

# **TESTING ON TROPOSPHERIC MODELLING FOR GPS TRACKING STATIONS IN TAIWAN**

**Chia-Chyang Chang**

Department of Surveying and Mapping Engineering  
Chung Cheng Institute of Technology  
Tahsi, Taoyuan 335, Taiwan, ROC

**Ching-Liang Tseng**

Satellite Geoinformatics Research Center  
National Cheng Kung University  
Tainan 701, Taiwan, ROC

## **ABSTRACT**

The effect of the atmosphere has become one of the major accuracy limiting factors for GPS positioning, since a high level of positioning precision has been achieved with the improvement of observation instruments and data processing strategies. Due to the fact that the atmospheric effect caused by the tropospheric error can only be treated by using a number of atmospheric models associated with proper estimation techniques, it is important to practically test the tropospheric modelling used in the GPS data processing. This paper details some of the network tests carried out using different atmospheric models and estimation techniques for two GPS data sets made at the tracking stations in Taiwan. These tests aim to investigate the effect of the tropospheric modelling by applying standard atmospheric models, surface measured meteorological data, polynomial scale factors, and stochastic estimation of a Kalman Filter respectively, for GPS positioning. The repeatability in three components, regarded as an indicator of coordinate precision for each GPS data set, was used to realise the effectiveness of those tests based on the different tropospheric modelling conducted.

## 1. INTRODUCTION

As a fact that a radio signal passing through the earth's atmosphere and received by an antenna located on the surface of the earth will be bent and delayed, most geodetic space techniques require an accurate estimation for such atmospheric delay error induced in the measurement. The propagation media which affect the GPS signal are mainly divided into two layers, namely the ionosphere and troposphere.

Tropospheric delay errors can significantly degrade the GPS accuracy of the height component. It has also been proved that an error of 1 cm in modelling the tropospheric zenith delay can result in a height error of around 3 cm [Brunner and Welsch, 1993]. In order to improve the GPS accuracy, a standard atmospheric model is normally used in the GPS data processing to provide an estimate of the tropospheric zenith delay at a site. This estimate is then 'mapped down' to the required elevation angle using a so-called mapping function [Davis et al., 1985]. The scale factors, such as one scale factor for every station and for each observation session, or a time-varying polynomial scale factor, can then be introduced into the least squares adjustment and solved for the extra unknowns, in order to compensate for any residual error in the estimate of the tropospheric delay using only a standard atmospheric model [Shardlow, 1994].

The standard atmospheric models provide a broad approximation of expected tropospheric conditions, but they ignore the actual atmospheric conditions. Alternatively, surface measured meteorological data can then be introduced into the models to practically estimate the tropospheric delay errors. However, surface measured meteorological data is easily affected by calibration problems and is not completely representative of the atmospheric conditions dominating the upper troposphere. More recently, the measurement of water vapour using a water vapour radiometer (WVR) and stochastic estimation techniques conducted for modelling the tropospheric delay correction are progressively used to improve the feasibility of the tropospheric modelling [Tralli and Lichten, 1990].

As various atmospheric models and estimation techniques have been developed to practically determine the tropospheric delay errors, some strategies implemented to estimate the tropospheric delay corrections were carried out with the data made at some GPS tracking stations in Taiwan, and tested for their effectiveness.

## 2. EFFECTS OF PROPAGATION ERRORS

The signal delay introduced by the atmosphere depends on the refractive index along the actual path traveled from the radio source in space to the ground receiver. This propagation delay is commonly described by the refractivity (N) through the relationship

$$N = (n-1) \times 10^6 \quad (1)$$

where n is the varying refractive index of the atmosphere.

The excess path length of the signal ( $\Delta S$ ) due to the propagation delay is, thus, given by

$$\Delta S = 10^{-6} \times \int_s N ds \quad (2)$$

It can be seen that a model to determine the integral of refractivity along the path is required to estimate the propagation delay in the atmosphere, although it is believed to be difficult due to refraction and variations in atmospheric pressure, temperature, and relative humidity.

The propagation media affect radio signals at all frequencies and cause refraction with a time delay of the arriving signals. The propagation media are mainly divided into the ionosphere and troposphere. The ionosphere is the upper of the two layers, ranging between approximately 100 km and 1,000 km above the earth's surface and containing free electrons. The troposphere is the region below this, extending from the earth's

surface to a height of about 50 km, in which most of the common climatic variations occur.

