# Evaluation of a GPS-Based Approach for Rapid and Precise Determination of Geodetic/Astronomical Azimuth

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**Abstract:** The rapid and precise determination of the azimuth information, purely based on the satellite positioning technology of global positioning system (GPS), is tested and evaluated by this paper. The main consideration of using GPS for the azimuth determination is expected to replace the traditional astronomical azimuth measurement, which is believed to be time-consuming and weather-dependent. A GPS approach is simply based on setting up the GPS receivers at the two ends of a baseline, recording and processing the phase observable, and converting its coordinate solution into an azimuth with the inverse geodetic formulas. This type of azimuth obtained by the GPS static solution has been assessed to be well consistent with the astronomical azimuth by a level of better than ±1″. The GPS kinematic mode of azimuth value, however, is biased from the standard value for a RMS error of ±9″. The correction of the deflection of the vertical, theoretically required by the geodetic and astronomical azimuth conversion, is also implemented and found to be only effective at the observation site where the $\eta \tan \varphi$ value is higher than the estimation accuracy of the $\eta$ value provided by a gravimetric geoid model.

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## Introduction

The information of azimuth provided by a precise measuring process, such as the traditional type of astronomical observation, has played an important role in both surveying engineering and navigation systems. One of these applications can be seen by the definition of a traditional geodetic datum, in which an initial azimuth is one of the basic elements and normally working with the triangular networks for geodetic computations (Chang and Tseng 1999). One other application in surveying is the Evötvös correction applied to airborne or shipboard gravity measurements with the information of latitude, velocity, and azimuth of the moving platform to compensate the inertial accelerations due to the earth rotation (Torge 1989).

The information of azimuth is also required by the satellite ground control antenna and the military searching radar for target trajectory tracking in near real time (Cicci 1999). In addition, as an effective navigation tool providing uninterrupted heading information for the vehicle, the gyroscope needs to be calibrated at a baseline where the azimuth is well maintained. It is also the initiative for this paper to propose and test a high efficiency and accuracy of azimuth measurement at any independent standalone vector, based on any up-to-date technique, for a variety of applications in surveying and navigation.

Although the new types of geodetic instrument, such as total station and global positioning system (GPS) receiver, have been adopted to improve the precision of the angle and time measurements (Balodimos et al. 2003), it is generally believed that astronomical observations are tedious on both fieldwork and computation. Therefore, an astronomical method for azimuth determination has not been developed by any study to ensure a "rapid" observation. Due to the fact that the azimuth values can be rigorously defined with the three-dimensional (3D) coordinates of the two ends of a baseline (Burkholder 1997), there is room for using GPS to conveniently determine the azimuth from one point to another with an accuracy of better than 1 arcsec (Evans et al. 1989; Evans and Stein, U.S. Patent No. 4,954,833, 1990). It is also known that different types of azimuth can be connected by the geometric formulas; it is also possible to convert the GPS-derived azimuth for various applications (Robitaille 1997).

Since the GPS-based 3D coordinates can be theoretically applied to compute the accurate azimuth value, the meridian convergence errors in geodetic problems can be detected using such a concept (Soler and Fury 2000). It is also found that a GPS-based multiantenna system has been widely installed to provide necessary information on position, velocity, time, and attitude (including azimuth-related parameter) for ship guidance and control (Ueno 2000). As the orientation information is always required by the military for many weapon platforms, some demonstrations have proved that GPS is capable of determining azimuth in real time (Hawker 1991; Vinnins and Gallop 1998).

The so-called GPS-based approach for azimuth determination, using GPS-derived coordinates referred to the WGS-84 ellipsoid at the two ends of a baseline, is comprehensively discussed in this paper. The test items consisted of the following:

1. Computation model based on topocentric (local) or geocentric geodetic coordinates;
2. Accuracy derived from static, rapid static, or kinematic GPS positioning solutions;

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3. Performance on different baseline lengths for 0.2, 7, and 18 km; and
4. Geodetic and astronomical azimuth conversion by using or not using corrections.

This GPS-based approach is expected to provide an acceptable accuracy for geodetic/astronomical azimuth measurements. When this proposed method is tested and confirmed, the GPS can rapidly provide high accuracy 3D coordinates and precisely determine the azimuth to effectively extend its applications in surveying and navigation.

