

PERFORMANCE OF HIGH RATE INTERPOLATED DATA APPLIED TO GPS KINEMATIC POSITIONING

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ABSTRACT

The GPS observables collected at reference stations for the post-processing type of kinematic positioning are likely to be provided by a public data service on a routine basis with a large sampling interval of 30 seconds. This low-rate data might lead to a non-matching observation with the rovers recording the trajectories with a high data rate of 1 Hz or even greater. To ensure the differential solutions achieved, GPS observables are required to be the same data interval for the two baseline stations. A two-step interpolation technique is proposed to work for GPS data densification to strengthen data availability and reduce the cost of taking any supplementary observation. A curve fitting function, with a linear correction, is applied to interpolate both the phase and the range observables into a higher rate data set for GPS kinematic positioning. It is found that a third order polynomial and a linear correction can work properly to increase the applicable data rate from 30 sec to 1 sec for kinematic positioning. An external accuracy of 0.5 cm in plan and 2.4 cm in height is obtained from a short baseline solution. Another medium baseline solution also confirms that the external accuracies of 1.3 cm in plan and 7.6 cm in height can be achieved using a 1 sec interval of interpolated observables as opposed to raw data. A two-hour static data set is also applied to test for the stand-alone kinematic positioning, namely the precise point positioning (PPP). The PPP solution demonstrates that the N-S component of the accuracy is significantly improved from 34 cm to 15 cm, when the interpolated data is used. Based on the kinematic tests, the accuracies of the solutions using the original data and the interpolated data are generally in good agreement.

KEYWORDS: GPS observables. Data interpolation. Kinematic positioning. Accuracy assessment.

INTRODUCTION

GPS has shown the capacity to deliver much improved positioning services and accuracy due to the modernization of the satellite system, significantly upgraded error estimation models, much increased mobility for the end-user receivers, and more reliable and rapid integer ambiguity resolution approaches [6]. For kinematic GPS positioning, a typical real time kinematic (RTK) system consists of three components, namely a reference station located on a precisely measured ground mark, a rover receiver to be determined for its position, and a radio-communication link device. Following the constantly improved kinematic data processing algorithms and new generation of communication technologies, a network-based RTK or virtual reference station (VRS) system has been recently developed to reduce operation fees, remove spatially correlated systematic errors, increase system reliability, and extend inter-receiver distances up to many tens of kilometres [3],[4],[8].

Kinematic GPS is certainly the most effective technique for high precision positioning. Its solution can be used for time-sensitive applications, such as precision navigation, engineer stake-out, cadastral boundary surveys, etc. It is also popular in dealing with the recorded GPS observables from both reference and rover stations, with the precise ephemerides, in post-processing differential mode to increase its successful rate and achieve high accuracy for the applications of trajectory determination, hydrographic surveys, airborne surveys, etc.

As more and more GPS data collected from the continuously tracking stations are

available to the public, those data can directly serve as reference data for kinematic GPS. However, publicly available GPS tracking data is normally recorded at intervals as large as 30 seconds, which is insufficient to meet the general requirement of 1 Hz sampling rate in kinematic positioning [9]. High-rate data may be collected by GPS permanent stations, but it is difficult to understand these data provided on a routine basis because many agencies don't distribute these data for free. As a result, the rover data may not be synchronously processed with the reference data to obtain all epoch solutions by the differential RTK technique. Therefore, it would be helpful if an interpolation algorithm can be adopted to artificially densify the low-rate data into a coherent high-rate data set for post-processing kinematic GPS. When both reference and rover data are available with the same temporal resolution, there might be no need to carry on with any supplementary GPS campaigns, especially using high-cost moving platforms, like aircrafts.

Besides the relative mode of kinematic GPS, the development of a stand-alone mode of precise point positioning (PPP) technique has drawn more attention in recent years. In the PPP un-differential algorithm, the dual-frequency GPS pseudo-range and carrier phase measurements are commonly used, with the availability of precise GPS satellite orbit and clock products, to solve the unknown vectors, for example, the position coordinates, receiver clock error, tropospheric delay, and phase ambiguity. This technique brings great flexibility to field operations mainly because the need for a reference station is removed and the accuracy for static and kinematic applications can reach the centimetre to decimetre level [1].

