Testing a Medium-Range DGPS Network for the Taiwan Area

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The DGPS technique can be used for navigation over baselines up to approximate 100 km. However, DGPS positioning accuracy is highly correlated and degraded with the extension of its operational range. It is, hence, inproper to simply carry on the conventional DGPS for navigation over a medium scale of range. The approach aimed at providing sufficient accuracy with the use of a less dense network of reference stations and the modelling of GPS errors has been proposed and tested for the regional navigation over the Taiwan area. It has been shown with the test results that medium-range DGPS based on using multi-reference stations is capable of improving the RMS coordinate differences by 35%, as opposed to using of single reference station. The RMS values can be further improved by 14% and 6% in plan coordinates and height respectively, when the modelling of atmospheric delay is aplied to the differential corrections based on the pseudo-range observables collected from the multi-reference stations.

1. INTRODUCTION. The GPS has been demonstrated to be a high accuracy global positioning system in a wide variety of applications for surveying and navigation. However, it is also believed that the accuracy provided by stand-alone GPS positioning is not sufficient for some precise applications. The technique of DGPS (Differential GPS) is, hence, developed and used to improve the positioning accuracy for navigation. The basic assumption of using DGPS is that GPS error sources, such as orbital and atmospheric propagation errors, are highly correlated between the reference and user stations. Theoretically, most of the GPS errors can be effectively eliminated or reduced by using such differential corrections.

As the spatial correlation of GPS errors is degraded with the increment of the baseline length, the accuracy of DGPS is subject to the lengths between the reference and user stations. The concept of so-called Wide Area DGPS (WADGPS), designed by modelling more varieties of GPS errors to reduce the space-related errors for DGPS, has been developed and tested [Kee et al., 1991][Chao, 1996]. The benefits of employing WADGPS include the minimising of the number of reference stations and the cost of the system operation, comparing to the conventional DGPS.

Taiwan is located along the western coast of the Pacific Ocean, and plays a key role on the air and sea transportation around the Asian Pacific area. The system of conventional DGPS is theoretically not suggested for the Taiwan area due to its medium scale of coverage. Instead, a medium-range DGPS system designed by using multi-reference stations and modelling GPS errors, similar to the concept of WADGPS, is more feasible for the navigation activities around this area. Such a DGPS system, aiming to be effectively operated for medium-range navigation positioning, is proposed and tested in this paper.

2. OPERATION OF MEDIUM-RANGE DGPS. The medium-range DGPS designed for the Taiwan area consists of four components, namely the master station, reference stations, communication device, and user stations. The operation of this medium-range DGPS is mainly based on the WADGPS approach, in which the master station collects GPS observations from three reference stations, and broadcasts only one set of differential corrections to the user stations. The GPS observations, based on L1 C/A code pseudo-range observables for real-time positioning, are processed at the master station with the modelling of space-related errors and the weighted averaging of the differential corrections. The user stations can, therefore, apply these differential corrections to their own GPS code range observables to improve the positioning accuracy. The procedure of operating such a medium-range DGPS system is briefly described as follows:

- (a) Three reference stations, whose accurate three-dimensional coordinates have been well-defined, collect GPS observations and broadcast ephemeris;
- (b) The data collected are then transmitted to the master station via cables;
- (c) The modelling of GPS errors, including atmospheric delay, satellite orbits, and clock errors, is carried out at the master station;
- (d) One set of differential range corrections and correction vectors is broadcast to the user stations via radio station with enough power;
- (e) The user stations can then apply these differential corrections to improve the positioning accuracy.

2.1. REFERENCE STATIONS. For testing the medium-range DGPS designed for the Taiwan area, three GPS tracking stations, equipped with high quality GPS receivers, are selected as the reference stations. These three GPS stations, located at TAIW (Taipei), KDNM (Kending), and KMNM (Kinmen), are part of fiducial stations set up for the GPS fundamental network in Taiwan (see Figure 1). High accuracy three-dimensional geocentric coordinates have been well-defined in the ITRF94 for these three reference stations [Chang and Tseng, 1998]. The selection of these three reference stations is based on the consideration of GPS data quality and the largest coverage for DGPS network working in the Taiwan area.

