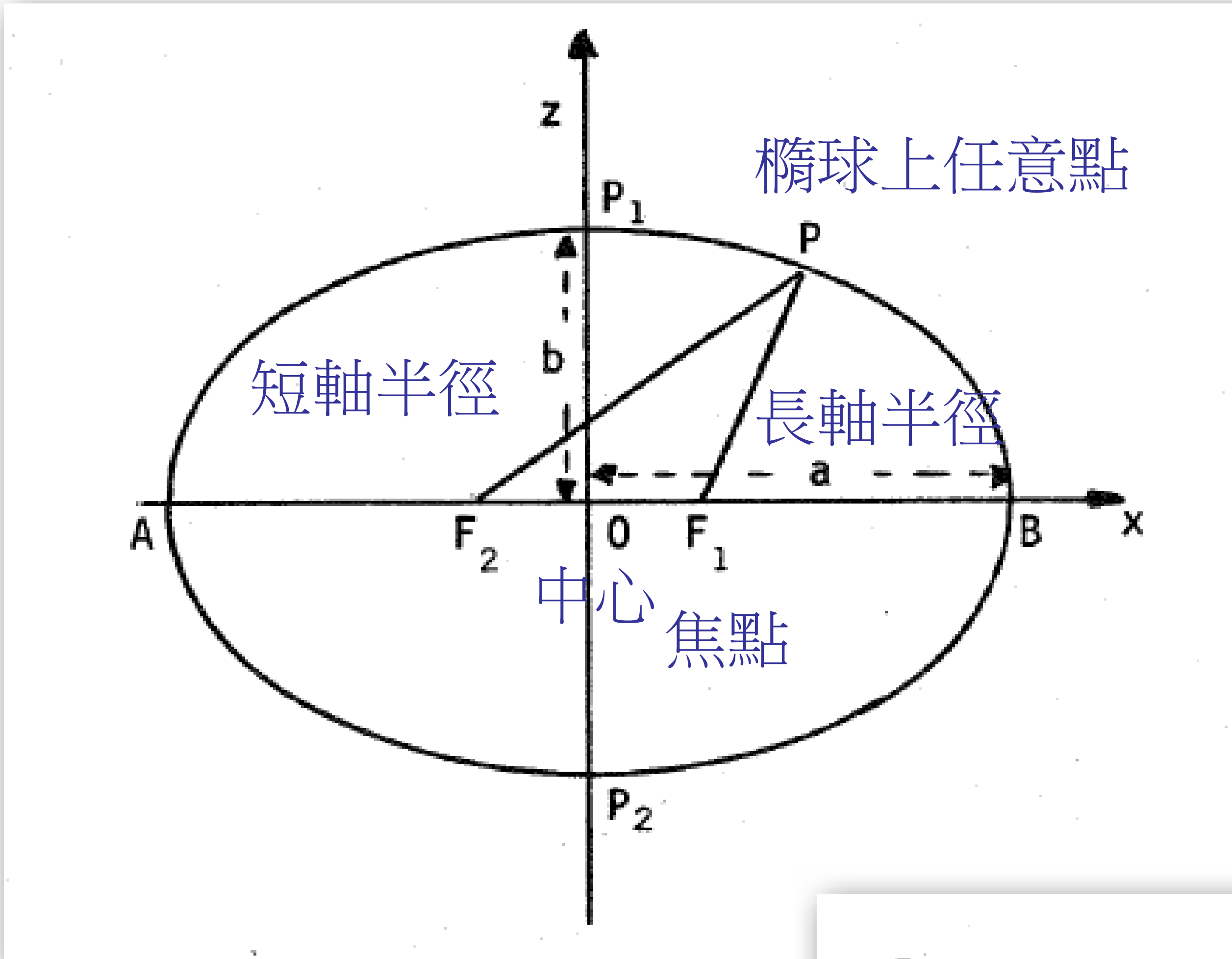
橢球體的特性



$$F_2P + F_1P = 2a$$

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橢球相關量

扁率

$$f \equiv \frac{a-b}{a}$$

第一離心率

$$e \equiv \frac{OF_1}{a} = \frac{\sqrt{a^2-b^2}}{a} \;;\quad e^2 = \frac{a^2-b^2}{a^2}$$

第二離心率

$$e' \equiv \frac{OF_1}{b} = \frac{\sqrt{a^2-b^2}}{b} \;;\quad e'^2 = \frac{a^2-b^2}{b^2}$$

$$e^2 = 2f - f^2$$

扁率與離心率之關聯

子午橢圓與大地緯度

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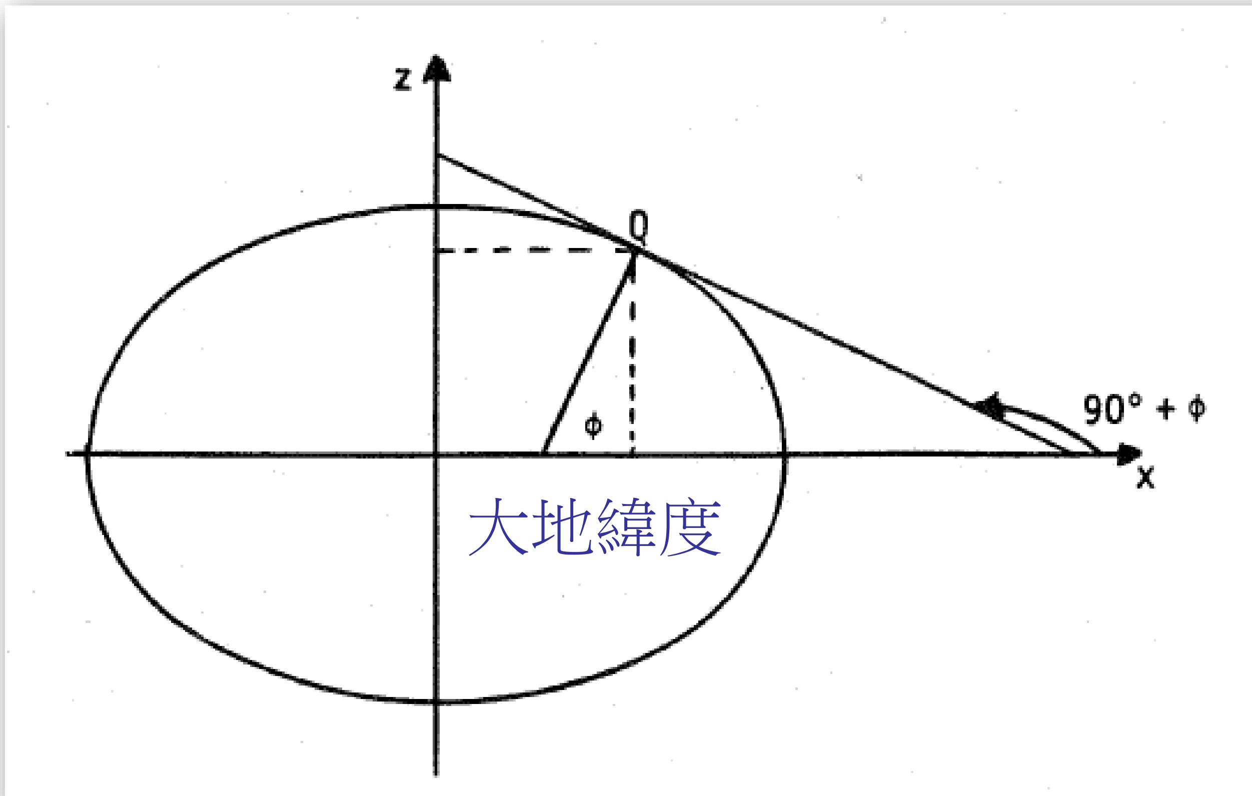
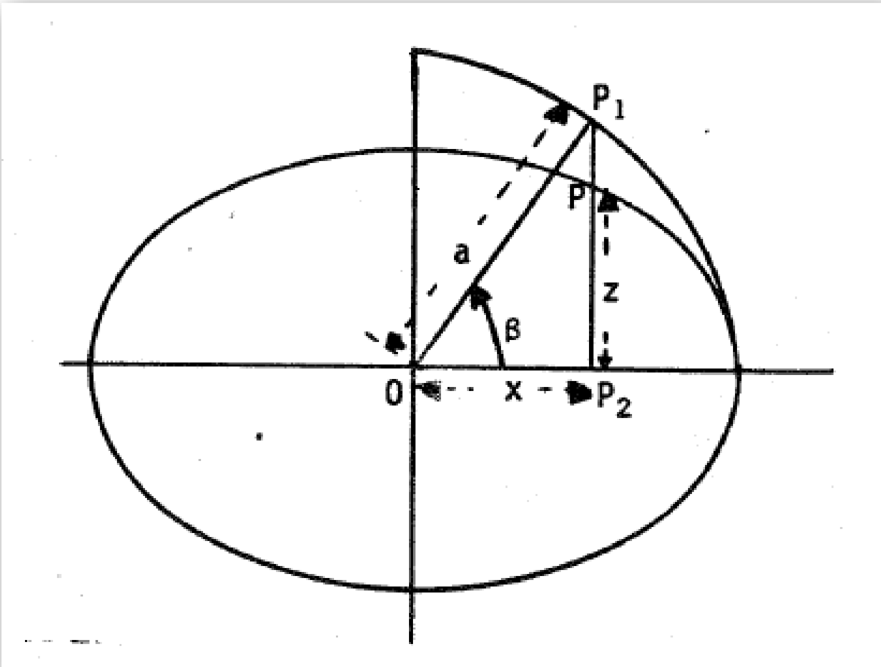
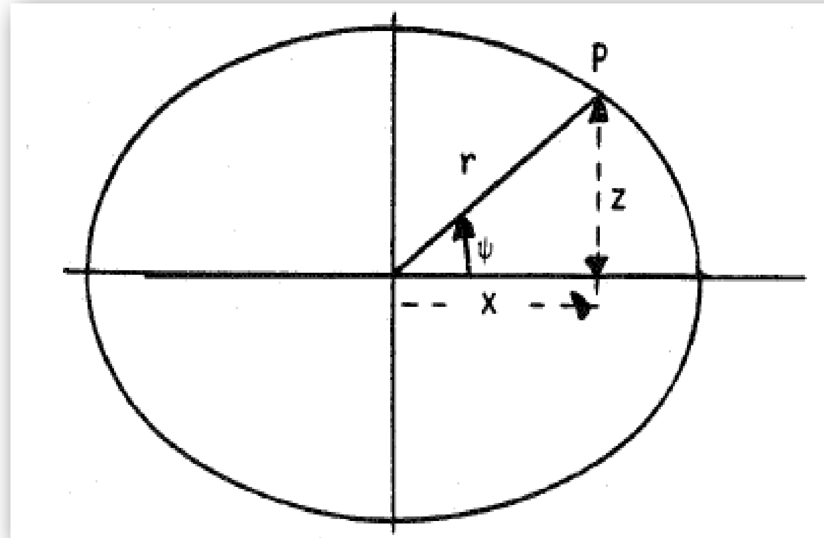