For both the relative and stand-alone mode of kinematic GPS, the epoch solutions are expected to be successive and high density. The precondition for that is using high-rate GPS data collected at both reference and rover stations in RTK or at the rover only in PPP. When those baseline data sets are not observed with a consistent small sampling interval, or standalone receiver is not operated with a higher sampling rate, the high density of kinematic solutions will not be achieved. If an easily implemented mathematical method for high-rate data interpolation is demonstrated to be possible for high accuracy positioning in post-processing kinematic GPS, it can be an effective way to add value to the originally low-rate data for high-density kinematic positioning.

This paper attempts to apply a simple form of polynomial for time-varying data curve fitting to the static GPS data, along with a linear interpolation for fitting data correction at the densified epoch. Further, a one-kilometre short baseline and a fifteen-kilometre medium baseline are measured to test the kinematic accuracy using different time scales of the interpolated data, based on the reference station's original L1, L2 phase observables and C1, P2 range observables. For stand-alone kinematic positioning, a static GPS data set is also interpolated from a lower rate to a higher rate and tested for its positioning accuracy using the PPP technique.

INTERPOLATION ALGORITHM

GPS kinematic positioning normally requires higher sampling frequencies to archive continuously received data for many scientific applications. If only low-frequency data are available, or the rover and the reference station data are not received synchronously, it is necessary to interpolate GPS measurements in the time domain to generate a consistently higher-rate data in post-processed kinematic positioning.

Two main interpolation algorithms were proposed and implemented to produce 1 second densified data from lower-rate data. The first used a high-order polynomial

fitted by eight sequential raw data points to interpolate the GPS phase and range observables separately [5]. The second mainly relied on the observation residuals, treated with the known satellite orbit etc, of two subsequent epochs to interpolate the data with a linear function [5],[10].

For the sake of simple calculation, the interpolation is carried out directly using the raw observables, under the condition of no SA effects and cycle slips. To generate a higher rate of GPS interpolated data from a lower rate of GPS raw data, a two-step interpolation scheme is practically applied. In the first step, a polynomial is fitted using some sequential data points, where the polynomial order and the data point number are relevantly tested and selected. When the coefficients of a polynomial are derived by the least squares estimation, the interpolated observables can be simply calculated with a dense data interval, where the interpolation epoch is centered at the data points.

In practice, the phase and range data are interpolated separately for each observable. Here, the primary carrier phase of L1(Φ) is presented by a function of time (t) with a k^{th} -order polynomial as

$$\Phi = a_0 + a_1t + \dots + a_k t^k \quad (1)$$

To solve the polynomial coefficients (a_0, a_1, \dots, a_k), n sequential data points ($n \geq k$) are used to form a matrix equation as

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \vdots \\ \Phi_n \end{bmatrix} = \begin{bmatrix} 1 & t_1 & t_1^2 & \dots & t_1^k \\ 1 & t_2 & t_2^2 & \dots & t_2^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & t_n & t_n^2 & \dots & t_n^k \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} \quad (2)$$

In the polynomial fitting procedures, the unknown parameters (a) can be determined using the least squares principle. When such a k^{th} -order polynomial is established, this function is expected to describe the data variation trend over this period of GPS observation and can interpolate the GPS observables (Φ_i') by simply giving the epochs (t_i) inserted for a higher data rate as

$$\Phi_i' = a_0 + a_1t_i + \dots + a_k t_i^k \quad (3)$$

To enhance the precision of the interpolation data, the second stage of data correction is further adopted by estimating the k^{th} -order polynomial fitting residuals ($\delta\Phi_{obs}$) at the sequential raw data points (t_{obs}) with the differences between the GPS raw observables (Φ_{obs}) and the polynomial estimates based on equation (3), that is

$$\delta\Phi_{obs} = \Phi_{obs} - (a_0 + a_1t_{obs} + \dots + a_k t_{obs}^k) \quad (4)$$

The data corrections ($\delta\Phi_i'$) made at the interpolated epochs (t_i') are determined by a two point linear operation of the fitting residuals ($\delta\Phi_{obs(i-1)}$, $\delta\Phi_{obs(i+1)}$) between the two adjacent raw data epochs ($t_{obs(i-1)}$, $t_{obs(i+1)}$) covering the interpolated epochs as

$$\delta\Phi_i' = (\delta\Phi_{obs(i+1)} - \delta\Phi_{obs(i-1)}) / (t_{obs(i+1)} - t_{obs(i-1)}) \times (t_i' - t_{obs(i-1)}) \quad (5)$$

The interpolated GPS observables (Φ_i) can be finally generated by

$$\Phi_i = \Phi_i' + \delta\Phi_i' = (a_0 + a_1t_i + \dots + a_k t_i^k) + \delta\Phi_i' \quad (6)$$

and combined with the GPS raw data to form a full set of GPS observables with a higher data rate for kinematic positioning.