Within the ionosphere, the first-order ionospheric time delay caused by its dispersive nature is inversely proportional to the square of the frequency of the radio signal. It is, therefore, to be possible to evaluate the refractivity of the ionosphere by modelling these different effects to two different frequencies transmitted from the GPS satellites. When GPS observations are made by dual-frequency receivers, normally used for geodetic purposes, this ionospheric delay error can be largely eliminated by using the so-called L1/L2 ionospheric-free observable, which is a combination of the L1 and L2 carrier phase observables.

Nevertheless, the troposphere is a non-dispersive medium for signal frequencies below 30 GHz, which is the case with the radio signals of GPS satellites. The troposphere delays the arrival of both the L1 and L2 signals by the same amount. Thus, the effect caused by the tropospheric error cannot be effectively reduced by dual-frequency GPS observations as used to eliminate the ionospheric error. Instead, estimation of the tropospheric delay is required by properly modelling.

### **3. TROPOSPHERIC DELAY MODELLING**

The error of tropospheric delay can be reduced by attempting to model the refractivity of the atmosphere along the signal path, using a common expression of

$$N = 77.642 (P/T) - 12.92(e/T) + 371900(e/T^2) \quad (3)$$

where  $P$  is the total pressure (mbar),  $T$  is the absolute temperature ( $^{\circ}\text{K}$ ), and  $e$  is the partial pressure of water vapour (mbar) [Saastamoinen, 1973]. The refractivity  $N$  can be commonly viewed as the sum of a dry component, i.e.  $N_d = 77.642 (P/T)$ , and a wet component, i.e.  $N_w = - 12.92(e/T) + 371900(e/T^2)$ .

The dry component contributes around 90% of the total tropospheric refraction, in which the zenith delay error is typically up to around 200 - 230 cm for an altitude near sea level and decreases approximately as the function of  $\csc(\vartheta)$  at other elevation angles ( $\vartheta$ ). The contribution of the wet component is smaller but more difficult to predict accurately because the domination of partial water vapour pressure ( $e$ ) varies along the path of the signal.

### 3.1 Empirical Atmospheric Models

One of the typical empirical models proposed for modelling the tropospheric delay is called Saastamoinen model [Saastamoinen, 1973], in which the tropospheric delay ( $dt$ ) can be expressed as the tropospheric zenith delay,  $\tau$ , times the mapping function,  $m(\theta)$ , i.e.

$$dt = \tau_d m_d(\theta) + \tau_w m_w(\theta) \quad (4)$$

where

$$\tau_d = 0.2277 \times 10^{-2} P \quad (4-1)$$

$$m_d(\theta) = \csc\theta [ 1 - ( B / P ) \cot^2\theta ] \quad (4-2)$$

$$\tau_w = 0.2277 \times 10^{-2} ( 1225 + 0.05 T ) e / T \quad (4-3)$$

$$m_w(\theta) = \csc\theta \quad (4-4)$$

where  $\theta$  is the elevation angle to the satellite at the antenna,  $B$  is a small correction term, and  $P$ ,  $T$  as well as  $e$  are pressure, temperature, and water vapour pressure, respectively. As it can be seen, when surface meteorological data are observed at sites, the tropospheric delay errors can be estimated using this model.

As a remedy, meteorological values can also be substituted by a standard atmospheric condition, in which the reference pressure, temperature, and humidity at sea level are introduced. For some GPS processing softwares, the theoretical meteorological values depending only on the heights of the GPS sites can then be calculated and put into Saastamoinen model to estimate the tropospheric delay errors.

One other type of empirical atmospheric models assuming a standard atmosphere, namely MAGNET, is also one example of such model that requires no any observed meteorological data or even the assumptions about the behaviour of the atmosphere [Curley, 1988]. This kind of empirical atmospheric model could effectively remove the biases introduced from poorly calibrated meteorological instruments.