歸化緯度與地心緯度





三種緯度關聯式

$$\tan \psi = (1-e^2)^{\frac{1}{2}} \tan \beta = (1 - e^2) \tan \phi$$

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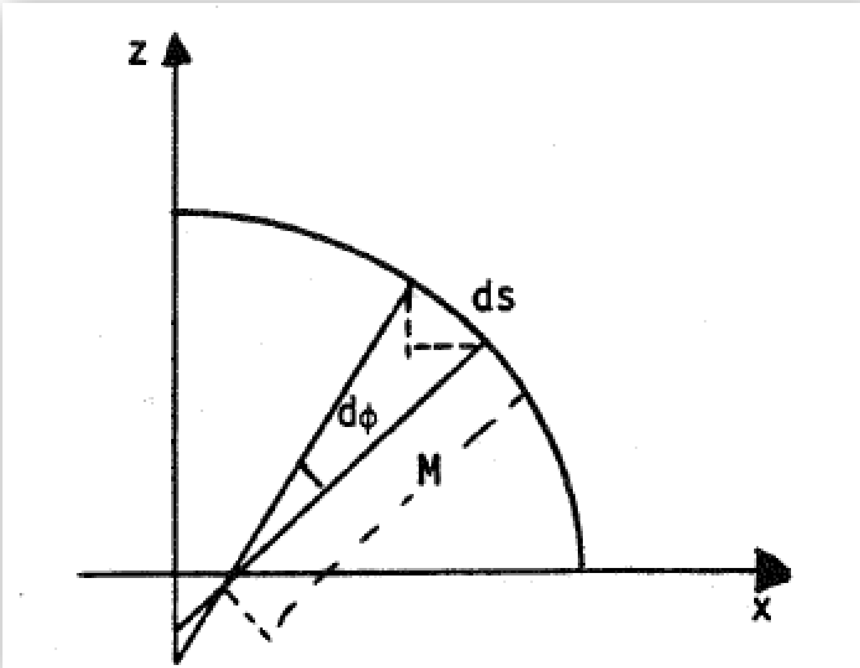
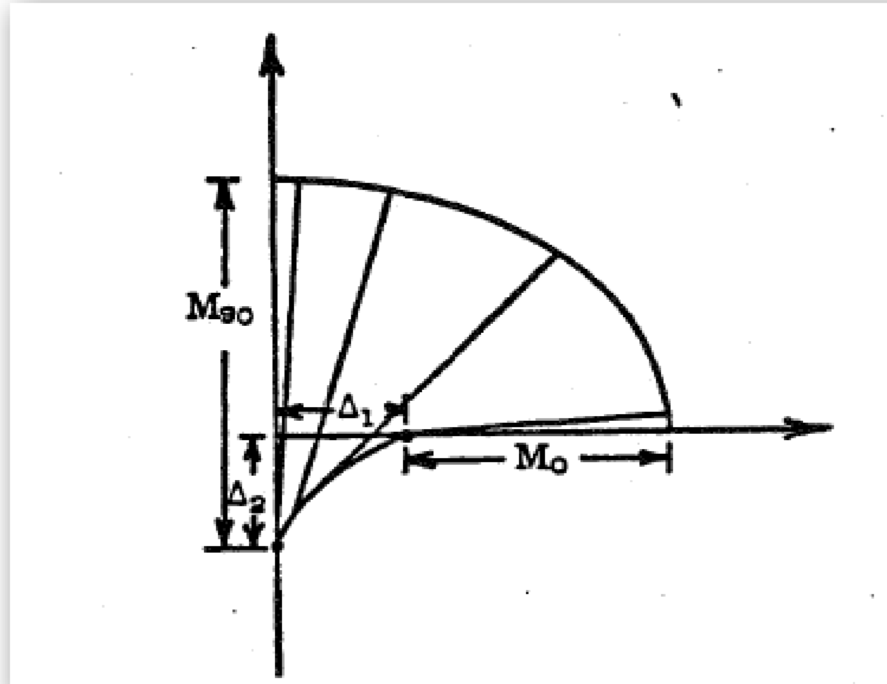
主截面與曲率半徑-1