TEST TRIAL ON SHORT BASELINE

One short baseline with a 160-metre distance on the campus was selected for the tests. The two ends of the baseline included one GPS tracking site, as the reference station, and one rover station. This test campaign was carried out using dual frequency GPS receivers over 2-hour observations with a 15 degrees elevation cut-off angle and a one second data interval used. For data verifications, GPS data with interval of 30 seconds were also collected at the reference station.

To analyse the effect of interpolating the GPS data from a 30 sec to 1 sec interval for the reference station, different successive data epochs, from 2 to 5, and different orders of polynomial, from the 1st to the 5th, were accordingly tested. The 1 sec fitting data was then applied to the static baseline solutions using TTC (Trimble Total Control) software. The 3D coordinate precision of the baseline solution, based on using the fitting data sets, was examined by a standard deviation of less than 0.2 cm in each component as a threshold.

The GPS raw data was also interpolated to a 15 sec data interval using the fitting functions and solved for the kinematic solutions. The 3D coordinate precision of the rover, processed using the interpolated data and treated as one of the assessment indications, was requested to be better than 1 cm and 3 cm in the plan and vertical components, respectively.

After two stages of the tests, the interpolated data fitted by using the 3rd- or 4th order of polynomial with 4 to 6 epochs of data points can provide acceptable coordinate precisions for both static and kinematic GPS positioning. To reduce the computation burden, a 3rd-order polynomial along with 4 epochs of data point was finally adopted to generate the interpolated type of high rate data for the tests of kinematic GPS positioning. The precisions of using different intervals of interpolated data sets in kinematic positioning are listed in Table 1 and compared in Figure 1.

Table 1. *Short baseline kinematic precisions based on the interpolated data*

Data source	Data interval (sec)	σ_E (cm)	σ_N (cm)	σ_U (cm)
Raw	30	0.5	0.5	1.8
Interpolated	15	0.7	1.0	3.1
	10	0.7	1.0	3.2
	5	0.7	1.0	3.3
	3	0.7	1.0	3.3
	1	0.7	1.0	3.3

As seen in Table 1 or Figure 1, there are a 0.5 cm and 1 cm levels of jump in positioning errors in the plan and vertical components, respectively, for the interpolated data solutions. These precisions confirm that GPS raw data still offers a better solution than the interpolated data for kinematic positioning. However, the kinematic precisions are similar for five different intervals of interpolated data solutions and somewhat effective for the GPS data densification. One example of the kinematic solution based on the highest rate of interpolated data, i.e. with 1 sec data interval, is displayed in Figure 2 for the three coordinate components.

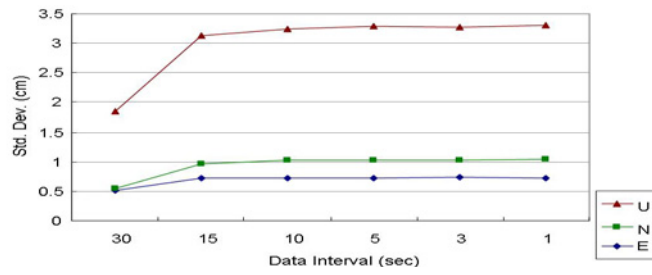


Fig. 1. Comparison of kinematic precisions for different interpolation intervals

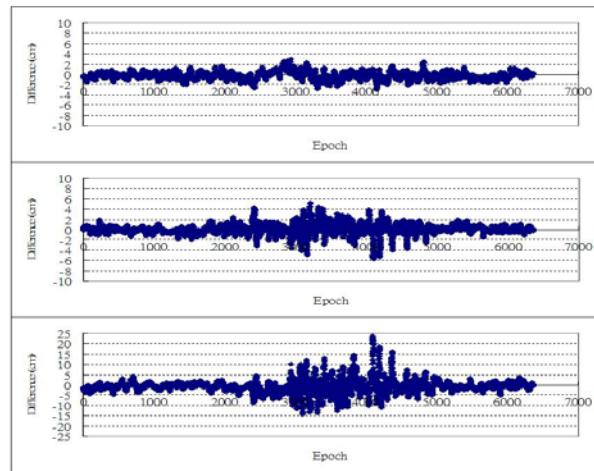


Fig. 2. Short baseline kinematic solutions based on 1 sec interpolated data in the E, N and U components