Figure 1. Reference Stations used for Medium-range DGPS in the Taiwan Area.

2.2. ERROR MODELLING. As the fact that GPS signal is delayed by the atmosphere, depending on the refractive index along the actual path from the radio source in space to the receiver, an atmospheric model is normally required to estimate the excess path length of the signal in the atmosphere. However, it is believed to be difficult to define such models as the refraction varies with atmospheric pressure, temperature, and relative humidity.

The design of such a medium-range DGPS system is aimed to improve the positioning accuracy, based on not only the elimination of the effect of SA (Selective Availability), but also the modelling of some space-related DGPS errors. One set of differential corrections, in which the atmospheric delay errors have been taken into account, will be generated at the master station and tested by the user stations for this medium-range DGPS system in the Taiwan area.

2.2.1. *Ionospheric delay.* Within the ionosphere, the first-order ionospheric time delay caused by its dispersive nature is inversely proportional to the square of the frequency of the radio signal. It is, therefore, to be possible to evaluate the refractivity of the ionosphere by modelling these different effects to two different frequencies transmitted from the GPS satellites. Unfortunately, dual-frequency GPS receivers are mainly used for geodetic purposes. The low-cost GPS receivers, acquiring only L1 C/A code pseudo-range, are still equipped on most of the vehicles for navigation.

An ionospheric model is, hence, required to estimate the delay errors for the users equipped with single-frequency GPS receivers. One of the easy-use ionospheric model, namely the Klobuchar model, will be tested for the medium-range DGPS in the Taiwan area. In the Klobuchar model, the day-time ionospheric zenith delay is treated as the middle part of the cosine wave, and the night-time delay is described as a constant term [Klobuchar, 1982]. The eight coefficients, included in the satellite broadcast message, can be used by the single frequency users to estimate the ionospheric delay. The delay is expressed by

$$
T_g = DC + A \cos[2\pi (t - T_p)/P]
$$
\n(1)

where

- T_g is the ionospheric zenith delay at epoch t
- *DC* is the night-time constant (=5 \times 10⁻⁹ sec)
- *A* is the amplitude term of the cosine function
- *P* is the period term of the cosine wave
- T_p is the phase term of the cosine wave $(=14^h$ local time)

The vales for *DC* and T_p are constant. The terms for *A* and *P* are normally represented by the third-order polynomials, which are the function of geomagnetic latitude (ϕ_m) of the sub-ionospheric point. The polynomials can be given by

$$
A = \alpha_{0} + \alpha_{1} \phi_{m} + \alpha_{2} \phi_{m}^{2} + \alpha_{3} \phi_{m}^{3}
$$
 (1-1)

$$
p = \beta_{0} + \beta_{1}\phi_{m} + \beta_{2}\phi_{m}^{2} + \beta_{3}\phi_{m}^{3}
$$
 (1-2)

where α_0 , α_1 , α_2 , α_3 , and β_0 , β_1 , β_2 , β_3 are the coefficients that determine the amplitude and period of the cosine wave, respectively defined in equation (1). The ionospheric zenith delay (T_g) , estimated by using equation (1), is then mapped to the slant delay based on the satellite elevation.

The ionospheric model described as above is basically an empirical estimation of the delay. As only one set of coefficients is broadcast to the global users by GPS satellites, the ionospheric delay is estimated to be a global average value and not the real reflection of local ionospheric status. However, it is simple to use and believed to be effective to reduce 50% of ionospheric delay error for single-frequency GPS receivers at mid-latitudes [Klobuchar, 1982].