- 子午圈:通過測站與兩極之平面

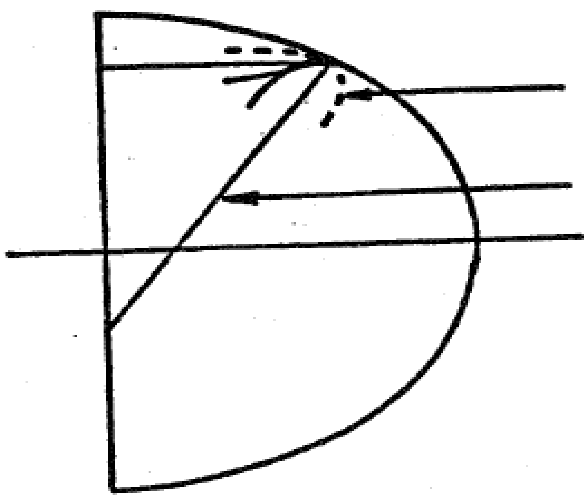




$$M = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$$

主截面與曲率半徑-2

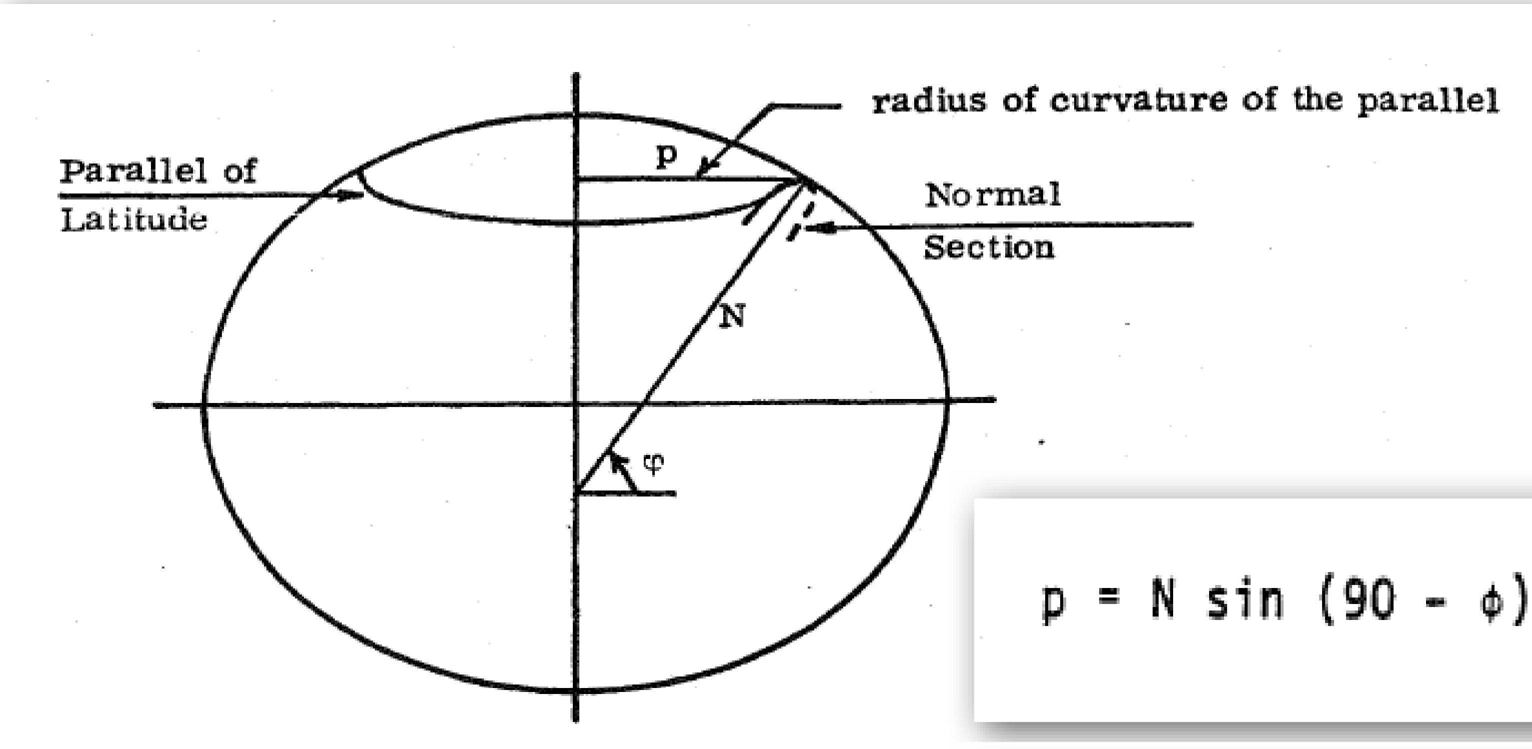
- 卯酉圈:通過測站並垂直於子午圈之平面



$$N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$$

平行圈與曲率半徑

- 平行圈:平行緯圈之平面



$$p = N \sin(90 - \phi) = N \cos\phi$$

- 任意截面(方位角 α 處)之曲率半徑

$$\frac{1}{R_\alpha} = \frac{\sin^2\alpha}{N} + \frac{\cos^2\alpha}{M}$$

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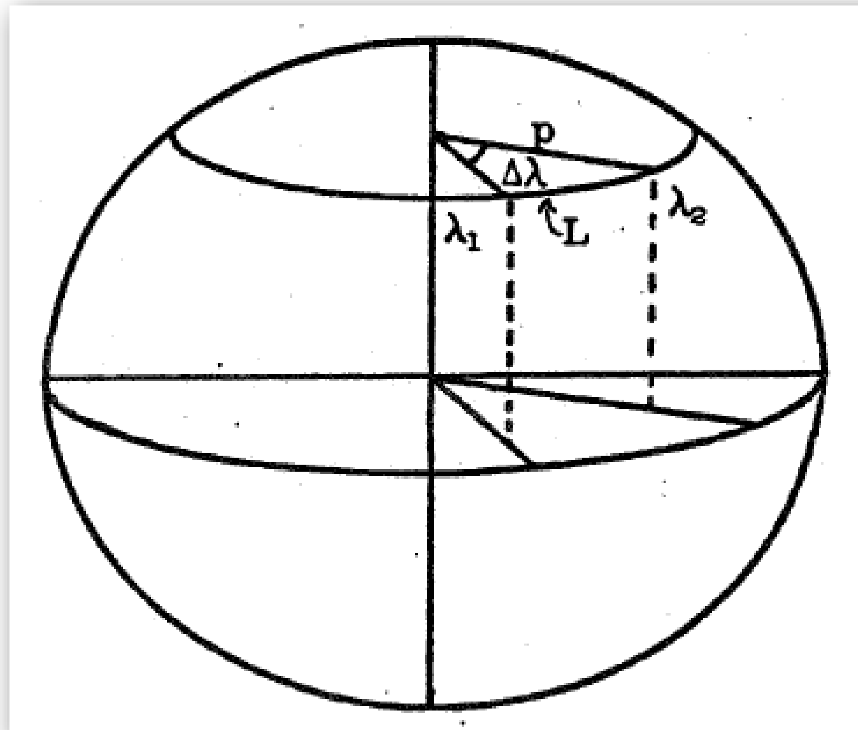
弧長的計算

- 子午圈

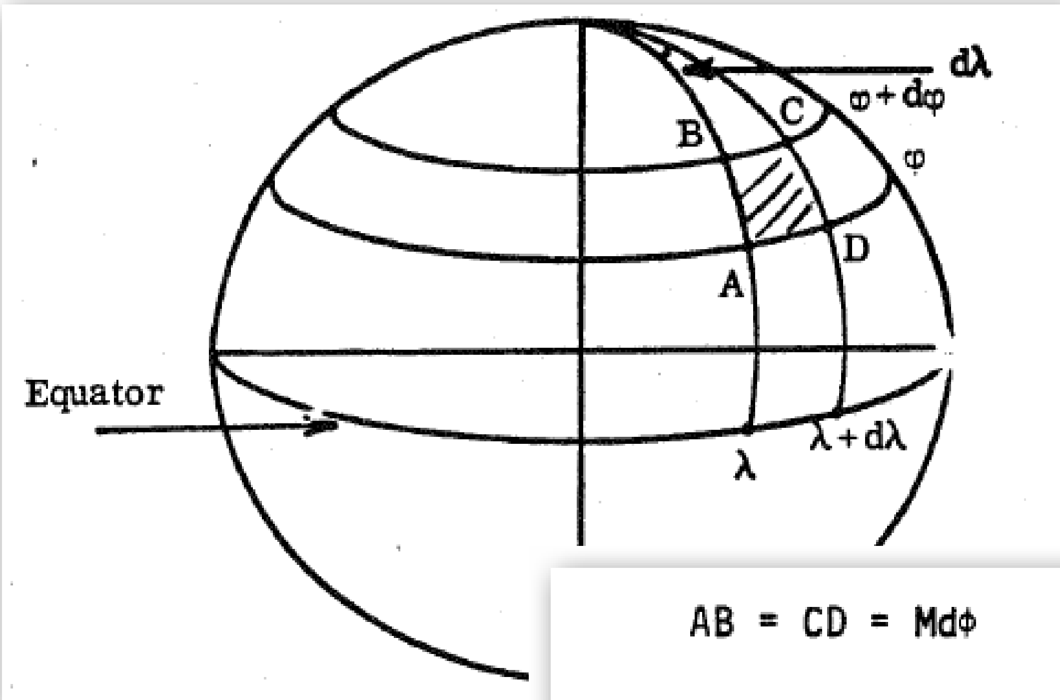
$$ds = Md\phi$$

- 平行圈

$$L = p\Delta\lambda = N\cos\phi\Delta\lambda$$



橢球表面積的計算



$$AB = CD = Md\phi$$

$$AD = BC = N\cos\phi d\lambda$$

Letting the area of the differential figure be dZ we have:

$$dZ = AD \cdot AB = MN\cos\phi d\phi d\lambda$$

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近似球體的平均半徑

- 高斯(Gauss)平均半徑

$$R = \sqrt{MN} = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\phi}$$

- 三軸平均半徑

$$R_m = \frac{a+a+b}{3}$$

- 等面積平均半徑

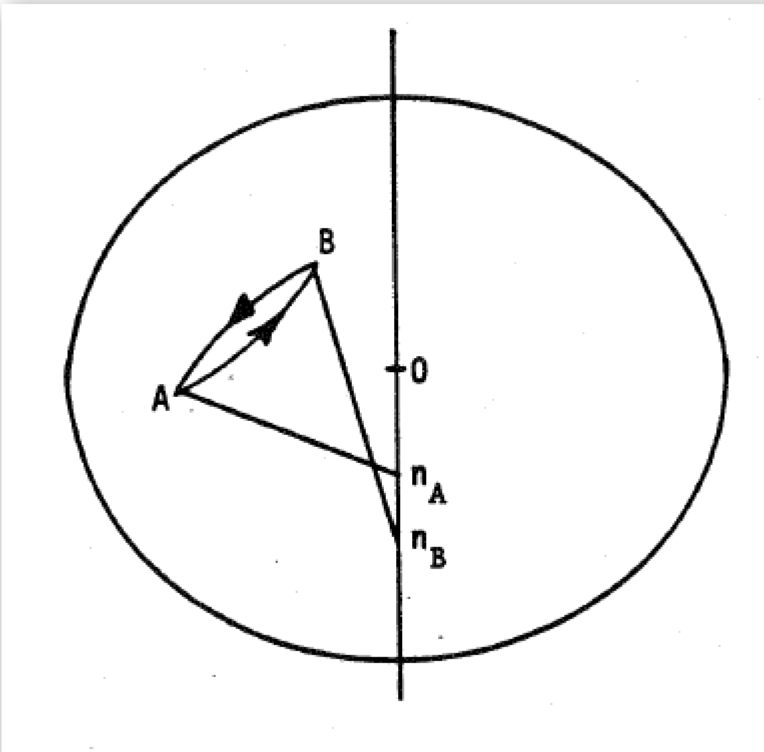
$$R_A = \sqrt{\frac{\Sigma}{4\pi}}$$

- 等體積平均半徑

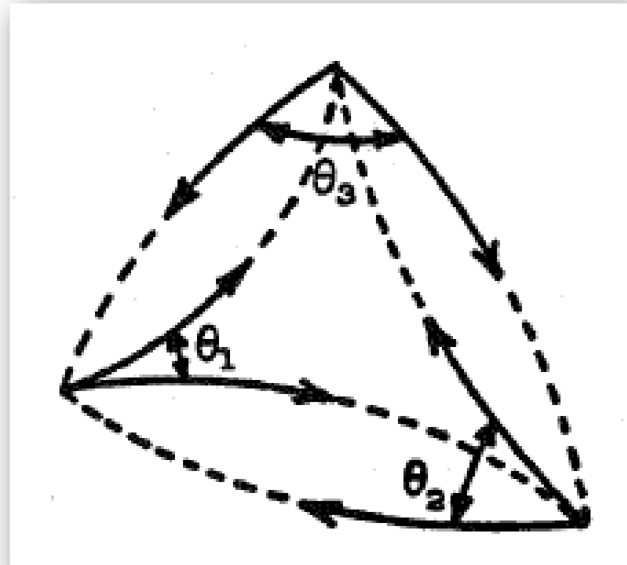
$$R_V = \sqrt[3]{a^2b}$$

直截面的特性

- 直截面:包含測站法線之切平面



不同緯度之兩點會形成正反直截面



三角網形成之直截面

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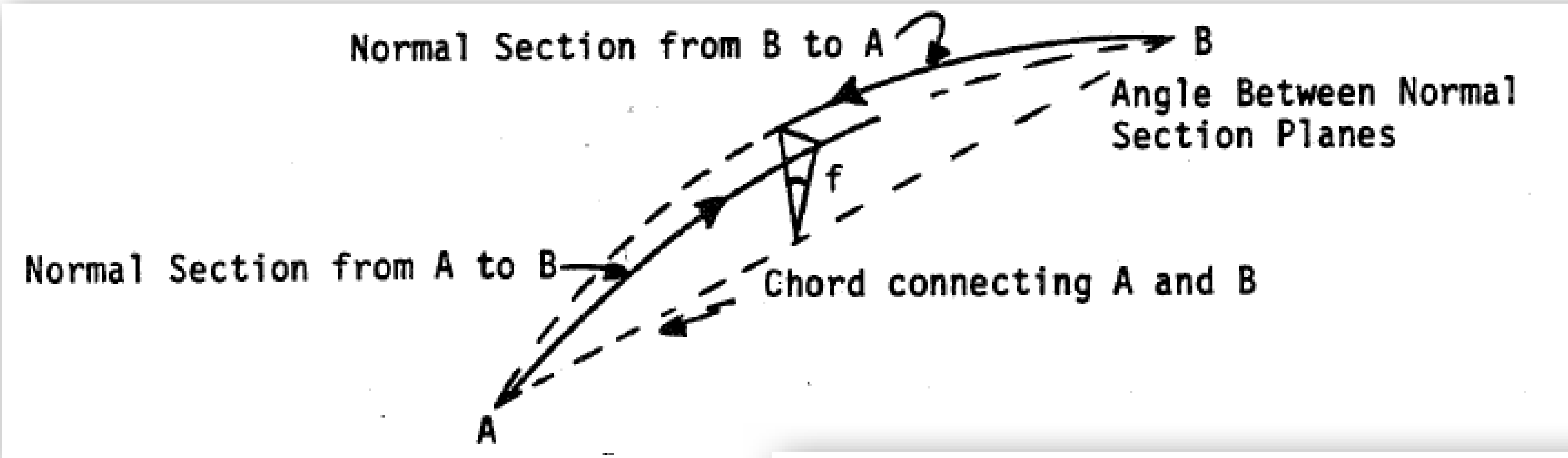
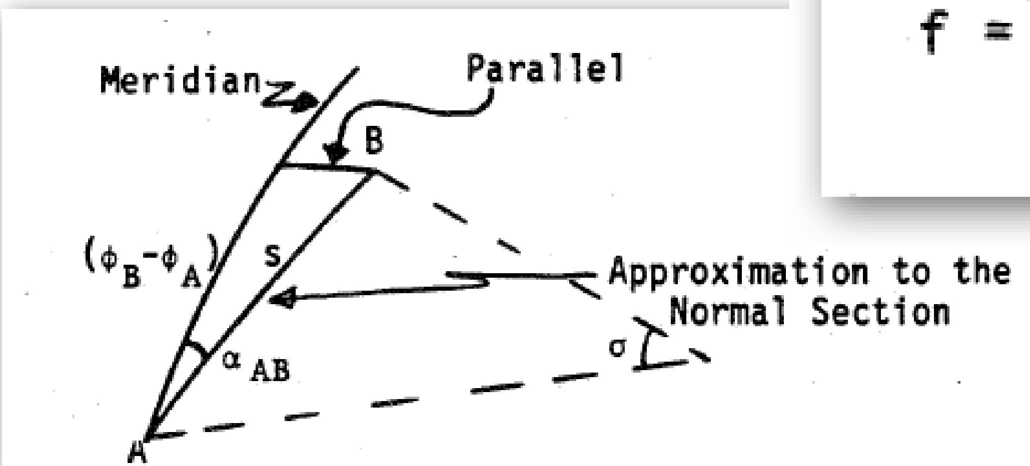
正反直截面之間的角距



$$f = \tfrac{1}{2}e^2\left(\frac{S}{N_A}\right)\cos^2\phi_m \sin 2\alpha_{AB}$$



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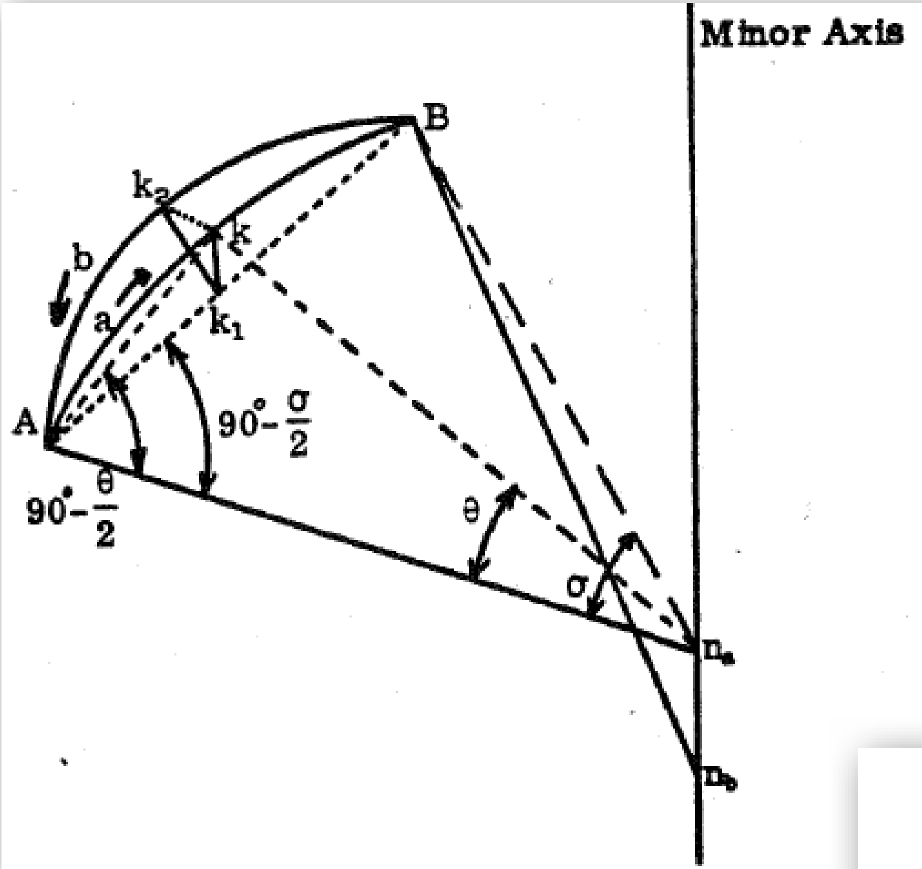
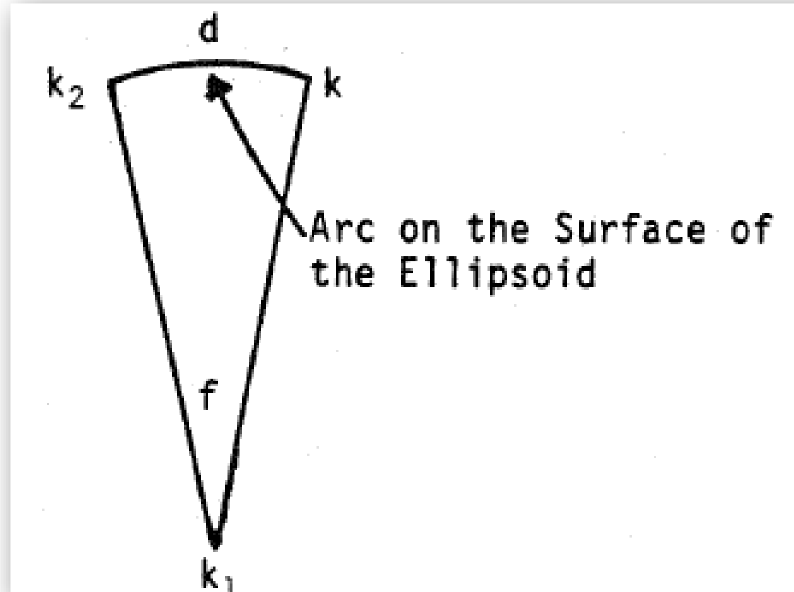
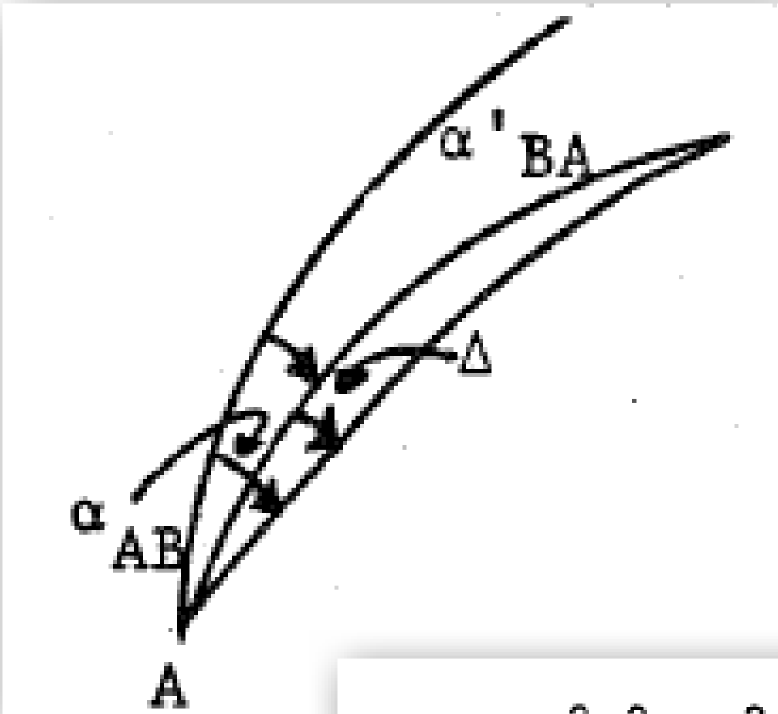
正反直截面之間的線距





$$d = \frac{e^2}{4}\, s\theta(\sigma-\theta)\cos^2\phi_m \sin 2\alpha_{AB}$$

正反方位角差



$$\Delta = \frac{e^2\sigma^2\cos^2\phi_m\sin2\alpha_{AB}}{4} = \frac{e^2}{4}\left(\frac{S}{N_A}\right)^2\cos^2\phi_m\sin2\alpha_{AB}$$