Since the external accuracy is also an important quality assessment indication for the solutions obtained from the interpolated GPS data sets, the kinematic baseline vectors can be compared with the same baselines whose vectors have been determined by a more accurate measurement, for example, high accuracy static GPS. The plan and vertical vectors defined by $\Delta L = (\Delta E^2 + \Delta N^2)^{1/2}$ and $\Delta h = \Delta U$, respectively, in the solution series were adopted to calculate the root mean square (RMS) difference as the external accuracy indication. The RMS differences obtained from a 15 sec interval of GPS raw data and different intervals of interpolated data are listed in Table 2 and shown in Figure 3.

Table 2. Short baseline kinematic accuracies based on the interpolated data

Data source	Data interval (sec)	RMS _{ΔL} (cm)	RMS _{Δh} (cm)
Raw	30	0.4	2.0
Interpolated	15	0.7	3.2
	10	0.7	3.3
	5	0.7	3.3
	3	0.7	3.3
	1	0.7	3.4

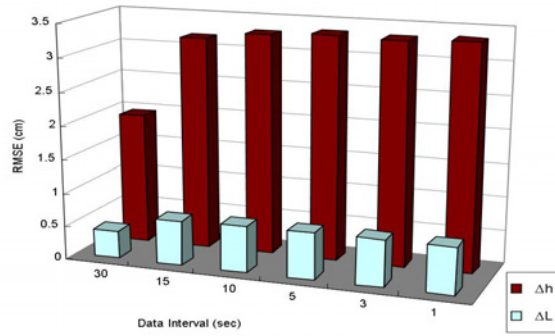


Fig. 3. Comparison of kinematic accuracies for different interpolation intervals

The kinematic accuracies shown in Table 2 and Figure 3 further confirm the good agreement of the interpolated data from the short baseline test, with RMS differences in the plan and vertical vectors of better than 0.7 cm and 3.4 cm, respectively. The RMS vector differences are consistent for five different intervals of the interpolated data, although the 30 sec raw data still provides the best RMS differences.

TEST TRIAL ON MEDIUM-RANGE BASELINE

It is believed that a post-processing type of GPS kinematic positioning based on single reference station can be well operated in a medium range of less than 15 km. To test the interpolated data applied to such a baseline length, three suitable GPS permanent stations in Taoyuan County, Taiwan, were selected. These three stations include two university sites (CCSB and TCYU) and one IGS (International GNSS Service) site (TWTF), whose 3D- coordinates were all accurately computed with a long term data set. The fixed station selected was CCSB, chosen because of its operation of two data sampling rates, i.e. 30 sec and 1 sec, whereas the other two rovers (TCYU and TWTF) were only observed with a lower rate of 30 sec interval. The baseline stations and the baseline lengths are shown in Figure 4.

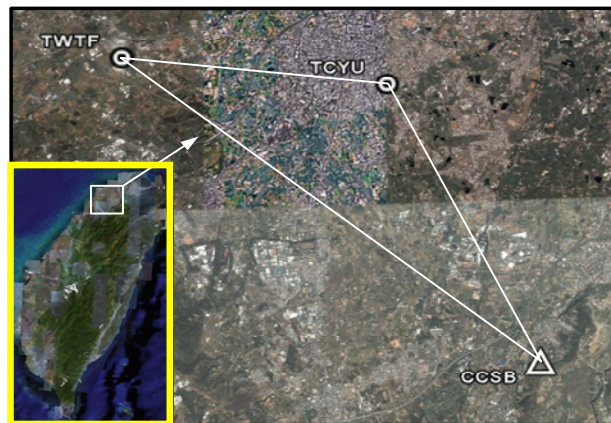


Fig. 4. Medium-range baselines located in Taoyuan, Taiwan for kinematic positioning