## Azimuth Observations and Computation Models

### Astronomical Observation and Specification

The astronomical azimuth can be determined by a well-known type of observation called the time hour method and adopted by this study for fieldwork. This observation method is carried on using precise theodolite to take angle measurements between a reference star, such as the polaris, and a terrestrial target, and using the clock to mark the corresponding time. Based on knowing star position and angles at the time marked, the astronomical azimuth between the north celestial pole and the observed object can be obtained from two measurements, namely the polar azimuth and horizontal angle between the polaris and the target. It is noted that clear weather is restrictedly required by astronomical observation in order to sight stars at night. It is generally believed that its procedure is time consuming and demanding in both measurement and computation, leading to a difficult operation and a limited scale of application.

In practice, the corrections are also required for astronomical observation during the azimuth calculation. Those items normally include the correction of curvature, level error, diurnal aberration, height of the observed point, and polar motion. The measurements of astronomical azimuth must be made for at least two nights with 16 sets of observations each. When a rejection limit is set to be ±5″ for observation deviated from the set mean, at least 12 sets of acceptable observation are recommended for each night. Based on the national first-order specification, the astronomical azimuth computed by each night of observation is only allowed to differ from the two night average by less than ±1″. This value, hence, can be served as a useful quality control standard for any GPS-derived azimuth solution.

### Global Positioning System-Based Approach

It has been common to carry on positioning with GPS technique for its advantages of high accuracy, easy operation, and independence of weather conditions, which may be all beneficial to the azimuth determination if a computation model is also developed. Consequently, a GPS-based approach for determining azimuth, which is expected to be identical to the astronomical observation, is tested and evaluated by this paper.

From a technical point of view, this GPS-based approach has the advantages of shortening the observation duration, simplifying the data processing, and achieving a high accuracy, compared to the traditional astronomical observation. This method should also permit more than one of the targets observing at the same time for azimuth determination, based on GPS relative positioning. Fig. 1 depicts a typical configuration of three GPS receivers for use in the method determining a geodetic type of azimuth ($\alpha_G$). The GPS antennas set up at the standpoint and the target form a baseline to solve for its azimuth, in which their coordinates are originally provided and referred to the reference site through GPS data processing software. It is noted that the azimuth baseline distance is relatively not restricted, and the GPS receiver number can even be reduced to two if the coordinate of standpoint is accurately known.

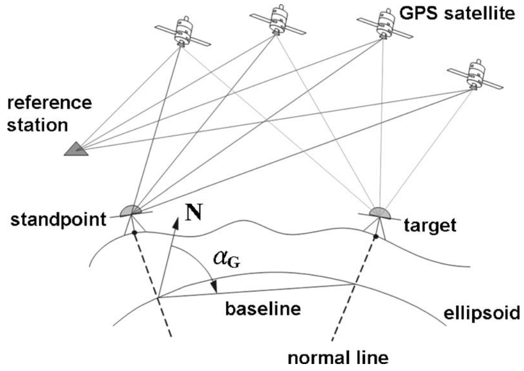

**Fig. 1.** Schematic diagram to global positioning system-based approach for azimuth determination

Once the earth-centered earth-fixed (ECEF) geocentric coordinate differences $(\Delta X, \Delta Y, \Delta Z)$ between the standpoint and target site are obtained, the components can be rotated to the local geodetic coordinates $(\Delta n, \Delta e, \Delta u)$ centered at standpoint. A plane type of azimuth $(\alpha_P)$ based on the topocentric coordinates can then be given as

$$\alpha_p = \tan^{-1}\frac{\Delta e}{\Delta n} \tag{1}$$

in which

$$\begin{bmatrix} \Delta n \\ \Delta e \\ \Delta u \end{bmatrix} = \begin{bmatrix} -\sin\varphi\cos\lambda & -\sin\varphi\sin\lambda & \cos\varphi \\ -\sin\lambda & \cos\lambda & 0 \\ \cos\varphi\cos\lambda & \cos\varphi\sin\lambda & \sin\varphi \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{2}$$

It is also believed that several small corrections are needed to convert this plane type of azimuth to a so-called geodetic azimuth, along the geodesic on the ellipsoid, by implementing a target height correction and the geodesic correction (Burkholder 1997).