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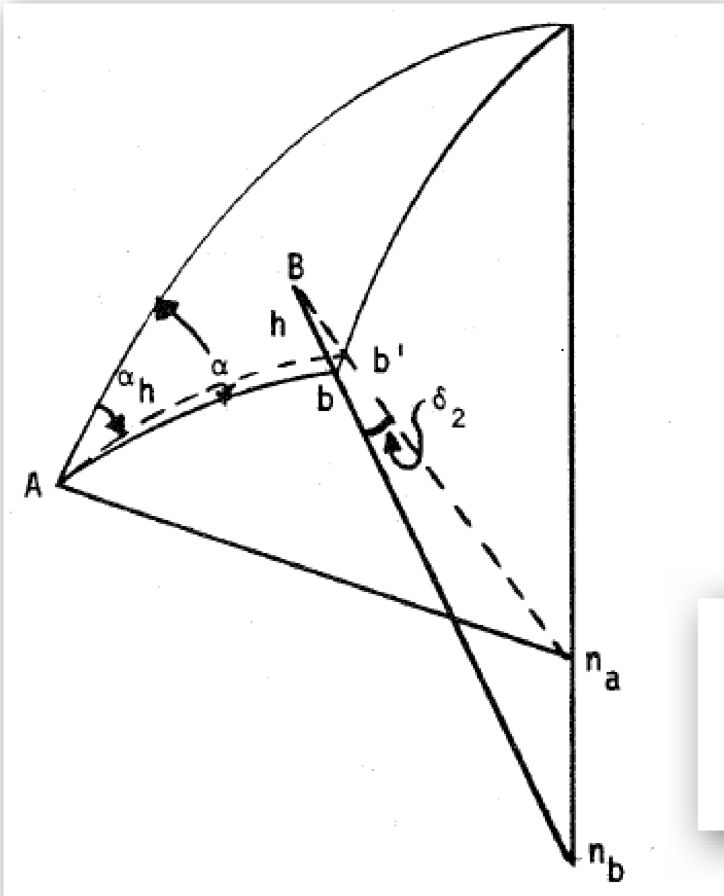
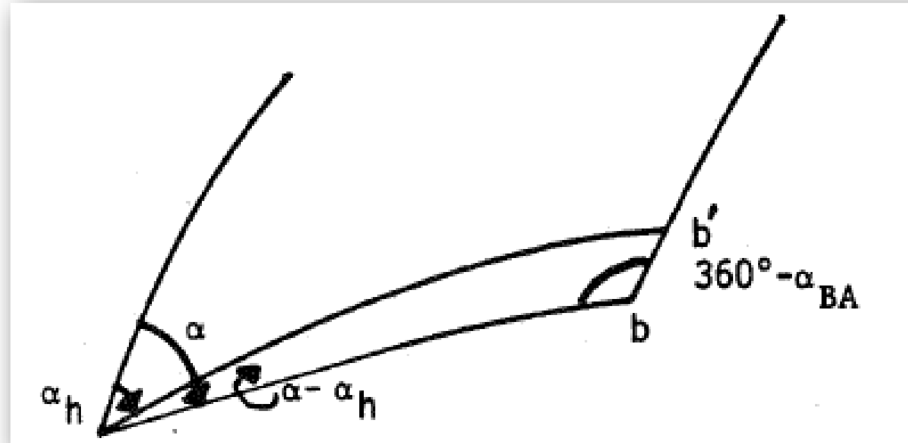
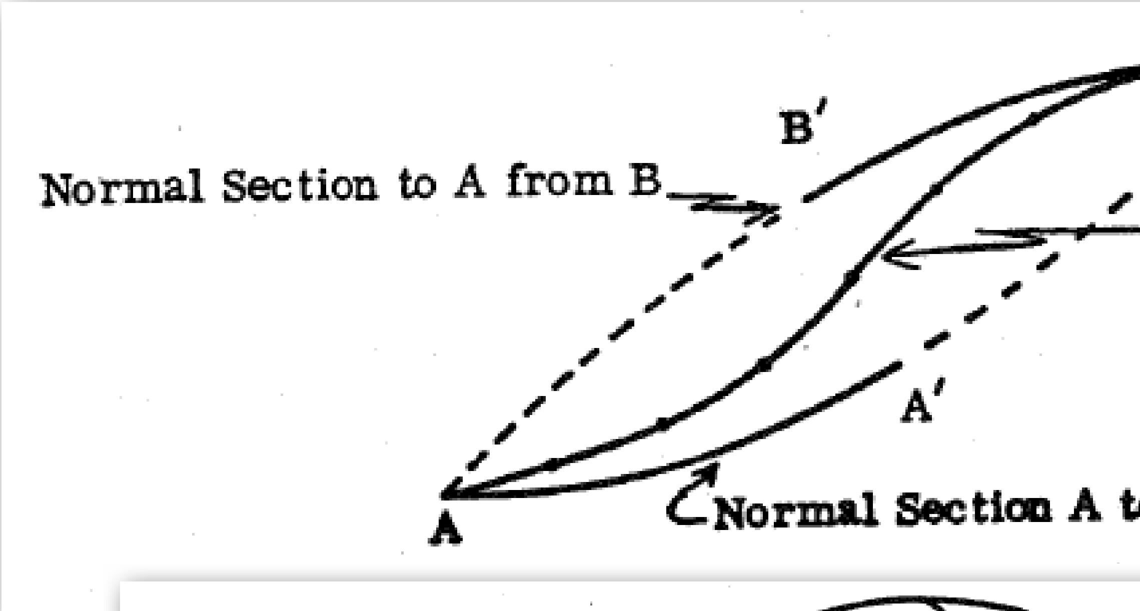
標高差