2.2.2. *Tropospheric delay.* The troposphere is a non-dispersive medium for signal frequencies below 30 GHz, which is the case with the radio signals of GPS satellites. The troposphere delays the arrival of both the L1 and L2 signals by the same amount. Thus, the effect caused by the tropospheric error cannot be effectively reduced by even dual-frequency GPS observations as used to eliminate the ionospheric error. Practically, the estimation of the tropospheric delay is required by properly modelling.

One of the typical empirical models proposed for modelling the tropospheric delay is the Hopfield model [Hofmann-Wellenhof et al., 1992], in which the tropospheric delay (*dt*) can be expressed as the tropospheric zenith delay (τ) times the mapping function, $m(E)$, for dry and wet components respectively, as

$$
dt = \tau_d m_d(E) + \tau_w m_w(E) \tag{2}
$$

where

$$
\tau_d = 155.2 \times 10^{-7} \times \frac{P}{T} (h_d - h_s) \tag{2-1}
$$

$$
m_d(E) = \frac{1}{\sqrt{\sin(E^2 + 6.25)}}\tag{2-2}
$$

$$
\tau_{w} = 155.2 \times 10^{-7} \times \frac{4810}{T^2} e(h_w - h_s)
$$
 (2-3)

$$
m_W(E) = \frac{1}{\sqrt{\sin(E^2 + 2.25)}}
$$
\n(2-4)

$$
h_d = 40136 + 148.72(T - 273.16) \tag{2-5}
$$

$$
h_w = 11000 \tag{2-6}
$$

where E is the elevation angle to the satellite at the antenna, h_s is the height of station, h_d and h_w are the heights of dry and wet troposphere respectively, and *P*, *T* as well as *e* are pressure, temperature, and water vapour pressure, respectively. As it can be seen, when surface meteorological data are observed at sites, the tropospheric delay errors can be estimated using this model.

As a remedy, meteorological values can also be substituted by a standard atmospheric condition, in which the reference pressure, temperature, and humidity at sea level are introduced. The meteorological parameters are, hence, simply related to the height of station, and the Hopfield model can be practically applied for the estimation of troposperic delay during the processing of differential range corrections.

3. TEST RESULTS. As mentioned above, three GPS stations located at TAIW, KDNM, and KMNM were tested as the reference stations in the medium-range DGPS for the area around Taiwan. Some other tracking stations shown in Figure 1, such as YMSM, FLNM, PKGM, TMAM, and MZUM, were simulated as the user stations in the tests. One of the IGS station located at SHAO was also included in the tests. The 72-hour GPS data were collected at totally nine GPS tracking stations from 24 to 26 July 1997, in which the days in summer were expected to have stronger atmospheric condition.

3.1. USING SINGLE AND MULTI- REFERENCE STATIONS. The basic principle of conventional DGPS is that the data used to obtain the differential corrections are collected and computed at only one reference station. The differential corrections are then sent to the local user stations in order to improve the positioning accuracy by mainly reducing the error of SA. A DGPS network, composed of at least three reference stations, can also be used to obtain one set of differential corrections, computed by the weighted mean of the range corrections from different reference stations. This set of differential corrections is then provided to the regional user stations to reduce the space-related errors and to effectively expand the working area to a medium scale of range for DGPS.

3.1.1. *Results based on single reference station.* The results listed in Tables 1-3, were tested by using TAIW, KDNM, and KMNM, respectively, as the single reference station for the conventional DGPS. The user stations shown in tables are listed with the baseline lengths to clearly display the relationship between the lengths and accuracies for the results carried out by DGPS. The accuracy indicator shown for the horizontal and vertical components is the RMS values based on the coordinate differences between the DGPS results and those coordinates previously defined, and computed by selecting values lay inside an interval of $\pm 3\sigma$.

| User station | -0 Baseline length | RMS (m) | | |
|--------------|------------------------------|------------|----------|--|
| | (km) | Horizontal | Vertical | |
| YMSM | 16 | 0.6 | 3.1 | |
| FLNM | 141 | 1.2 | 2.7 | |
| PKGM | 203 | 1.7 | 3.7 | |
| MZUM | 204 | 1.3 | 2.3 | |
| TMAM | 272 | 2.1 | 3.7 | |
| KMNM | 324 | 1.9 | 3.0 | |
| KDNM | 349 | 2.3 | 4.0 | |
| SHAO | 674 | 3.6 | 4.7 | |

Table 1. DGPS Results using TAIW as the Reference Station.