### 3.2 Scale Factor Parameter Estimation

As it has been mentioned, the simplest way to carry on tropospheric modelling is to apply a standard atmospheric model, such as Saastamoinen or MAGNET, and then solve for scale factors to account for the estimated 5 - 10% error in the tropospheric modelling. This strategy assumes that the tropospheric delay changes depend only on changes in the elevation angle from the receiver to satellite. This has the effect of removing the average residual tropospheric delay error from the measurements, thus leading to improved results.

The residual tropospheric delay, which is not modelled by a standard atmospheric model, can be estimated as part of the least squares procedures. When a double difference observation equation is formed with phase observable for a baseline solution, it can be expressed as

$$\phi_{ab}^{ij} = \frac{f}{c} \rho_{ab}^{ij} + N_{ab}^{ij} + (1 + \alpha_{ab}) T_{ab}^{ij} \quad (5)$$

where  $\phi$  is the phase observable,  $\rho$  is range,  $N$  is the integer ambiguity,  $T$  is the tropospheric zenith delay, and  $\alpha$  is a scale factor to be estimated.

Typically, this scale factor parameter is solved per station per session. However, this estimated parameter is only a constant offset to the standard atmospheric model, and thus is not allowed to reflect the time-varying nature of the atmosphere. Alternatively, a polynomial tropospheric zenith delay scale factor can be introduced, such as

$$\alpha = \alpha_0 + \alpha_1(t_i - t_0) + \alpha_2(t_i - t_0)^2 + \dots + \alpha_n(t_i - t_0)^n \quad (6)$$

This polynomial model is then correlated through time via the connection of a start epoch ( $t_0$ ) and the current epoch ( $t_i$ ). As the behaviour of the model is dictated by the order of polynomial, the choice of the polynomial order will depend on the atmospheric conditions [Dodson et al., 1996]. Moreover, some other error sources affecting high precision GPS, such as ocean tide loading effects, will also be absorbed by the polynomial parameters [Chang, 1995].

### 3.3 Stochastic Estimation

An alternative technique to estimate the tropospheric delay error is to characterize atmosphere with probabilistic laws or statistical models. The effects of the troposphere on radio wave propagation can then be predicted over varying spatial dimensions and temporal scales according to a given probability density function or stochastically [Tralli and Lichten, 1990]. Simply, the mathematical model can be solved for a piecewise constant tropospheric zenith delay, based on the criteria that the zenith delay is correlated in time by some stochastic process.

The tropospheric zenith delay (TZD) can be estimated as a first-order Gauss-Markov process or a random walk process by expressing the correlation of the zenith delays between adjacent discrete time periods [Dodson et al., 1996]. The difference between the adjacent zenith delay values (DAZ) can then be written as

$$DAZ_i = (m - 1)TZD_{i-1} \pm \sigma_i \quad (7)$$

where  $m$  is a measure of exponential correlation between adjacent measurements for the sampling interval, and defined by the type of stochastic process. For the tests shown in this paper, a random walk process, in which  $m = 1$ , was used. The stochastic process noise ( $\sigma$ ) is also required in this estimation technique.

A further modification of this stochastic estimation technique can be made to solve for the zenith delay as a continuously varying stochastic parameter within a Kalman filter, which is able to produce a minimum error estimate for this technique.

#### **4. TESTS AND RESULTS**

The growth of continuously operating regional GPS network has been one of the most recent trends in GPS. In order to re-establish a high accuracy geodetic network purely based on the GPS observation, a regional GPS network consisting of eight permanent GPS tracking stations in Taiwan was set up and operated by the government from 1995. These regional GPS tracking stations also aim to provide their high accuracy three-dimensional coordinates for some high accuracy geodetic, geodynamic and navigation applications.

In order to test the effectiveness of the tropospheric modelling, the network solutions based on the data collected from part of the GPS tracking stations in Taiwan were carried out to test for their coordinate repeatabilities.

##### **4.1 Data Availability**

A total of four GPS stations, as a part of the continuously operating GPS tracking stations in Taiwan, were selected to test the effect of the tropospheric delay modelling. These four GPS stations, located at YMSM, FLNM, PKGM, and MZUM, were chosen mainly based on the consideration of their geographic distributions, which represent the locations in the North, South, and East part of Taiwan, as well as the offshore island. The full network of stations is shown in Figure 1.