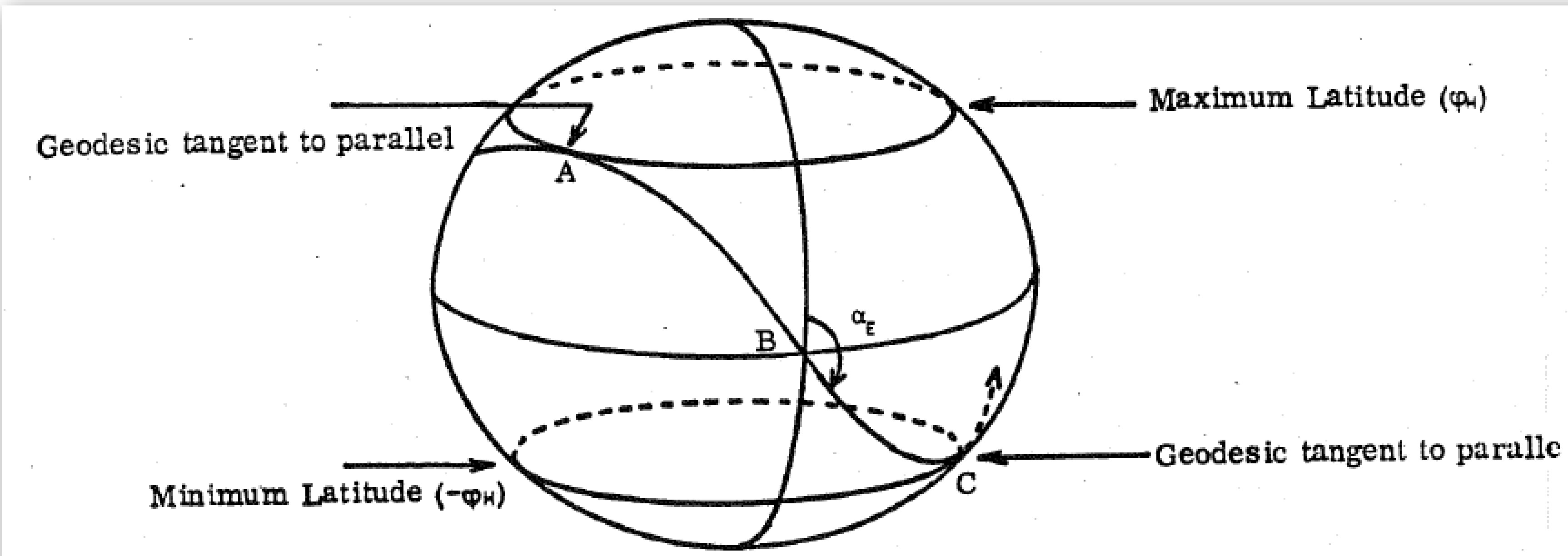


$$\alpha - \alpha_h = \frac{h}{2M_m} e^2 \cos^2\phi_m \sin 2\alpha_{AB}$$

大地線



$$p \sin\alpha = \text{constant}$$



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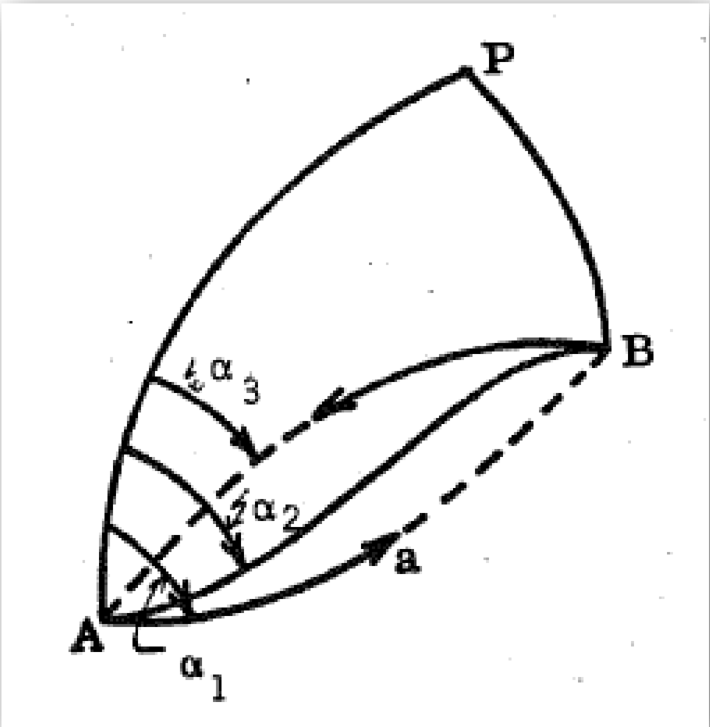
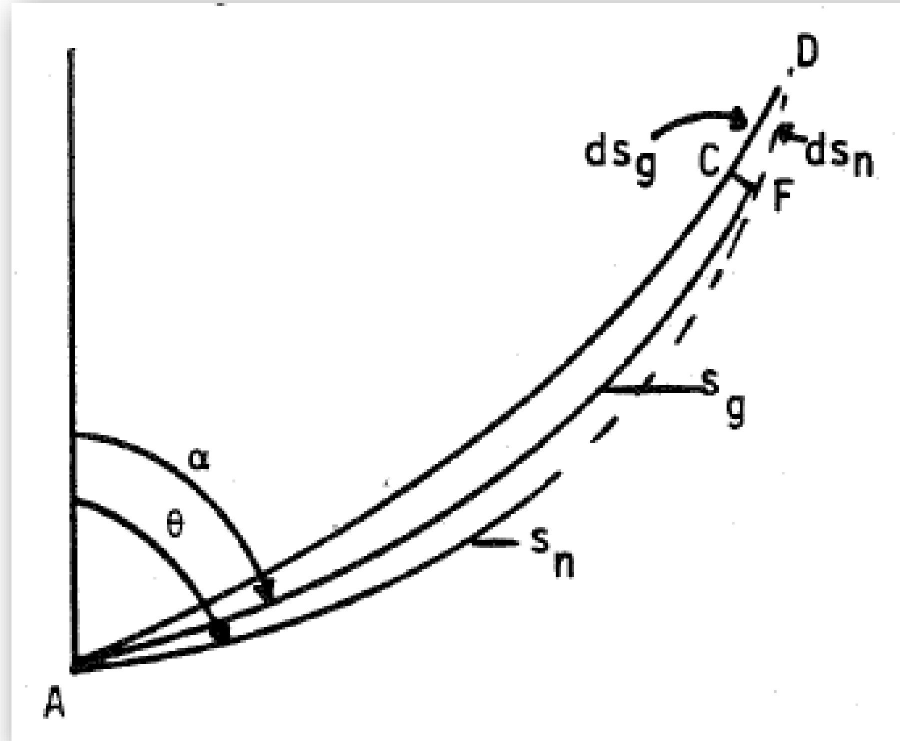
截面差



$$(\alpha_1-\alpha_2)\approx\frac{1}{3}(\alpha_1-\alpha_3)$$

方位角的改正
(正反方位角差的1/3)



$$s_n - s_g = s\frac{e'^4\cos^4\phi}{360}\left(\frac{s}{N}\right)^4\sin^2 2\alpha$$

距離的改正

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方位角的改正

截面差

$$\delta_1 = \frac{-e^2}{12}\left(\frac{S}{N}\right)^2 \cos^2\phi_m \sin 2\alpha_{AB}$$

標高差

$$\delta_2 = \frac{h}{2M_m} e^2 \cos^2\phi_m \sin 2\alpha_{AB}$$

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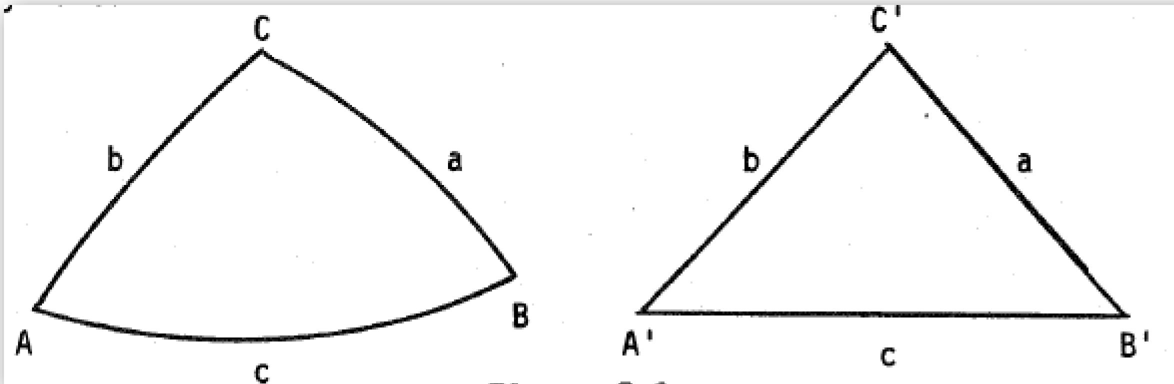
球面角超



$$\varepsilon = A^\circ + B^\circ + C^\circ - 180^\circ$$

$$\varepsilon = \frac{F}{R^2}$$

$$\tan\frac{\varepsilon}{4} = \sqrt{\tan\frac{s}{2}\tan\frac{s-a}{2}\tan\frac{s-b}{2}\tan\frac{s-c}{2}}$$

where $a + b + c = 2s$

$$A - A' = \frac{\varepsilon}{3}$$

$$B - B' = \frac{\varepsilon}{3}$$

$$C - C' = \frac{\varepsilon}{3}$$

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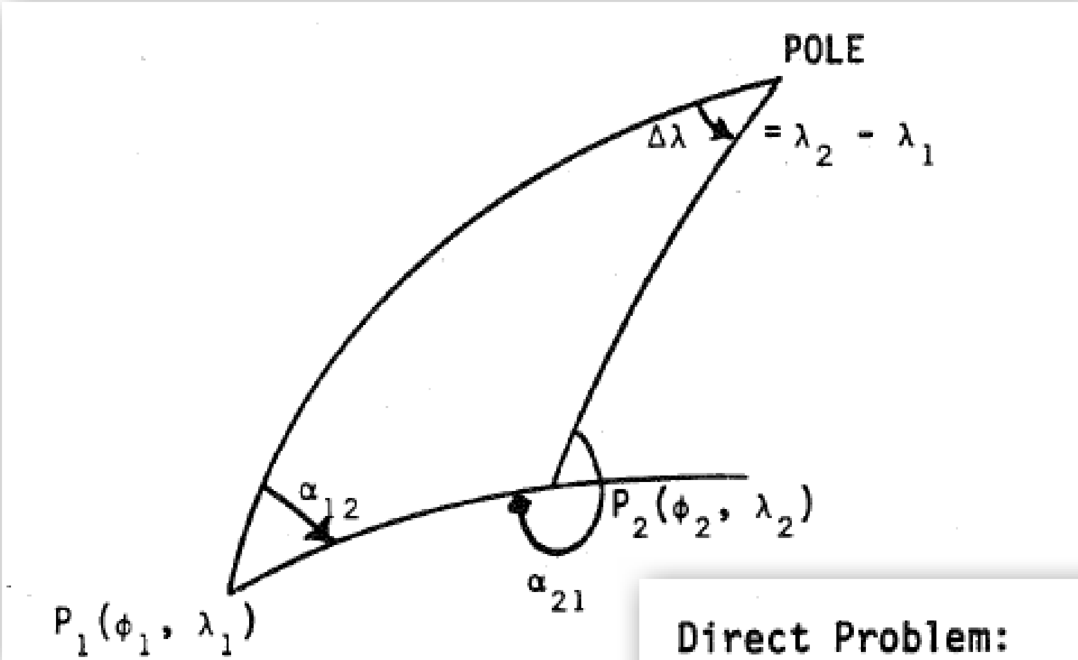
大地位置計算



Direct Problem: 正算

$$\phi_2 = f_1(\phi_1, \lambda_1, \alpha_{12}, s)$$
$$\lambda_2 = f_2(\phi_1, \lambda_1, \alpha_{12}, s)$$
$$\alpha_{21} = f_3(\phi_1, \lambda_1, \alpha_{12}, s)$$

Inverse Problem: 反算

$$s = f_4(\phi_1, \lambda_1, \phi_2, \lambda_2)$$
$$\alpha_{12} = f_5(\phi_1, \lambda_1, \phi_2, \lambda_2)$$
$$\alpha_{21} = f_6(\phi_1, \lambda_1, \phi_2, \lambda_2)$$

布桑(Puissant)正算

$$\Delta\phi = s\cos\alpha_{12}B - s^2\sin^2\alpha_{12}C - hs^2\sin^2\alpha_{12}E - (\delta\phi)^2D$$

$$B = \frac{1}{M_1}, \quad C = \frac{\tan\phi_1}{2M_1N_1}, \quad D = \frac{3e^2\sin\phi_1\cos\phi_1}{2(1-e^2\sin^2\phi_1)},$$

$$E = \frac{1+3\tan^2\phi_1}{6N_1^2}, \quad h = \frac{s\cos\alpha_{12}}{M_1}$$

$$\Delta\lambda = \frac{s}{N_2}\sin\alpha_{12}\sec\phi_2\left[1 - \frac{s^2}{6N_2^2}(1 - \sin^2\alpha_{12}\sec^2\phi_2)\right]$$

$$\Delta\alpha = \Delta\lambda\sin\phi_m\sec\frac{\Delta\phi}{2} + \frac{\Delta\lambda^3}{12}\left(\sin\phi_m\sec\frac{\Delta\phi}{2} - \sin^3\phi_m\sec^3\frac{\Delta\phi}{2}\right)$$

