Estimation of Horizontal Movement Function for Geodetic- or Mapping-Oriented Maintenance in the Taiwan Area

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Abstract. The concept of dynamic datum, in which the coordinates of sites change as a function of time, was relied on to estimate the regional land movement functions for any geodetic or mapping applications. The test area was selected for a deformation active island, Taiwan, and categorized into several regions depending on the similar pattern of the horizontal land movement. The velocity field computed for the Taiwan area was based on the coordinates of first-order control stations measured in 1997 and 2000, both using high precision GPS. The regional movement functions derived from the velocity information can then be estimated with several mathematical models, e.g. averaging, weighted mean and surface fitting. Furthermore, those regional functions were practically applied to estimate and examine the horizontal coordinate movement in the test regions. It is further expected to utilize those regional functions to assess the coordinate qualities of the control stations over a period of time, in order to determine an applicable duration for the geodetic maintenance. It is realized from the internal errors that the horizontal movement velocity obtained using the regional functions based on a second-degree surface fitting model are tested to be better than 0.4 cm/yr. When these regional functions were tested with another independent set of GPS-based coordinates measured in 2002, the external estimation errors of those regional functions were assessed to be approximately 1.7 cm/yr in all test regions. It is, hence, suggested that the coordinate accuracy of the first-order GPS control stations in the Taiwan area can only be maintained for no more than 4 years. In addition, the revision of 1/1,000 digital maps is estimated to be probably required for every 10 years when regional land movement, instead of the topographic changes, is taken into account.

Keywords. Velocity field, land movement, regional function

1 Introduction

For the establishment of the geodetic networks, the Global Positioning System (GPS) is particularly important for its great performances on the working scale from local, regional, to global. In addition, the GPS receivers are relatively easy to operate so that the geodetic observations using GPS can be effectively implemented. Most importantly, millimeter to centimeter level positioning accuracy has been widely demonstrated over the baseline lengths from tens to thousands of kilometers. With this space geodetic technique, it has been proved to be feasible to establish a regional or continental scale of reference system by GPS observations.

It is generally believed that once a GPS-based reference system is established in a crustal deformation or tectonic plate motion area, such as Taiwan, a procedure linked to its regular maintenance must be implemented. Its purpose is to ensure the position accuracy of GPS control station that might be degraded by any natural hazards or effects, e.g. earthquakes. If regular maintenance is not properly carried out for those GPS sites, the geocentric reference system based on this fundamental GPS network will be distorted or may even be no longer reliable for any geodetic or mapping applications. Additionally, if the operation of geodetic maintenance is frequently made for those GPS sites, it would jeopardize the public administration works for land planning and management, as the coordinates of the control stations are changed too often. Therefore, the land movement in the area of interest is investigated, in order to determine a suitable frequency for maintaining the fundamental GPS network to ensure accurate positions.

The geocentric reference system of Taiwan Geodetic Datum 1997 (TWD97), whose three-dimensional coordinates were established using GPS and connected to sixty-four IGS (International GNSS Service) sites and referred to the ITRF94 (International Terrestrial Reference Frame 1994), has acted as the geodetic datum in the Taiwan area since 1997 (Chang and Tseng (1999)). In addition, one other data set measured at the TWD97 first-order control stations was provided in 2000, as a part of government re-construction work following the 1999 Taiwan earthquake (Mw=7.6). It is possible to realize the time evolution of this reference system and to study its maintenance period through the estimation of the regional land movement functions for the Taiwan area.

2 Regional Movement Function

A dynamic mode of geodetic datum is believed to be capable of describing the time evolution of land movement for an active deforming area (Blick et al. (2003)). Based on this concept, it can offer users the spatial and temporal information for three-dimensional coordinates with time variants for the geodetic control stations. For its definition, three components are required to represent such a dynamic datum: a set of geodetic observations for the network of sites; a rigorous adjustment of the geodetic observations; and a procedure for modeling a time varying function of the site coordinates (Tregoning and Jackson (1999)).

To investigate this four-dimensional type of datum, the velocity field calculated from the geodetic observations must be accurately determined. In such cases, it may be more appropriate to adopt discrete site velocities estimated from GPS measurements made at well-distributed control stations. It is also noted that when any regional movement function is available to well represent the trend of horizontal movement within an area, it can allow users to compute site displacement at any epoch by simply inputting the site's geographic location and time span. The information provided by the regional movement function would be helpful to easily judge how long the coordinate accuracy can be maintained and what duration the geodetic measurements need to be re-observed.