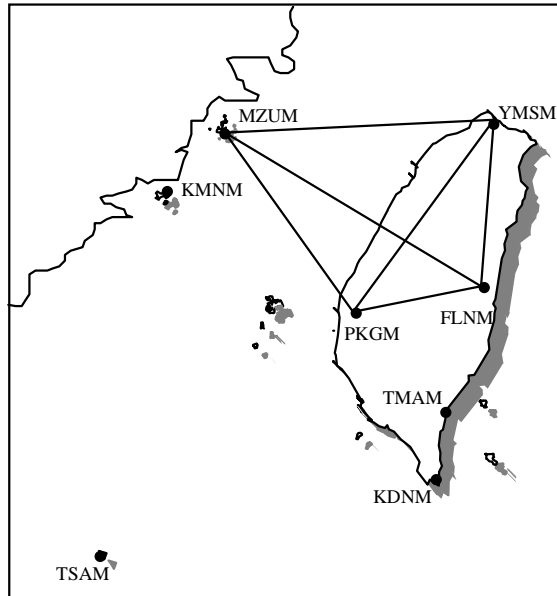


Figure 1 The GPS network tested for tropospheric modelling

Two GPS data sets archived for the tropospheric tests were obtained from 8 January to 10 January 1996, and from 22 July to 24 July 1996, respectively. These two data sets were both carried out with 3 days of 24-hour observations, and measured in the two different seasons, namely winter and summer in the same year, to represent the most different atmospheric conditions. Two 12-hour sessions in one day were independently treated during the data processing. Therefore, the results of 6-set of single-session network solutions can be used to assess the precision of coordinate determinations based on the repeatability for each GPS data set. Surface measured meteorological data were also collected at the GPS sites, provided at one hour intervals for the measurements of temperature, pressure, and relative humidity.

#### 4.2 Test Models and Techniques

All the tests carried out for the tropospheric modelling were performed by using the double difference algorithm to obtain the network adjustment solutions, based on a precise ephemeris from the IGS. The GAS (GPS Analysis Software) was used in the

data processing for the tests of the tropospheric modelling [Steward et al., 1994]. The procedure used is classified into following five main stages:

- Data converting and filtering
- Cycle slip editing
- Tropospheric modelling
- Network adjustment
- Precision assessment

During the data processing, cycle slip editing was performed by a single baseline solution where the satellite positions were held fixed by a precise ephemeris. GPS data was processed by using a combination of different frequencies, i.e. L1, L2, L1-L2, and L1/L2. A network adjustment was then computed with one 12-hour session of GPS data for all the baselines, where FLNM was fixed to its coordinates defined by a long term of GPS observations. Detailed descriptions of the atmospheric models and estimation techniques used in the GPS network adjustments are now given as follows:

- MAGNET standard atmospheric model
- Saastamoinen standard atmospheric model
- Software-defined meteorological data
- Surface measured meteorological data
- Polynomial scale factor
- Random walk process Kalman Filter

Once the GPS network solutions associated with different atmospheric models or estimation techniques are carried out, a series of adjustment output files containing the adjusted Cartesian vectors and relevant variance-covariance matrix can be used to compute the final coordinates for the network stations. The repeatabilities of three coordinate components are then available by calculating the root mean square deviation of each single-session solution from the final weighted mean.

### 4.3 Results and Analysis

Different atmospheric models and estimation techniques were assessed over the two sets of GPS observations. The repeatability tests on 6 successive single-session solutions for each GPS data set, namely Winter 96 and Summer 96, can be viewed as an indicator of coordinate precision. The results based on the tests for each atmospheric model and estimation technique are now detailed as follows.

#### 1) Standard Atmospheric Model Applied or Not Applied

It is generally believed that the precision of GPS positioning in terms of height can be effectively improved when a standard atmospheric model (AM) is introduced into the GPS data processing. In order to prove the effectiveness of applying such a standard atmospheric model, the GPS network adjustment results obtained by applying, or not applying, a standard atmospheric model, i.e. MAGNET, to solve for the tropospheric delay were compared. The scale factor, which is normally used to absorb the residual errors of tropospheric delay estimated by a standard atmospheric model, was not solved for at this stage. The repeatabilities in three-dimensional components tested for the two GPS data sets are listed in Table 1.