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高斯中緯度反算

$$s = s_1 \, (s_1/2N_m) / \sin(s_1/2N_m)$$

$$\alpha_{12} = \tan^{-1}(X_1/X_2) - \Delta\alpha/2$$

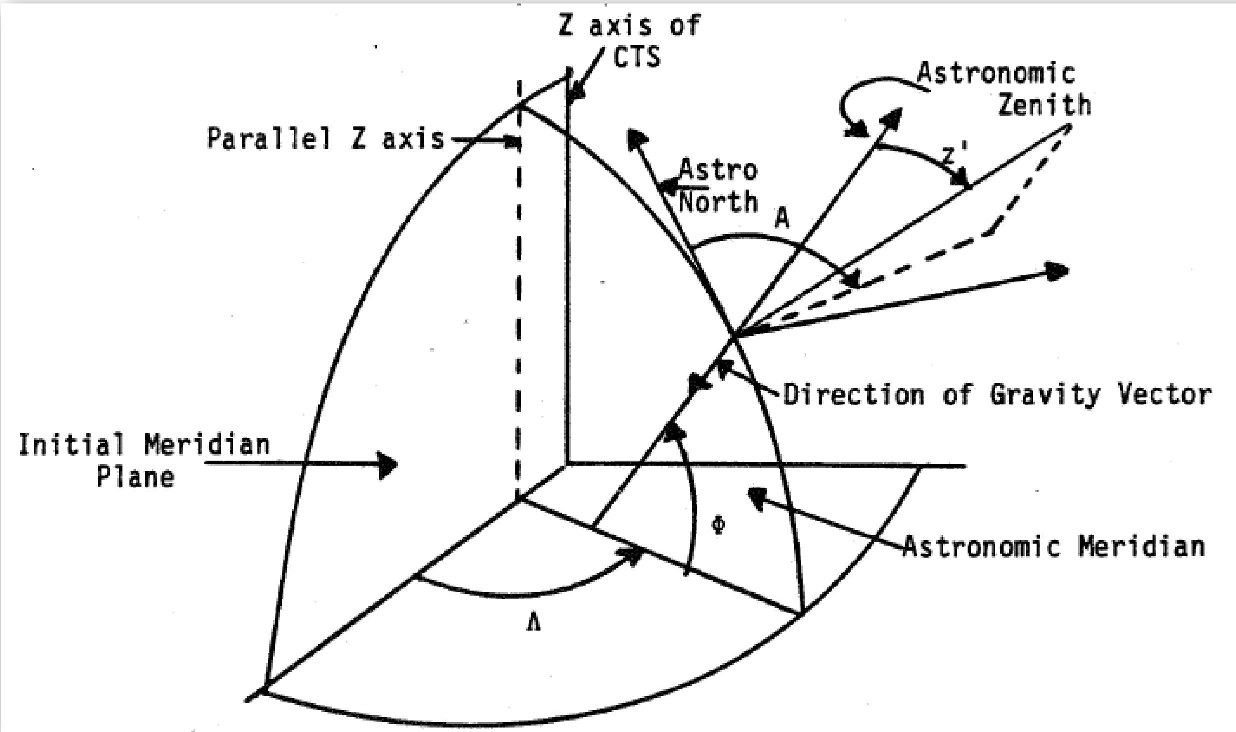
$$\alpha_{21} = \alpha_{12} + \Delta\alpha \pm 180^\circ$$

$$X_1 = s_1 \sin\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right) = N_m \Delta\lambda' \cos\phi_m$$

$$X_2 = s_1 \cos\left(\alpha_{12} + \frac{\Delta\alpha}{2}\right) = M_m \Delta\phi' \cos(\Delta\lambda/2)$$

$$s_1 = (X_1^2 + X_2^2)^{\frac{1}{2}}$$

天文方位角



瞬時與平均之化算

$$\Lambda_M = \Lambda_I - (x_p \sin\Lambda + y_p \cos\Lambda)\tan\Phi$$

$$\Phi_M = \Phi_I + y_p \sin\Lambda - x_p \cos\Lambda$$

$$A_M = A_I - (x_p \sin\Lambda + y_p \cos\Lambda)\sec\Phi$$

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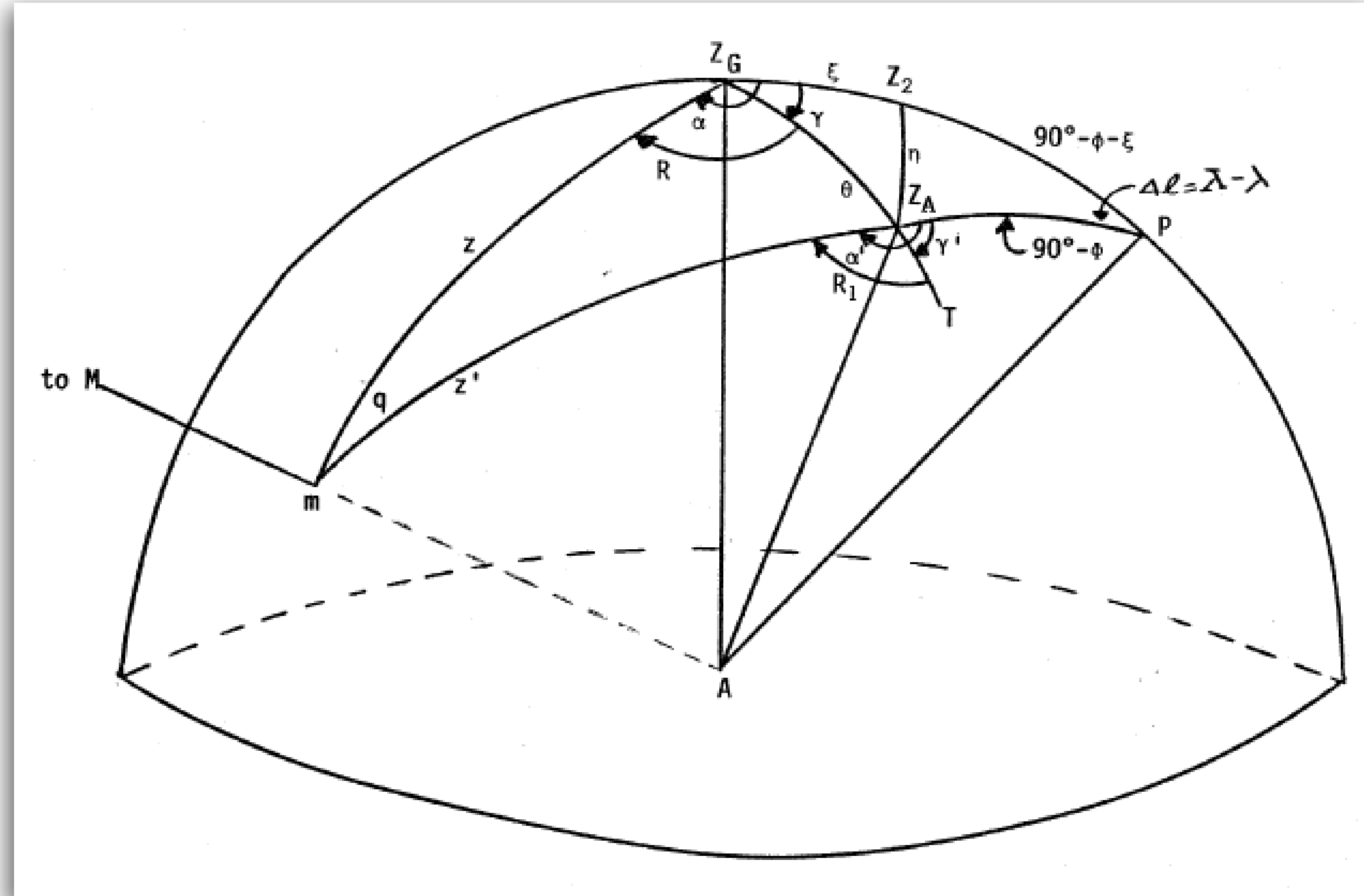
天文與大地觀測量之關係



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天文與大地觀測量之間的函數

$\xi = \Phi - \varphi$

$\eta = (\Lambda - \lambda)\cos\varphi$

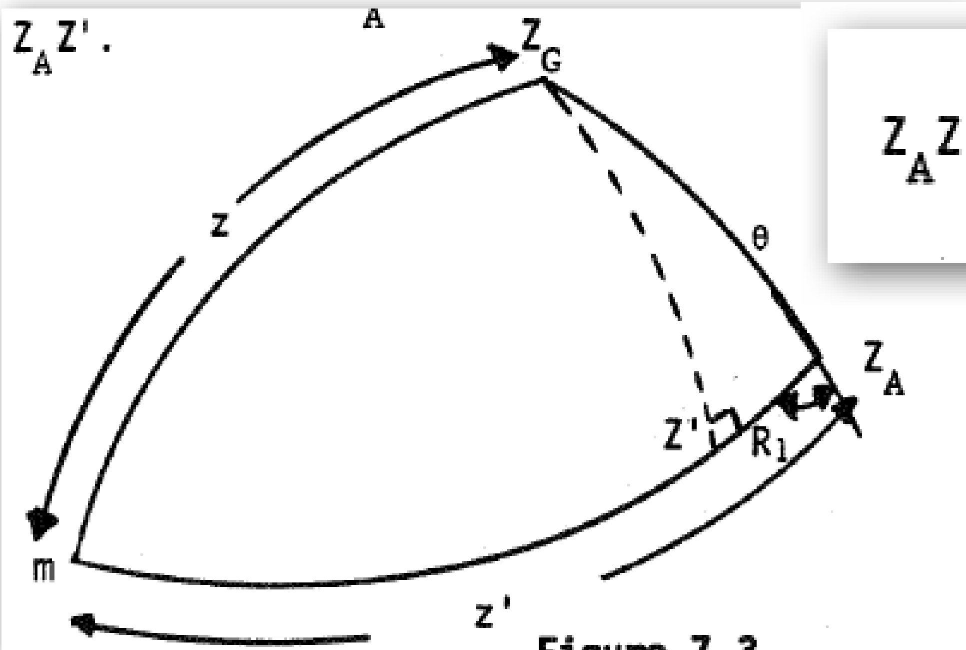
大地天文法
-垂線偏差(分量)

$$A - \alpha = (\Lambda - \lambda)\sin\phi + (\xi\sin\alpha - \eta\cos\alpha)\cot z$$

$$\alpha = A - \sin\alpha\cot z\,(\Phi - \phi) - (\sin\phi - \cos\phi\cos\alpha\cot z)(\Lambda - \lambda)$$