To ensure raw data quality, one hour tracking data observed with a standard data interval of 30 sec was tested for a post-processing kinematic positioning. The standard deviations of the 3D coordinate solutions over one-hour session for the two rovers are listed in Table 3. The standard deviations are indicators of precision and show the reasonable quality of data collected at the test sites. The plan coordinate precisions are better than 2 cm and the height precisions are better than 6 cm for both two rovers. However, the precisions point out an inconsistent performance in that the vertical precision derived from TCYU is slightly worse than that obtained from TWTF, an IGS site, by approximately 2 cm.

Table 3. *Medium-range kinematic precisions based on 30 sec raw data set*

Baseline	σ_E (cm)	σ_N (cm)	σ_U (cm)
CCSB-TCYU	0.8	1.6	5.5
CCSB-TWTF	0.8	1.3	3.7

As the reference site was selected for CCSB during the medium-range kinematic tests, its high rate data, i.e. the 1 sec data set, was directly adopted for the computation. For the two rovers, their raw data sets measured with 30 sec data interval were interpolated to be a higher rate of 15 sec, 10 sec, 5 sec, 3 sec and 1 sec data, individually, using the proposed interpolation algorithm. An example of the kinematic solutions using the 1 sec data set for the 13.4 km baseline CCSB-TWTF is shown in Figure 5. The accuracy assessments defined by the RMS differences of the baseline vectors between the computed and precisely measured values and made at the baseline TCYU-TWTF are listed in Table 4 and compared in Figure 6.

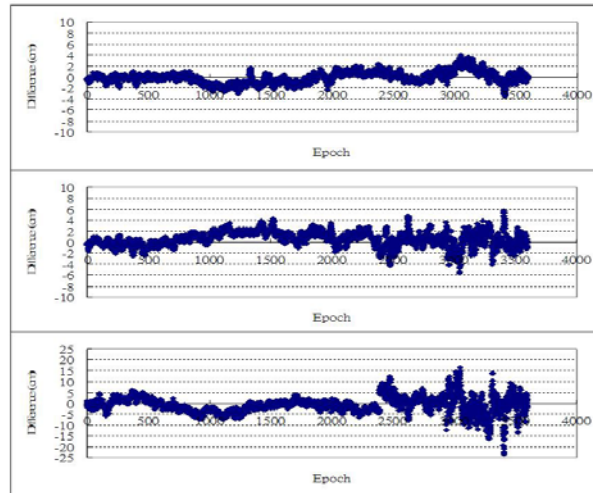


Fig. 5. CCSB-TWTF kinematic solutions based on 1 sec interpolated data in the E, N and U components

Table 4. *Medium-range kinematic accuracies based on the interpolated data*

Data source	Data interval (sec)	RMS _{ΔL} (cm)	RMS _{Δh} (cm)
Raw	30	1.1	7.2
Interpolated	15	1.0	7.2
	10	1.3	7.6
	5	1.3	7.5
	3	1.3	7.6
	1	1.3	7.6

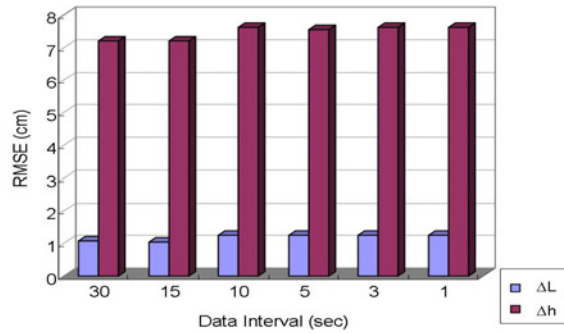


Fig. 6. Comparison of medium-range kinematic accuracies for different interpolation intervals

The RMS vector differences demonstrate the quality of the kinematic solutions from the medium-range observations consisting of the interpolated GPS data, with RMS around 1.5 cm and 7.5 cm in the plan and vertical vectors, respectively. The RMS vector differences for the kinematic solutions using the higher rates of interpolated data show the best accuracy when using 15 sec interpolated data. This level of accuracy is identical to using the original 30 sec data set, representing the possibility of using the data interpolation technique for GPS data filtering to improve the kinematic data quality. From Figure 6, it can be seen that all the RMS differences show almost the same level of accuracy for different intervals of interpolated data, demonstrating a stable performance of the GPS interpolated data from the operation of the data interpolation.