Table 1 Coordinate repeatabilities for GPS solutions based on using standard atmospheric model

Coordinate Component	Winter 96		Summer 96	
	AM Not Applied	AM Applied	AM Not Applied	AM Applied
N (mm)	28	7	34	12
E (mm)	77	31	72	36
H (mm)	56	28	64	68

It can be seen from Table 1 that a standard atmospheric model basically improves the GPS positioning precision over a regional scale of network solution, despite a little worse result shown in height for the Summer 96 solution. A very significant improvement can

be found from the precision of the Winter 96 in all three coordinate components, where more than half of the tropospheric errors are reduced. However, this level of improvement is only shown on the horizontal components for the Summer 96 solution.

### 2) Standard Atmospheric Model Applied with Scale Factor

The precision in height degraded in the result of the Summer 96, shown in Table 1, could be the reason that the scale factor is not associated with the standard atmospheric model. This can be proved by Table 2, where a scale factor (SF) was applied with the MAGNET standard atmospheric model attempting to absorb the residual tropospheric modelling errors.

Table 2 Coordinate repeatabilities for GPS solutions based on using a standard atmospheric model and scale factor

Coordinate Component	Winter 96		Summer 96	
	AM and SF Not Applied	AM and SF Applied	AM and SF Not Applied	AM and SF Applied
N (mm)	28	10	34	11
E (mm)	77	28	72	33
H (mm)	56	36	64	31

It is shown from the results of both GPS data sets that the use of scale factor is effective to improve the precision in all three coordinate components. The coordinate repeatabilities are impressively enhanced by a level of around 15 - 20 mm in North, 40 - 50 mm in East, and 20 - 30 mm in Height component from the results of both two GPS data sets.

### 3) MAGNET or Saastamoinen Standard Atmospheric Model

In order to realise the performance of different standard atmospheric models acting on the tropospheric modelling, two empirical atmosphere models, namely MAGNET and Saastamoinen, were applied to compare their repeatabilities. As surface measured

meteorological data was not used during this stage of GPS processing, a standard meteorological condition defined by the Bernese software was introduced into the Saastamoinen model. A scale factor was applied with both standard atmospheric models for the tropospheric modelling tests. The network adjustment results are then shown in Table 3.

Table 3 Coordinate repeatabilities for GPS solutions based on using MAGNET or Saastamoinen standard atmospheric model

Coordinate Component	Winter 96		Summer 96	
	MAGNET	Saastamoinen	MAGNET	Saastamoinen
N (mm)	10	10	11	12
E (mm)	28	28	33	33
H (mm)	36	36	31	31

It can be found that the use of two standard atmospheric models does not make any difference between the repeatabilities based on the two GPS data sets in all three coordinate components. It is, thus, an evidence to show the reliability of using the empirical atmospheric models, such as MAGNET and Saastamoinen, in the tropospheric modelling.

#### 4) Polynomial Scale Factor

Typically, one-parameter scale factor is solved for per station per session during the GPS data processing. It is implicated that only a constant offset to the standard atmospheric model is determined. It is also believed that the time-varying nature of the atmosphere is not well-considered by using only an offset instead of using a constant and a rate. The following tests based on the use of a polynomial scale factor, whose order varies from 0, i.e. solving for one parameter, to 2, i.e. solving for three parameters, were carried out with the Saastamoinen model for the two GPS data sets. The results are displayed in Table 4.

Table 4 Coordinate repeatabilities for GPS solutions based on

using different orders of polynomial scale factor

Coordinate Component	Winter 96			Summer 96		
	Order 0	Order 1	Order 2	Order 0	Order 1	Order 2
N (mm)	10	10	10	12	13	10
E (mm)	28	23	23	33	35	34
H (mm)	36	38	39	31	30	34

A level of 0 to 5 mm difference in three coordinate components can be found from the results obtained by using different orders of polynomial scale factor for the two GPS data sets. However, the most effective order of polynomial scale factor can not be easily made clear since the performance of their improved values were irregularly shown in the different coordinate components. As the choice of the optimal polynomial order might be relevant to the weather conditions, there is still a room for any further research on this tropospheric estimation technique.