To directly determine the geodetic azimuth, a set of computation formulas referred to the ellipsoid, such as the Gauss midlatitude method solving for the inverse geodetic problem (Rapp 1980), is adopted by this paper and carried out for the tests. The coordinate parameters used in the formulas are two points' geocentric geodetic coordinates $(\varphi, \lambda, h)$, which can be typically transformed from their ECEF coordinates $(X, Y, Z)$. The calculation of the geodetic azimuth $(\alpha_G)$ can then be expressed as

$$\alpha_G = \tan^{-1}\left(\frac{N_m \Delta\lambda' \cos\varphi_m}{M_m \Delta\varphi' \cos\dfrac{\Delta\lambda}{2}}\right) - \frac{1}{2}\left(\Delta\lambda \sin\varphi_m \sec\frac{\Delta\varphi}{2} + F\Delta\lambda^3\right) \tag{3a}$$

where

$$\Delta\varphi = \varphi_2 - \varphi_1; \quad \Delta\lambda = \lambda_2 - \lambda_1; \quad \varphi_m = (\varphi_1 + \varphi_2)/2 \tag{3b}$$

$$N_m = \frac{a}{(1 - e^2 \sin^2 \varphi_m)^{1/2}}; \quad M_m = \frac{a(1 - e^2)}{(1 - e^2 \sin^2 \varphi_m)^{3/2}} \tag{3c}$$

$$\Delta\varphi' = \Delta\varphi\left[\sin\left(\frac{\Delta\varphi}{2}\right)\Big/\left(\frac{\Delta\varphi}{2}\right)\right];$$

$$\Delta\lambda' = \Delta\lambda\left[\sin\left(\frac{\Delta\lambda}{2}\right)\Big/\left(\frac{\Delta\lambda}{2}\right)\right] \tag{3d}$$

$$F = \frac{1}{12}\sin\varphi_m\cos^2\varphi_m \tag{3e}$$

### Connection between Geodetic and Astronomical Azimuth

The azimuth obtained by astronomical observation is theoretically defined as the angle between vertical planes containing the north celestial pole and the observed object, whereas the geodetic azimuth is based on the angle in the plane perpendicular to the ellipsoid normal at the standpoint. However, it is realized that the two types of azimuth can be related through the so-called Laplace equation (Heiskanen and Moritz 1984). Once the north-south component ($\xi$) and the east-west component ($\eta$) of the deflection of the vertical are known at the standpoint, the astronomical azimuth ($\alpha_A$) and the geodetic azimuth ($\alpha_G$) can be connected by

$$\alpha_A = \alpha_G + \Delta\alpha \tag{4}$$

in detail

$$\Delta\alpha = \eta\tan\varphi + (\xi\sin\alpha - \eta\cos\alpha)\cot Z \tag{5}$$

in which $\varphi$=geodetic latitude obtained by GPS; and $Z$=angle representing the zenith distance. In many cases, $Z$ is close to 90°, leading the second term to be a small quantity and normally neglected. The azimuth conversion is, hence, written as

$$\alpha_A = \alpha_G + \eta\tan\varphi \tag{6}$$

Since the deflection of the vertical plays an important role in connecting the two types of azimuth, its two main components at the standpoint must be well provided. Theoretically, these values can be obtained by an operation of difference between the astronomical and geodetic coordinates. In addition, it is practical to simply calculate these values from a gravimetric geoid model by only inputting the points' geodetic coordinates. More recently, the accurate deflections of the vertical can also be obtained by combining GPS positioning with geodetic leveling (Soler et al. 1989; Evans 1991).

## Test Results and Analysis

### Global Positioning System Observation for Short Baseline

The GPS measurements made at one reference site and the two ends of a short baseline were carried out to enable the azimuth determination. This was to be achieved by testing the computation models which relied on a plane type and an ellipsoid type of geodetic coordinates, and using GPS solutions based on static, rapid static, and kinematic positioning techniques.