Table 2. DGPS Results using KDNM as the Reference Station.

| User station | Baseline length | RMS (m) | | |
|--------------|-----------------|------------|----------|--|
| | (km) | Horizontal | Vertical | |
| TMAM | 77 | 0.8 | 1.4 | |
| PKGM | 187 | 1.4 | 2.5 | |
| FLNM | 211 | 1.3 | 2.2 | |
| TAIW | 349 | 2.3 | 4.0 | |
| YMSM | 365 | 2.3 | 4.2 | |
| KMNM | 371 | 2.0 | 3.0 | |
| MZUM | 474 | 2.6 | 4.2 | |
| SHAO | 1014 | 4.9 | 5.7 | |

Table 3. DGPS Results using KMNM as the Reference Station.

The results show that different levels of positioning accuracy are appeared if single reference station located at different sites is used in the conventional DGPS for medium scale of range. As seen from the case for the user station at PKGM, whose baseline lengths are approximately 200 km to all the three single reference stations, the RMS values are significantly varied from 1.4 m to 1.7 m in horizontal coordinate differences and from 1.9 m to 3.7 m in vertical coordinate differences, based on different sites of single reference station used for DGPS.

It is also an evidence to show that the positioning accuracy of DGPS using single reference station for medium range is highly related to the baseline lengths between the reference and user stations. For the lengths between 15 km and 1,000 km around the Taiwan area, the variations of RMS coordinate differences are shown to be not consistent with the values of 0.6 to 4.9 m in plan coordinates and 1.4 to 5.7 m in height, where a level of around 4 m difference can be found.

Further investigation has also been carried out to realise the relationship between the DGPS accuracy and the baseline length. The variations of RMS coordinate differences are plotted with different lengths of baseline, and shown in Figure 2 and Figure 3 for plan coordinates and height, respectively.

Figure 3. Horizontal Accuracies Varied with Baseline Lengths.

Figure 4. Vertical Accuracies Varied with Baseline Lengths.

The degree of correlation between the RMS coordinate differences and the baseline lengths was also estimated by using a linear regression model for both horizontal and vertical components. The regression models, tested with the values of RMS (unit: m) and baseline length (S, unit: km), are listed as follows:

The correlation coefficients (*r*) for horizontal and vertical components are both larger than 0.7, which means a higher level of correlation between the DGPS accuracy and baseline length. These regression models can also be provided to simply estimate the accuracy of conventional DGPS using single reference station around the Taiwan area.

3.1.2. *Results based on multi-reference stations.* Instead of using single reference station, a DGPS network consisting of three reference stations, located at TAIW, KDNM, and KMNM, was also tested for the medium-range DGPS. When multi-reference stations were used, the final differential corrections to each common satellite were computed with those range corrections received from three different reference stations. In order to realise the accuracy of DGPS improved by using multi-reference stations, the results tested for the user stations are listed in Table 4 and Table 5.