拉普拉斯(Laplace)方位角: 糾正業已扭曲之大地網系

天頂距與方向角的垂線偏差改正



$$Z_AZ' = z' - z = -(\xi \cos\alpha + \eta \sin\alpha)$$

大地量(法線系統)與
觀測量(垂線系統)之差異

$$D = D' + \delta$$

where

$$\delta = -(\xi \sin\alpha - \eta \cos\alpha) \cot z$$

截面差+
標高差+
垂線偏差=總改正

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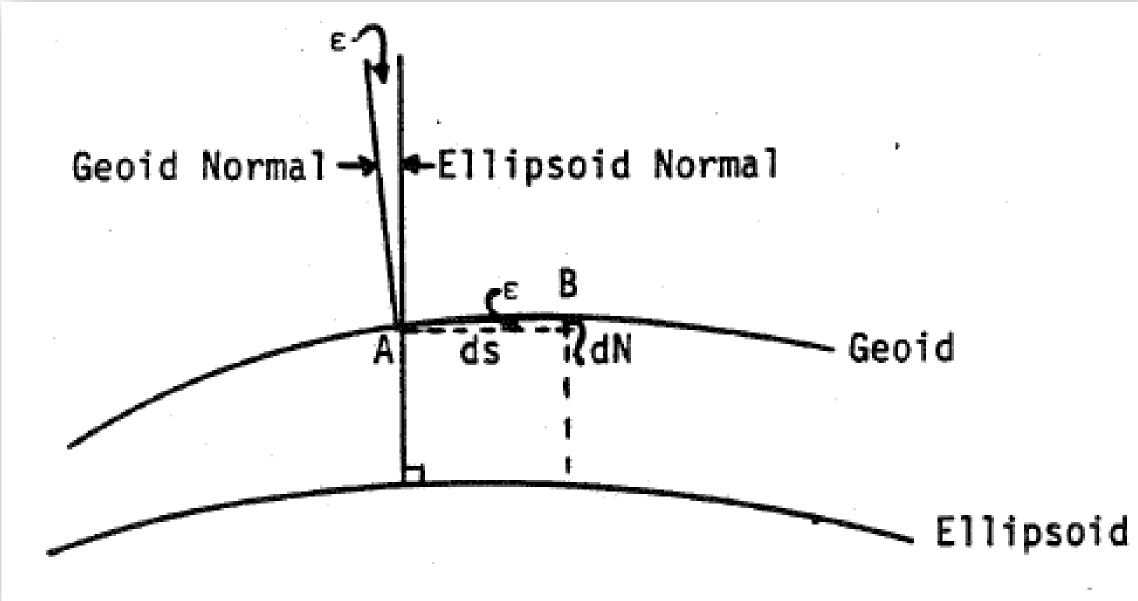
大地天文法之大地起伏

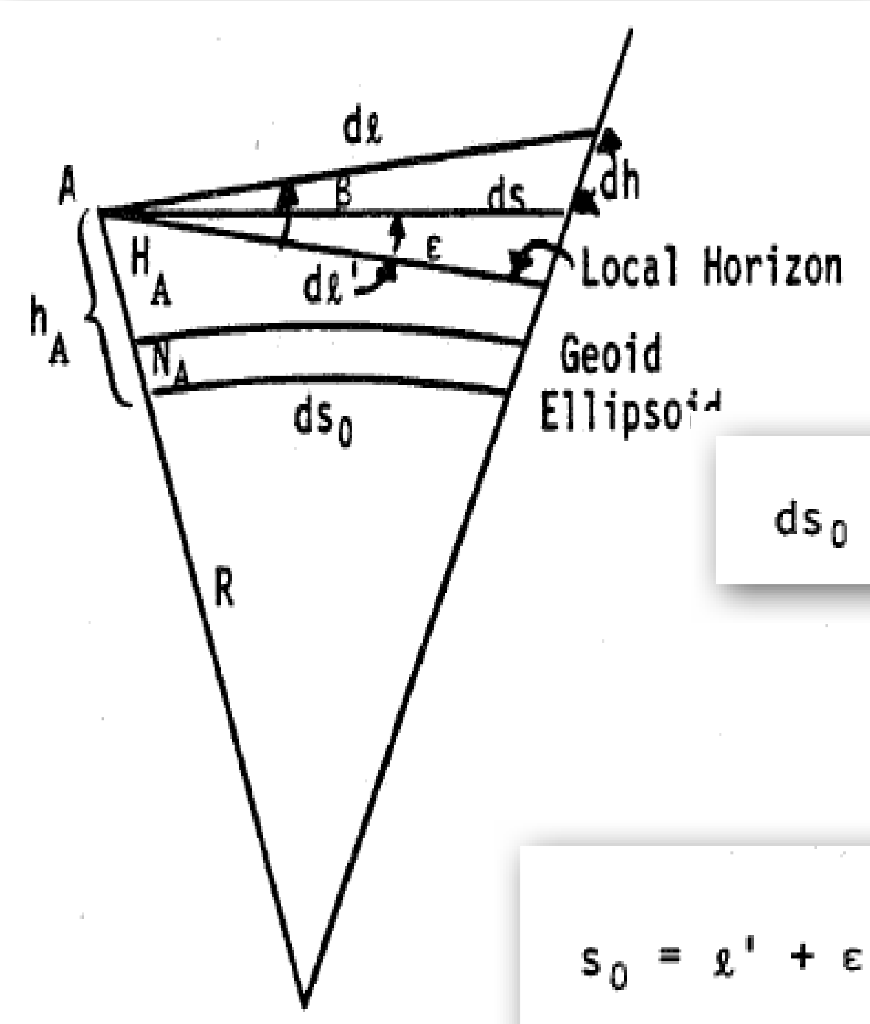


$$dN = -\varepsilon ds$$

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha$$

$$N_{ij} = -\frac{S_{ij}}{2}\left((\xi_i+\xi_j)\cos\alpha_{ij} + (\eta_i+\eta_j)\sin\alpha_{ij}\right)$$

觀測距離(基線)的化算



$$d\ell' = d\ell \cos\beta$$

$$ds_0 = ds - \frac{h}{R} ds_0 = d\ell' + \varepsilon dh - \frac{h}{R} ds_0$$

$$\varepsilon = \xi \cos\alpha + \eta \sin\alpha$$

$$s_0 = \ell' + \varepsilon_B h_B - \varepsilon_A h_A - h_m(\varepsilon_B - \varepsilon_A) - \frac{h_m}{R} s_0$$

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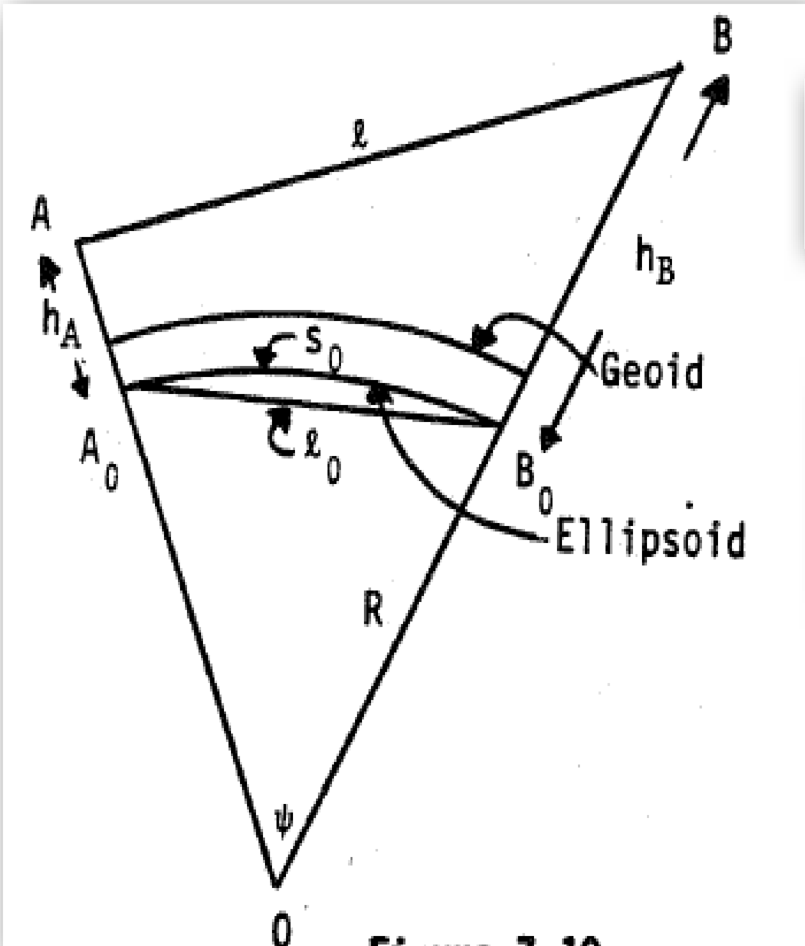
距離弦長的化算



$$\ell^2 = (R+h_A)^2 + (R+h_B)^2 - 2(R+h_A)(R+h_B)\cos\psi$$

$$\ell^2 = \Delta h^2 + \left(1 + \frac{h_A}{R}\right)\left(1 + \frac{h_B}{R}\right)\ell_0^2$$

$$\ell_0 = \sqrt{\frac{\ell^2 - \Delta h^2}{\left(1 + \frac{h_A}{R}\right)\left(1 + \frac{h_B}{R}\right)}}$$

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