A 200 m short baseline located at a defense research institute in northern Taiwan was comprehensively examined. The selection of this baseline was based on the requirement that its azimuth has been measured periodically, in order to perform calibration for the in-house developed gyroscope. As the correlation between azimuth quality and sight length might be significant for this 200 m short baseline, any observation carried out on it is precautionary in terms of the optical plummets and collimation of the GPS antennas over the points. It was expected that insignificant differences would have occurred between the observed astronomical azimuth and the GPS-based azimuth and thus the standard azimuth value could be maintained more conveniently.

The GPS measurements and astronomical observations both took place in August 2003. The GPS campaign was carried out over 3-h observations using a 1-s data interval and a 10° elevation cutoff angle. The baseline observations were also linked to a nearby GPS tracking station, which is 2 km away and treated as the reference site. The GPS data sets were processed with the use of IGS precise ephemeris, $L1/L2$ phase combination, a modified Hopfield atmospheric model, and antenna phase center correction.

**Table 1.** Comparisons between GPS-Based and Astronomical Azimuth for Short Baseline

| Azimuth type | Computation parameters | Azimuth value (deg) | (min) | (sec) |
|---|---|---|---|---|
| Astronomical | Observed $\alpha_A$ | 226 | 10 | 22.624 |
| GPS-based | $(n, e, u)$ | 226 | 10 | 20.639 |
| | $(\varphi, \lambda, h)$ | 226 | 10 | 22.100 |
| Bias | $(n, e, u)$ | 0 | 0 | −1.985 |
| (GPS–Ast.) | $(\varphi, \lambda, h)$ | 0 | 0 | −0.524 |

### Azimuth Computations and Evaluations

It is generally believed that the static GPS with 2–4-h observations is able to provide a reliable coordinate solution. As a part of this study, a 3-h GPS solution was used to investigate the bias of azimuth determined at the selected short baseline. The indicator of the bias was estimated by comparing the GPS-based azimuth value with that derived from the astronomical observations and satisfied with the first-order specification of the azimuth determination.

Considering the different types of geodetic coordinate applied for the azimuth computation, two sets of azimuth values computed by using topocentric and geocentric geodetic coordinates, based on Eqs. (1) and (3), respectively, were obtained and tested to estimate the biases. The results derived from the tests are listed in Table 1.

The comparisons between the azimuth values based on different types of geodetic coordinate show a relatively large difference of 2″ in computation error, for even a short baseline. It is also found that the lowest bias can be obtained by using geocentric geodetic coordinates, in which the computation bias is estimated to be around 0.5″. If an average difference of ±1″ suggested by the first-order specification for astronomical observation is adopted, the test results appear to show that the GPS-based geodetic azimuth computed on the ellipsoid with the inverse geodetic formulas enables a reliable azimuth value to be provided.

For logistical and financial reasons, many surveys are also possibly operated with a 15–20-min observation to carry on a so-called rapid static GPS. Twelve subsets of GPS data, consisting of 15-min observation each, have been organized from the completed set of 3-h (180-min) data collected at the short baseline. In order to realize the bias of using rapid static GPS solution for azimuth determination, the computation was carried out with the ellipsoid coordinates and the results were compared with the astronomical value. These biases are listed in Table 2.

**Table 2.** Azimuth Biases Based on Rapid Static Global Positioning System for Short Baseline Test (sec)

| Session | Azimuth bias | Session | Azimuth bias |
|---|---|---|---|
| 1 | −1.477 | 7 | 1.103 |
| 2 | 0.561 | 8 | 3.008 |
| 3 | −1.066 | 9 | −1.739 |
| 4 | 3.271 | 10 | −1.066 |
| 5 | 2.187 | 11 | 0.018 |
| 6 | 0.018 | 12 | 1.103 |
| Absolute average bias | | 1.385 | |

For those biases estimated using a 15-min GPS solution, the values ranging ±3″ generally show a larger level of variation than that of using 3-h GPS observation. The average bias is estimated to be around 1.5″ for the absolute discrepancies, which is slightly worse than that of the astronomical observation requested by the specification. However, this level of performance still reveals its partial validation of determining azimuth with a rapid static mode of GPS positioning.

Based on the same GPS data set collected with a rate of 0.1 Hz at the short baseline, it is also possible to adopt a postprocessing mode of on-the-fly (OTF) technique to carry out ambiguity resolution and epoch solution. The baseline epoch solutions over the entire observation are shown in Fig. 2 for the deviations in three components.