| | RMS (m) | | | | | |
|--------------|--------------------------|---------------------------------------|-----|-----|---------------------|---------------------|
| User station | Single reference station | | | | Multi-refer ence | Improvement (%) |
| | TAIW | KMNM KDNM Average | | | | |
| FLNM | 1.2 | 1.3 | 1.5 | 1.3 | 0.9 | 36 |
| MZUM | 1.3 | 2.6 | 1.3 | 1.7 | 1.0 | 41 |
| YMSM | 0.6 | 2.3 | 1.8 | 1.6 | 1.6 | θ |
| TMAM | 2.1 | 0.8 | 1.6 | 1.5 | 0.8 | 47 |
| PKGM | 1.7 | 1.4 | 1.4 | 1.5 | 0.4 | 73 |
| SHAO | 3.6 | 4.9 | 3.9 | 4.1 | 3.0 | 27 |
| Average | 1.8 | 2.2 | 1.9 | 2.0 | 1.3 | 35 |

Table 4. Horizontal Accuracy Improved by using Multi-reference Stations.

Table 5. Vertical Accuracy Improved by using Multi-reference Stations.

| User station | Single reference station | | | | Multi-refer ence | Improvement (%) |
|--------------|------------------------------------------------------|-----|-----|-----|---------------------|---------------------|
| | KMNM KDNM TAIW Average | | | | | |
| FLNM | 2.7 | 2.2 | 2.2 | 2.4 | 1.9 | 21 |
| MZUM | 2.3 | 4.2 | 2.1 | 2.9 | 1.4 | 52 |
| YMSM | 3.1 | 4.2 | 2.9 | 3.4 | 3.4 | θ |
| TMAM | 3.7 | 1.4 | 2.1 | 2.4 | 1.3 | 46 |
| PKGM | 3.7 | 2.5 | 1.9 | 2.7 | 1.1 | 59 |
| SHAO | 4.7 | 5.7 | 4.7 | 5.0 | 3.0 | 40 |
| Average | 3.4 | 3.4 | 2.7 | 3.1 | 2.0 | 35 |

It can be seen from the results that the average RMS values are lowered from 2.0 to 1.3 m in plan coordinates and from 3.1 to 2.0 m in height, when multi-reference stations are used for DGPS. An average level of 35% accuracy improvement can be found from the RMS coordinate differences, with the largest improvement of around 70% in plan coordinates and around 60% in height for the user station located at PKGM, whose site is located within the network composed of the three DGPS reference stations to get the maximum benefit from the differential corrections. It is also clear to see that the average RMS coordinate differences for all the user stations can be improved, except the same RMS value only shown at YMSM. It is also an evidence to show that the RMS values obtained by using multi-reference stations for DGPS navigation are more reliable and consistent than those carried out with the single reference station.

3.2. APPLYING ATMOSPHERIC CORRECTIONS. It has been mentioned that the space-related errors, such as the atmospheric delay, are basically increased with the baseline length between the reference and user stations, as those errors will not be completely eliminated by using the technique of DGPS. That is also the reason why the DGPS accuracy achieved by using multi-reference stations can be improved for medium-range DGPS.

In order to further investigate the effectiveness of applying atmospheric corrections to the medium-range DGPS network, the results tested by modelling ionospheric and tropospheric delay errors were listed in Table 6 and Table 7 for horizontal and vertical RMS coordinate differences, respectively.

| | Atmospheric correction applied | | | | | |
|--------------|--------------------------------|-----------|--------------------|----------------------------------|--------------------|--|
| User station | None | | Ionospheric delay | Ionospheric & tropospheric delay | | |
| | RMS (m) | RMS (m) | Improvement $(\%)$ | RMS (m) | Improvement $(\%)$ | |
| FLNM | 0.9 | 0.9 | | 0.8 | Ħ | |
| MZUM | 1.0 | 0.9 | 16 | 0.8 | 20 | |
| YMSM | 1.6 | 1.5 | 6 | 1.4 | 12 | |
| TMAM | 0.8 | 0.7 | 12 | 0.7 | 12 | |
| PKGM | 0.4 | 0.4 | 0 | 0.4 | 0 | |
| SHAO | 3.0 | 2.6 | 13 | 2.2 | 27 | |
| Average | 1.3 | 1.2 | | 1.1 | 14 | |