TEST TRIAL ON PRECISE POINT POSITIONING

Precise point positioning (PPP) is a method that performs position determination using a single GPS receiver. This positioning technique can offer several advantages, over differential positioning techniques, such as removing the need for a reference station, increasing the spatial operating range, referring to a global reference frame, and providing a new way for time transfer and atmospheric parameter estimation [2]. With the improvement in the IGS precise orbit and satellite clock error products, there has been a tremendous interest over the past few years in using PPP technique to determine the absolute positions of a series of points with a single GPS receiver. It has been demonstrated that positioning accuracy to a few centimetres level can be achieved by the PPP in a static mode. Decimetre level of the PPP accuracy can also be obtained in a kinematic mode for moving vehicle positioning.

The Canadian Spatial Reference System Precise Point Positioning Service (CSRS-PPP) provides post-processed position estimates over the Internet from GPS observation files submitted by the user [7]. The online PPP positioning service can process raw GPS data from single or dual-frequency receivers in either static mode (output is a single position) or kinematic mode (each epoch is corrected individually) using precise GPS orbits and clocks. Precise position estimates can be referred to as the global ITRF (International Terrestrial Reference Frame).

There is no minimum length for a GPS observation session in using CSRS-PPP. However, the quality of a PPP computed positions will be optimal if the carrier phase ambiguities have converged using longer data sets. The 2-hour session data collected at one of the tracking station of CCSB was used to test for the PPP accuracies based on

the original 30 sec and 1 sec data sets as well as the higher rates of the interpolated data sets.

Although the data sets observed with fixed GPS receivers can be processed using both static and kinematic options, a static option was only adopted by the test. An attempt to do so is expected to increase precision by allowing time averaging over the full session length for optimal parameter estimation and result in a useful evaluation to the scatter of independent position estimates over time as the precision indication. The coordinate differences between the PPP estimates and the well known coordinates of the test station are compared to obtain the RMS in three coordinate components as the accuracy indication.

One of the solution plots obtained by processing the 1 sec interval of the interpolated data in static mode is shown in Figure 7 for the three coordinate components. Table 5 gives the accuracies of the PPP solutions using the raw data for 24-hour or 2-hour session with a 30 sec or 1 sec data interval. The PPP results of using interpolated data sets with data intervals of 15 sec, 10 sec, 5 sec, 3 sec and 1 sec, all interpolated from the 30 sec raw data set, for a 2-hour session were also obtained and listed in Table 6. To examine the accuracy performances of PPP for different intervals of the interpolated data, a comparison plot is also shown in Figure 8.

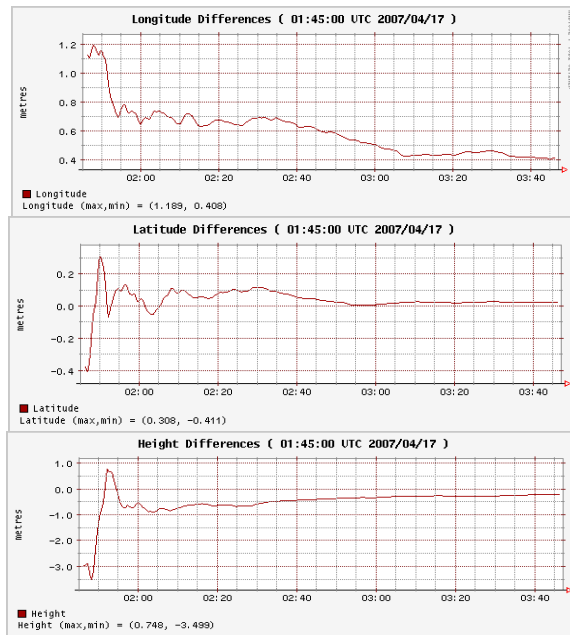


Fig. 7. PPP solutions using 1 sec interpolated data in the longitude, latitude and height components

Table 5. PPP accuracies based on the raw data sets

Session period (hour)	Data interval (sec)	RMS _E (cm)	RMS _N (cm)	RMS _U (cm)
24	30	4.4	4.7	14.8
2	30	13.3	24.2	39.9
2	1	21.3	26.7	43.1