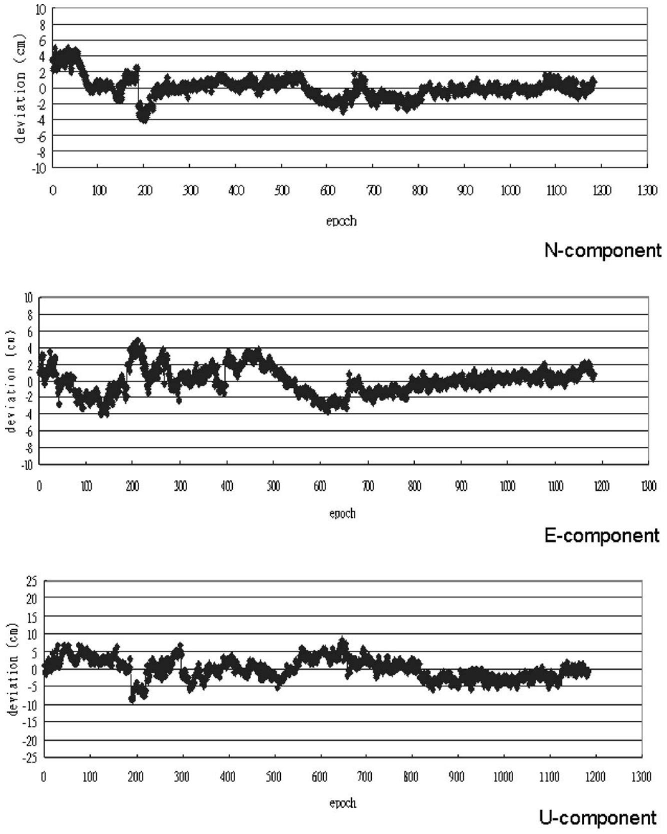

**Fig. 2.** On-the-fly epoch solutions in three components for short baseline test

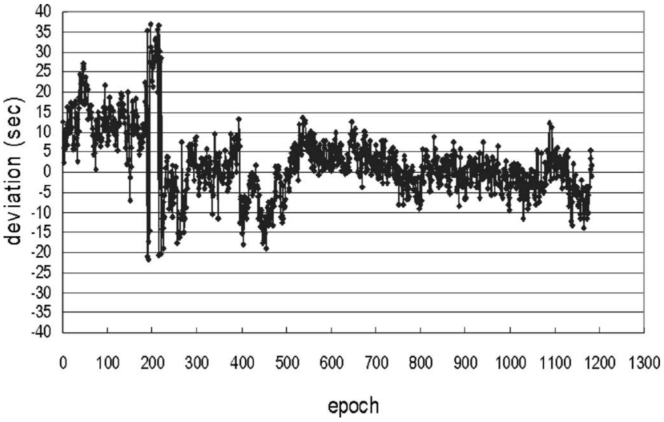

**Fig. 3.** Azimuth values determined by on-the-fly solution for short baseline test

It can be seen that the horizontal components of the OTF solution are varied within a ±5 cm level, and as expected the vertical precision of the epoch solution is slightly worse. As the static GPS baseline vector was also applied to make the comparisons, the root-mean-square (RMS) error of the OTF baseline solution can also be obtained. The average RMS errors have been estimated to be 1.7, 2.0, and 4.4 cm for the component in North, East, and Up, respectively. This level of accuracy demonstrates that the OTF epoch solution is generally realistic and enables one to determine azimuths for certain kinds of application.

The OTF epoch solution has been practically applied to evaluate the results based on the azimuth determination algorithm proposed. The GPS-based azimuth values computed by the OTF epoch solution are plotted in Fig. 3 for the deviations from the standard value. The statistical results are also listed in Table 3 to realize the accuracy performance.

It is noted that the scale in azimuth variation shows a range of ±37″, and the RMS error is estimated to be around ±9″. Table 3 containing the statistics for the azimuth bias clearly exhibits that more than 50% of the OTF azimuth solution are biased by less than ±5″, while 95% (2σ) of the biases are better than ±20″.

It is worth noting that the use of GPS solution in the azimuth determination would require careful consideration on its computation model, positioning mode, or accuracy requirement, as the accuracy performance of the GPS-based azimuth has been tested to be different even for a 200 m short baseline.