Table 6. Horizontal RMS Coordinate Differences for Atmospheric Modelling.

| | Atmospheric correction applied | | | | | |
|--------------|--------------------------------|-----------|--------------------|----------------------------------|-----------------|--|
| User station | None | | Ionospheric delay | Ionospheric & tropospheric delay | | |
| | RMS(m) | RMS (m) | Improvement $(\%)$ | RMS(m) | Improvement (%) | |
| FLNM | 1.9 | 1.9 | | 1.8 | | |
| MZUM | 1.4 | 1.4 | | 1.3 | | |
| YMSM | 3.4 | 3.4 | | 3.0 | 12 | |
| TMAM | 1.3 | 1.3 | | 1.3 | | |
| PKGM | 1.1 | 1.1 | | 1.1 | | |
| SHAO | 3.0 | 2.8 | | 2.7 | 10 | |
| Average | 2.0 | 2.0 | | 1.9 | | |

Table 7. Vertical RMS Coordinate Differences for Atmospheric Modelling.

It has been proved by the results that the RMS values can be further improved by modelling atmospheric delay errors, compared with those based on simply using multi-reference stations. When ionospheric delay model is applied to the differential range corrections, the average RMS values of coordinate differences can be reduced by 7% and 1% in plan coordinates and height, respectively. The average RMS coordinate differences obtained by applying both ionospheric and tropospheric delay models can be totally improved by 14% in horizontal coordinates and 6% in vertical coordinate. A level of up to 20% in plan coordinates and 10% in height can be even found from the RMS coordinate differences carried out with the modelling of atmospheric delay.

4. CONCLUSIONS AND SUGGESTIONS. Conclusions and suggestions on the study of medium-range DGPS using multi-reference stations and applying atmospheric corrections can be drawn as follows:

(1) The conventional DGPS accuracy was shown to be varied inconsistently for the user stations applying differential corrections, provided by different single reference stations, over a medium scale of range around the Taiwan area. The variations of RMS coordinate differences were also proved to be significantly correlated with the baseline length between the reference and user stations.

(2) The average RMS coordinate differences in both plan coordinates and height were significantly reduced by 35%, when a DGPS network composed of three reference stations was used for the Taiwan area. The RMS values tested from the user stations located within the DGPS network were even improved by a level of 60% in horizontal and vertical components.

(3) The effectiveness of applying atmospheric delay to the differential corrections, based on the modelling of ionospheric and tropospheric errors, has been proved by improving 14% and 6% of RMS coordinate differences in plan coordinates and height, respectively.

(4) The atmospheric delay models tested are simple to use, but based on a global scale. The effectiveness of using any empirical atmospheric model which is suitable to the area around the Taiwan, or developing any dynamic technique which is able to estimate the real-time atmospheric errors can be further investigated.

(5) Some other error sources for medium-range DGPS, such as the weighting algorithm to the differential corrections, the estimation of clock-related errors, and the assessment of computing satellite orbits or using IGS predict ephemeris, are all required to be further realised.

REFERENCES

- Chang, C. C. and Tseng C. L. (1998). A Geocentric Reference System in Taiwan. *Survey Review* (accepted).
- Chao, C. H. J. (1996). *Improved Modelling of High Precision Wide Area Differential GPS*. Ph.D. Thesis, University of Nottingham.
- Hofmann-Wellenhof, B., Lichtenegger H. and Collins J. (1992). *Global Positioning System: Theory and Practice*. Second Edition, Springer Verlag.
- Kee, C., Parkison B. W. and Axelrad P. (1991). Wide Area Differential GPS. *Navigation:* J*ournal of the Institute of Navigation*, Vol. 38, No. 2, pp. 123-143.
- Klobuchar, J. A. (1982). Ionospheric Corrections for Single Frequency User of the Global Positioning System. *National Telesystem Conference NTC'82, System for the 80s*, IEEE.

KEY WORDS

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