Since a variety of mathematical algorithms has been suggested by some surveying applications (Wielgosz, et al. (2003))(Yanalak and Baykal (2001)), four basic models, namely the averaging, weighted mean, first-degree and second-degree polynomial surface fitting, are applied and tested for their effectiveness.

For those models, the most simplified form can be based on the algorithm of averaging, i.e.

$$\overline{V} = \frac{V_1 + V_2 + \cdots + V_n}{n} \tag{1}$$

where $\overline{V}$ is an average value of velocity field in the region; V_i is the site velocity at the i-th data point; and n is the total number of data point in the region.

It is noted that the main advantage of this model is easy to be estimated and utilized, if a common pattern of the land movement is existed in the investigated area. However, it will be a location-independent value for all sites within the region.

The model of weighted mean can also be designed, based on the velocities known at the nearby data points, to obtain the site velocity at the computed point by means of an interpolation with the distance weight, i.e.

$$V_s = \frac{\sum_{i=1}^{n} P_i V_i}{\sum_{i=1}^{n} P_i} \tag{2}$$

where V_s is the estimated velocity at the interpolation point; V_i is the velocity of the i-th data point; n is the total number of data point within the region; and P_i is the weight which can be simply given by $P_i=1/d_i$ with the distance (d_i) between the data point and the interpolation point. This model, also named as distance weighting, has the advantages: (1) A regional pattern of the land movement can be presented since a limited range of data points has been applied to estimate the velocity information for the region; (2) The number of the data point in the velocity database can be flexibly adopted for the computations.

Moreover, it is able to apply a surface fitting model to establish a velocity-based regional function to estimate land movement in the area. Two basic types of surface fitting model are proposed for the study.

(1) First-degree Polynomial (Plane Fitting)

It is able to utilize a plane type of model to approximate the land movement for a small or flat area. It can be assumed that a relationship, as listed below, is existed between the velocity component (V_i) and the plane coordinate of data point (x_i, y_i), where a_0, a_1, a_2 are model parameters:

$$V_i = a_0 + a_1 x_i + a_2 y_i \tag{3}$$

(2) Second-degree Polynomial (Surface Fitting)
A curve type of model, based on a second-degree polynomial, is also available to estimate the land movement at expected site when those data points' velocities are well fitted. As listed below, six parameters (a_0, $a_1 \ldots a_5$) need to be defined by:

$$V_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i \tag{4}$$

3 Estimation of Horizontal Movement

3.1 Zoning Design

The data used to test models is mainly composed of the high accuracy 3-D coordinates measured by the government at a total of 105 first-order GPS control stations well distributed in the Taiwan area. The network adjusted coordinates were obtained from three independent GPS campaigns at epochs of 1997.0, 2000.4 and 2002.0. The coordinate differences in horizontal components, based on the comparisons between any two consecutive data sets, were then applied to either estimate the function parameters or test the estimation errors for the horizontal displacement (see Fig. 1). Due to the fact that a part of the control network was found to have significant displacement caused by the damaging earthquake that occurred in central Taiwan in September 1999 (Chang (2000)), this area was not included in the analysis. A total of five zones were categorized by the same geological regions and the similar trends of horizontal displacement (see Fig. 2). The first set of station displacement obtained for the period 1997.0 to 2000.4 at the selected sites are shown in Fig. 3.

It can be clearly seen in Fig. 3 that most of sites in Northern Taiwan, i.e. Zone I and II , shifted in a southeastern direction, whereas the sites to the South and Southeast appeared to have a more complicated displacement pattern. It does, however, indicate a similar trend of displacement stowards the southeast and southwest in Zone III and IV, respectively. It is also possible to see northwestern displacements in Zone V, although it is inconsistent in rate as the sites are located within a plate collision area.

It can be also seen that if site velocities are individually investigated, horizontal displacement trends might be too variable for a uniform model to be determined. For this reason this study attempts to simplify the estimation of the horizontal displacements using regional functions.