**Table 3.** Statistical Azimuth Biases Based on On-the-Fly Epoch Solution for Short Baseline

| Bias (″) | Occurrence (%) | Accumulation (%) |
|---|---|---|
| ±1.5 | 20.2 | 20.2 |
| ±3.0 | 16.8 | 37.0 |
| ±5.0 | 18.4 | 55.4 |
| ±10.0 | 24.5 | 79.9 |
| ±20.0 | 16.1 | 96.0 |
| ±35.0 | 3.0 | 99.0 |
| ±40.0 | 1.0 | 100.0 |
| RMS | ±8.693″ | |

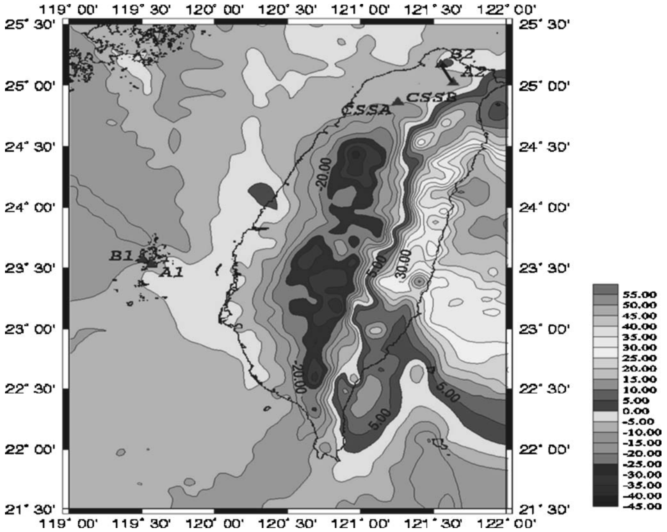

**Fig. 4.** Three test baseline locations and corresponding $\eta$ values in Taiwan

### Azimuth Conversion for Three Baselines

The azimuth determination has been proved to be effective by using a 3-h GPS data set recorded at the short baseline. It is also believed that the GPS-based approach, which relied on the accurate geodetic coordinates and the inverse geodetic formulas, is capable of determining the azimuth for certain lengths of the baseline. In order to test this, two other longer baselines, whose astronomical azimuths and GPS-based geodetic coordinates have been precisely observed, are also selected and investigated.

Considering the correction related to the deflection of the vertical and required by the conversion between the GPS-based geodetic azimuth and the observed astronomical azimuth, a further investigation was carried out by applying, and not applying, this correction to reduce the discrepancies between the two. To first realize the values of the deflection of the vertical in the east-west component ($\eta$) distributed in the Taiwan area, Fig. 4 is drawn and provided with the location of three baselines tested. The test results based on either applying, or not applying, the correction to the GPS-derived geodetic azimuth, as in Eq. (6), are presented with the bias estimates and listed in Table 4.

It is noted from Table 4 that a larger range of the bias, approximately 2–3″ caused by applying the correction to the GPS-derived geodetic azimuth, exists between the two types of azimuth at all three baselines. This group of azimuth discrepancies also appears to have a systematic-like bias, as those values are all negative when corrections were made. However, the results can still be found to be more random and better agreed on by less than ±1″, when the corrections were not practically made for azimuth conversion. It also briefly shows a trend that the bias would increase when the baseline length is extended.

The test results basically are not able to perfectly prove the effectiveness of the use of the deflection of the vertical in the azimuth conversion process. In view that a gravimetric geoid model has been used to compute the $\eta$ values, one might assume that the estimation of the deflection of the vertical is not accurate enough for azimuth conversion. This assumption could be approved as the precision of the geoid model was tested to be around 2–7″ over the Taiwan area (Hwang 1997).

To provide an accuracy analysis for the expected astronomical azimuth error as a function of the modeling error for the deflections of the vertical, the error propagation is worked with Eq. (6) and expressed as

$$\sigma^2_{\alpha_A} = \sigma^2_{\alpha_G} + (\eta \sec^2 \varphi)^2 \sigma^2_{\varphi} + (\tan \varphi)^2 \sigma^2_{\eta} \qquad (7)$$

in which $\sigma$ represents the standard deviation of the variables. It is clear to see that the differences between the astronomical and geodetic azimuths after transforming and comparing are mainly due to the errors introduced by the modeled deflections of the vertical ($\sigma_\eta$), as the GPS-based latitude error ($\sigma_\phi$) is relatively small.