Table 6. PPP accuracies based on 2-hour session of the interpolated data sets

Data interval (sec)	RMS _E (cm)	RMS _N (cm)	RMS _U (cm)
15	13.8	4.1	42.5
10	14.7	4.1	44.3
5	16.3	4.4	45.8
3	17.7	4.8	47.2
1	20.8	5.5	49.5

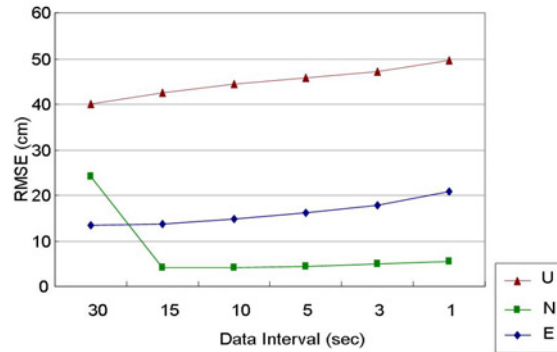


Fig. 8. Comparison of PPP accuracies for different intervals of the interpolated data

Table 5 shows that the static mode of PPP solutions provide the best accuracy of 5 cm and 15 cm in the plan and vertical components, respectively, based on the longest 24-hour session of the raw data. The test results demonstrate that PPP accuracy highly depends on the observation session length, mainly as the convergence of the phase ambiguity. For the 2-hour session data sets, the accuracies are significantly degraded to a 20 cm level in plan coordinates and a 40 cm level in height. Table 5 also shows the accuracies are slightly degraded, when a higher rate of 1 sec raw data set and the normal GPS clock information were applied.

As seen in Table 6 and Figure 8, the test results represent an inconsistent performance for the PPP accuracies obtained using five different rates of GPS interpolated data. A 7 cm level of accuracy difference can be found between the 15 sec and 1 sec data solutions. However, the accuracy difference in the North-South coordinate component is only to a few millimetres, when the data interval is increased. Moreover, the accuracy comparison between the PPP solutions using the same interval of 1 sec raw data and interpolated data is interesting. It is clear that the interpolated data sets work well for the PPP solution in one of the plan coordinate, i.e. the North-South coordinate component, as its accuracy is significantly improved by 21 cm. This phenomenon can be possibly explained by the filtering effect of using interpolated technique for GPS data densification.

CONCLUDING REMARKS

Based on the GPS kinematic data tests and accuracy assessments, some conclusions and suggestions on the subject of densifying GPS data to a higher data rate using the interpolation technique can be drawn as follows:

- (1) High accuracy post-processing kinematic positioning, based on either the

relative mode or the stand-alone mode, is of increasing importance in many geodetic and engineering applications. GPS observations collected at reference or rover stations must be synchronous or in a higher data rate. The data interpolation technique is expected to achieve the data densification of GPS observables by fulfilling the data set mathematically for its data processing.

(2) The data interpolation is designed to use GPS raw observables for a polynomial fitting along with a linear correction based on the fitting residuals. The plan coordinate accuracies were better than 1 cm, and the height accuracies were better than 3 cm for a short baseline kinematic test, in which the interpolated data estimated using the simplest form of a 3rd-order polynomial with 4 data epochs was applied. This test demonstrated that kinematic GPS was generally performed well, when the interpolated data sets were used.

(3) Compared to using GPS raw data, the interpolated data solutions showed an accuracy degradation of 0.5 cm and 1 cm in the plan and vertical components, respectively. It confirmed that GPS raw data generally offers a better solution than using interpolated data for kinematic positioning. However, the interpolated data solutions based on five different intervals, i.e. 15 sec, 10 sec, 5 sec, 3 sec and 1 sec, were found to perform at the same level of positioning accuracy.

(4) The RMS vector differences, obtained from both the medium-range kinematic GPS and the precise point positioning, proved the good agreement of the kinematic solutions based on the different intervals of interpolated data. It was also noted that the interpolated data sets could play a role in GPS data filtering as the horizontal accuracy was significantly improved by more than 20 cm when using interpolated data, as opposed to same interval of GPS raw data, in the PPP tests.

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