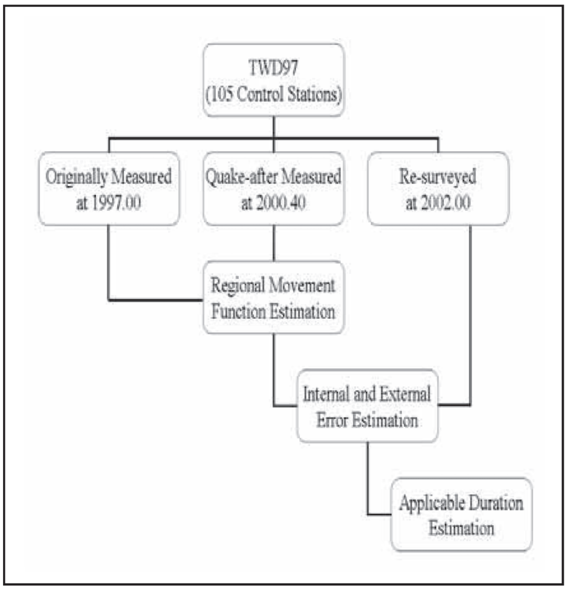

Fig. 1 Data Sets Applied and Analyzed

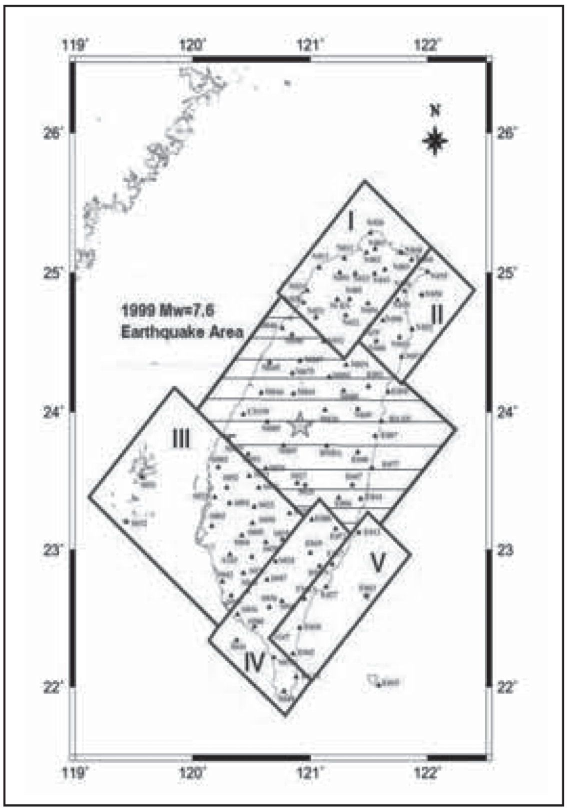

Fig. 2 Zoning Design for Five Test Regions (Excluding Earthquake Area in Central Taiwan)

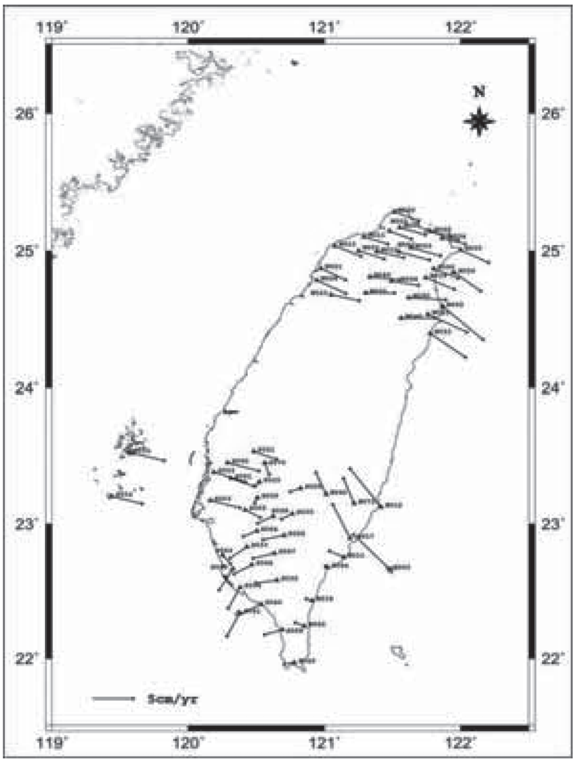

Fig. 3 Horizontal Displacement (1997.0-2000.4)

Table 1. Internal Errors of the Regional Functions (unit: cm/yr)

Zone		Averaging	Weighted Mean	Plane Fitting	Surface Fitting
I	E	0.53	0.35	0.24	0.24
	N	0.41	0.38	0.35	0.26
	L	0.47	0.35	0.24	0.24
II	E	0.68	0.56	0.44	0.38
	N	1.38	1.44	0.59	0.29
	L	1.00	0.94	0.56	0.44
III	E	2.50	2.15	1.21	0.88
	N	0.65	0.59	0.53	0.29
	L	1.09	1.03	0.79	0.68
IV	E	0.62	0.59	0.47	0.06
	N	2.09	1.68	0.35	0.24
	L	0.68	0.68	0.44	0.09
V	E	1.47	1.53	0.44	0.18
	N	2.26	2.24	1.06	0.65
	L	2.56	2.53	1.03	0.65
Avg	E	1.15	1.03	0.97	0.35
	N	1.35	1.26	1.09	0.35
	L	1.15	1.09	1.00	0.41