It is also realized from Fig. 4 that the east-west component of the deflection of the vertical is estimated to be 5–10″ for three test baselines. When an average latitude of 24° in the Taiwan area is applied to the correction item, the azimuth conversion based on $\eta \tan \varphi$ in Eq. (6) presents an even smaller range of the values, varying from 2.2–4.5″, for the three baselines. This level of correction might not be suitable for practical connecting the geodetic and astronomical azimuth, as a maximum error of 7″ could exist in the estimation of the $\eta$ values and lead to an error estimate of 3.1″, based on Eq. (7), in azimuth conversion. However, by applying the correction function a better agreement in azimuth conversion is still expected for some areas, such as the eastern and central part of the island. This corresponds to the fact that the areas have the largest amount (>30″) of the $\eta$ values, as shown in Fig. 4.

## Concluding Remarks

This paper has discussed the test results of the application of a GPS-based approach for the rapid and precise determinations of the geodetic or astronomical azimuth, in terms of the computation model, accuracy performance, and conversion effect. Some concluding remarks on this subject can be drawn as follows:

1. The short baseline test results showed that the computation model has a nonignored effect on the azimuth determination. A small bias of only 0.5″ was found for the GPS-based azi-

**Table 4.** Conversion between Astronomical and GPS-Based Azimuth for Three Baselines

| Item | | Baseline CSSA–CSSB | Baseline A1–B1 | Baseline A2–B2 |
|---|---|---|---|---|
| Baseline length (km) | | 0.3 | 6.9 | 18.3 |
| Astronomical azimuth | | 226° 10′ 22.624″ | 305° 27′ 31.780″ | 333° 09′ 16.332″ |
| Correction applied | GPS-based azimuth | 226° 10′ 19.277″ | 305° 27′ 29.742″ | 333° 09′ 14.056″ |
| | Bias (GPS–Ast.) | −3.347″ | −2.038″ | −2.276″ |
| Correction not applied | GPS-based azimuth | 226° 10′ 22.100″ | 305° 27′ 32.395″ | 333° 09′ 17.079″ |
| | Bias (GPS—Ast.) | −0.524″ | 0.615″ | 0.747″ |

muth when a computation model based on geocentric geodetic coordinates and inverse geodetic formulas was used.

2. Over a 3-h GPS measurement period made at the short baseline, there was a difference in azimuth solution by using static or rapid static GPS, with 180- or 15-min session lengths, respectively. The azimuth biases based on rapid static GPS solution were possibly up to 3″ and averaged to be 1.5″.
3. Based on a postprocessing OTF mode, the GPS epoch solution provided the azimuth with a bias variation of ±37″ and a RMS error of ±9″. It was also found that more than 50% of the OTF epoch solution shows an azimuth bias of less than ±5″, while 95% ($2\sigma$) of the biases are better than ±20″.
4. The test results based on two medium ranges of the baselines proved that the GPS approach offers a reliable azimuth solution with a bias of around 0.7″. However, the conversion between the geodetic and astronomical azimuth was not confirmed, as the agreement was degraded to approximately 2″ when the correction based on the east-west component of the deflection of the vertical ($\eta$) was applied.
5. The uncertainty of applying the correction to better improve the consistency between the geodetic and astronomical azimuth could be caused by not enough precision of the geoid model, significant gradients of the geoid, a small amount of the $\eta$ value, and mitigation factor ($\tan\varphi$) of the correction. It is known that high density of gravity data, ellipsoid heights and elevation differences, Laplace observations, or azimuth observations documented in this paper might all contribute to a better geoid model for the island. It is expected to ensure the conversion when a "newer" geoid model is developed soon to provide more accurate $\eta$ values.
6. If an estimation bias of ±1″ suggested by the first-order specification for astronomical observation is adopted, the effectiveness of using a GPS-based approach for the rapid and precise determination of the azimuth can be approved by the tests. However, it is worth noting that the use of GPS in azimuth determination would still require careful consideration on computation model, positioning mode, and conversion correction.

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