3.2 Estimation of Model Errors

Based on the data, the horizontal land displacements over a time span of 3.40 years, as shown in Fig. 3, can be used to estimate the site velocities for every data point. The four models were then tested using the data from each zone, i.e. a total of 21, 9, 20, 9 and 7 points in Zone I-V respectively. The internal errors of the regional functions established were further assessed by comparing data points' measured and estimated velocities. The RMS errors of the regional function estimates are listed in Table 1, in which the E, N and L stand for the horizontal component in easting, northing and $\sqrt{E^2+N^2}$, respectively.

It can be found in Table 1 that the averaging, weighted mean and first-degree surface fitting models are not satisfactory to provide the best velocity estimates due to their larger RMS errors. On the whole, the surface fitting model using the second-degree polynomial shows the smallest error of the displacement estimate in all test regions, in which an average of 0.4 cm/yr is obtained. However, it is also seen in Zone III that a larger error of around 0.9 cm/yr still occurred on the E-component.

Furthermore, to check with the external errors of the regional functions, one other independent GPS data set at epoch of 2002.0 was tested. The site velocities were computed using two sets of coordinate over a time span of 5.0 years, i.e. from 1997.0 to 2002.0. The site velocities were then also derived using the four regional functions, and compared with those measured to determine estimation errors. As those measured velocities are based on a period of 5 years, an extension of 1.6 years from the fitting function data, this set of external errors, as shown in Table 2, indicates the capability of using the regional functions to represent the land displacement to be occurred.

It can be seen in Table 2 that the second-degree polynomials once again provide the best fitted velocities for the points within the tests regions, although the differences between the model estimates are less significant. In addition, it is found that the external errors are generally larger than those of the internal, e.g. the smallest error of the function estimate is increased from 0.4 cm/yr to 1.7 cm/yr, due to the time span extension. For the increasing errors more significantly occurred in Zone II, it might show an irregular movement trend happened in this area.

Table 2. External Errors of the Regional Functions (unit: cm/yr)

Zone		Averaging	Weighted Mean	Plane Fitting	Surface Fitting
I	E	0.96	0.96	1.02	1.02
	N	0.86	0.86	0.86	0.80
	L	1.14	1.14	1.20	1.20
II	E	2.00	1.92	1.80	1.82
	N	2.50	2.52	2.16	2.06
	L	2.84	2.76	2.56	2.58
III	E	3.14	2.72	1.74	1.36
	N	1.48	1.46	1.36	1.26
	L	1.52	1.62	1.56	1.50
IV	E	1.42	1.30	1.34	1.26
	N	2.58	2.14	1.30	1.26
	L	1.58	1.62	1.60	1.64
V	E	2.38	2.52	1.54	1.44
	N	2.66	2.58	0.90	0.72
	L	3.22	3.32	1.74	1.50
Avg	E	1.98	1.88	1.50	1.38
	N	2.02	1.92	1.32	1.22
	L	2.06	2.10	1.74	1.68

4 Applicable Durations

4.1 Datum Maintenance

Based on the regional functions fitted to express the movement velocities in the test regions, the estimation of an applicable duration for the re-survey of the geodetic control stations has become possible. Using the elements of the variance-covariance matrix constructed by the network adjustment, the original coordinate precision, i.e. 1σ, in TWD97 can be realized to be 0.3 cm and 0.6 cm for the latitudinal and longitudinal components, respectively. When a 95% confidence circle describing the horizontal accuracy of points was introduced, its radius can serve as a boundary value to judge if any significant site movements occurred (Geomatics Canada (1996)). It has been calculated for the first-order GPS control stations in TWD97 that a tolerance value of 3.4 cm in horizontal component was requested to meet the original point accuracy.

Moreover, due to the existence of the estimation errors in the regional functions, the uncertainties have to be taken into account. When the best fitting model was applied to estimate the applicable duration for the first-order control stations, the movement allowances were designed to be between 3.4 cm + 0.41 cm/yr and 3.4 cm + 1.68 cm/yr for the inclusion of two types of estimation errors (see Fig. 4). The regional movement accumulated over 4 years in the test regions are also derived using the best fitting model and shown in Fig. 5.

Using information provided by Fig. 4 and Fig. 5, the two boundaries of the applicable duration can be estimated and listed in Table 3, along with the annual velocity, for the test regions in Taiwan.

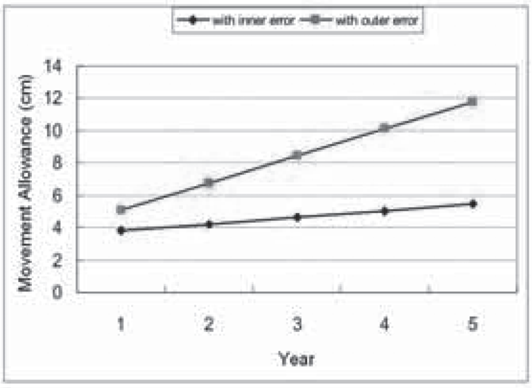

Fig. 4 Movement Allowances for the Control Stations

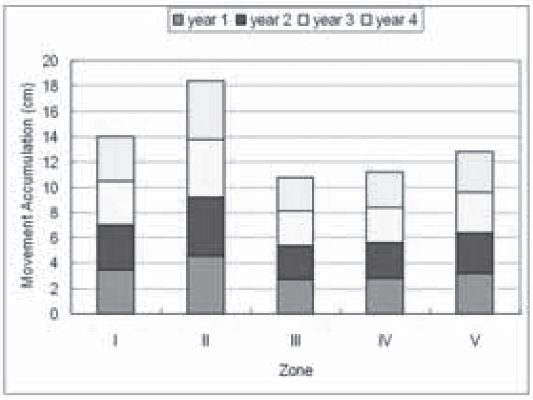

Fig. 5 Regional Movement Accumulated for Four Years

Table 3. Applicable Durations Estimated for First-order GPS Control Stations

Zone	Velocity (cm/yr)	Duration (yr)
I	3.5	2
II	4.6	1-2
III	2.7	2-4
IV	2.8	2-4
V	3.2	2-3

It can be generally seen in Table 3 that the first-order horizontal coordinate accuracy can be confirmed for less than 4 years in Taiwan. This is, of course, highly related to the significant site movement with an average velocity of more than 3 cm/yr. It is, hence, suggested to set up a maintenance procedure to ensure high accuracy of geodetic applications in this area

4.2 Map Revision

Similarly, the regional function based on the second-degree surface fitting model can also be applied to estimate the duration for map revision. According to the nation's mapping standard, any plane error over 0.2 mm shown on the maps is not allowed for all scales of mapping. In other words, a regional movement of 20 cm was realized to be the error allowance for digital mapping on a scale of 1/1,000. If the internal and external estimation errors were also counted, the allowances between 20 cm + 0.41 cm/yr and 20 cm + 1.68 cm/yr can be set up. Based on such information, the applicable durations for 1/1,000 digital maps are estimated and listed in Table 4 for the test regions in Taiwan.

Table 4. Applicable Durations Estimated for 1/1,000 Digital Maps

Zone	Velocity (cm/yr)	Duration (yr)
I	3.5	7-11
II	4.6	5-7
III	2.7	9-27
IV	2.8	9-24
V	3.2	8-14

When the most detailed topographic maps currently utilized in Taiwan were estimated for their applicable duration, it is known from Table 4 that the map revision cycles vary widely from 5 years in Zone II to probably 27 years in Zone III. If a lower estimation error, i.e. 20 cm + 0.41 cm/yr, is adopted, a reduced cycle of approximately 10 years is still necessary. However, it must be emphasized that such long cycle of map revision is only based on the horizontal land movement. When any topographic or geomorphic changes are included, the map revision cycle must be accordingly shortened to carry on the cartographic work.

5 Conclusions and Suggestions

Two sets of high precision GPS coordinate data measured in 1997.00 and 2000.40 were adopted to compute the site movement velocity for a part of the first-order control stations in Taiwan. The regional functions were then fitted using four mathematical models. Based on the precision indicators, the second-degree surface fitting model provided the best velocity estimates with an average of 3.4 cm/yr and the variations from 2.7 to 4.6 cm/yr in five categorized zones. Moreover, the lowest estimation error of the regional function was tested to be around 0.4 cm/yr or 1.7 cm/yr, depending on an internal (1997.0-2000.4) or external (1997.0-2002.0) data set assessed.

The regional functions were further applied to estimate the applicable durations specifically for the first-order control stations and 1/1,000 digital maps in Taiwan. Under a specific level of accuracy standard and estimation uncertainty, it is suggested that a suitable maintenance period of 4 years for the control stations and 10 years for the 1/1,000 digital maps is required, both based on the use of the regional movement functions estimated.

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