##### 5) Surface Measured Meteorological Data

As the surface meteorological data measured at sites are believed to be closer to reflect an actual atmospheric condition near ground, the tropospheric delay errors estimated using this type of measured data were tested. The surface measured meteorological data was applied with the Saastamoinen standard atmospheric model, and a first order polynomial scale factor was solved for in the tests. The results obtained by using the same tropospheric modelling but in conjunction with the Bernese-defined standard meteorological data were compared, and shown in Table 5.

It is clear to see that the repeatabilities in all three coordinate components are not significantly improved when surface measured meteorological data is used to estimate the tropospheric delay, where only 1 mm improvement is shown on the height of the Winter 96 solution. The reasons could be the small variations in atmospheric condition during the data collecting, or biased surface measured data caused by the calibration errors of the measurement equipment, systematic and random observation errors, radiation effects on

the equipment, and ground proximity effects [Brunner and Tregoning, 1994].

Table 5 Coordinate repeatabilities for GPS solutions based on using defined or measured meteorological data

Coordinate Component	Winter 96		Summer 96	
	Defined	Measured	Defined	Measured
N (mm)	10	10	13	13
E (mm)	23	23	35	35
H (mm)	38	37	30	30

#### 6) Random Walk Process Kalman Filter

As it has been mentioned, a Kalman Filter can be used to solve for the tropospheric zenith delay as a continuously varying stochastic parameter. A random walk process based only on a process noise, says  $\sigma=1$ , was implemented in the GPS data processing to model the tropospheric zenith delay. The results carried out by this stochastic estimation technique were compared with those obtained by introducing the surface measured meteorological data into the Saastamoinen model and solving for a first order polynomial scale factor. The results are shown in Table 6.

Table 6 Coordinate repeatabilities for GPS solutions based on using a Kalman Filter

Coordinate Component	Winter 96		Summer 96	
	Polynomial	Kalman Filter	Polynomial	Kalman Filter
N (mm)	10	10	13	9
E (mm)	23	27	35	27
H (mm)	37	63	30	48

A slight improvement in horizontal precision is shown in the results of the Summer 96

when a stochastic estimation technique of a Kalman Filter, opposing to a traditional estimation technique of polynomial scale factor in conjunction with a standard atmospheric model, is applied to estimate the tropospheric delay. The precision in height is, however, much degraded by using this stochastic estimation technique. It is hoped to be further investigated by applying a proper stochastic process to GPS data observed in different atmospheric conditions.

## **5. CONCLUSIONS AND SUGGESTIONS**

In summary, this paper has made the following conclusions and suggestions:

1. The use of a standard atmospheric model, such as MAGNET or Saastamoinen, can effectively estimate the tropospheric delay, where the GPS precision is significantly improved not only in height but also in all three components. However, the precision in height could be degraded if the scale factor is not solved for with the standard atmospheric model in the tropospheric modelling. It is also shown that the scale factor is effective to absorb the residual tropospheric modelling errors.
2. A level of 0 to 5 mm difference can be seen on the repeatabilities when a varied order of polynomial scale factor is solved for in the tropospheric modelling. However, it is not clear to decide which order of polynomial is more reliable for this tropospheric estimation technique. As its choice might depend on the weather conditions, more GPS data sets observed in different seasons and weather conditions are expected to be tested.
3. The use of surface measured meteorological data is not able to effectively improve the precision of the GPS result, compared with those obtained by only using a standard meteorological data set. The reason is possibly the small changes in weather conditions, any calibration errors of the surface meteorological measurement equipment, or some other systematic and random observation errors resulting in a bias from the actual atmospheric conditions.



4. Compared with the estimation technique of polynomial scale factors, only a slight improvement on the repeatability is seen in the horizontal component when a stochastic estimation technique of a Kalman Filter is applied. The precision in height is, however, notably degraded by using a random walk process Kalman Filter. Comprehensive tests on more GPS data sets are required, in order to set up a criteria for the use of a stochastic estimation technique.

5. WVR data is believed to be helpful to directly provide information on the spatial and temporal variability of the tropospheric wet delay, which can not be achieved by any other tropospheric modelling techniques. The feasibility of solving tropospheric zenith wet delay during the GPS data processing needs to be investigated to effectively enhance GPS positioning accuracy.

6. A wide variety of weather and climate conditions are expected to be comprehensively tested, in order to realise the validity of different tropospheric modelling techniques in GPS data processing. A more practical and effective strategy to implement the tropospheric modelling for different scale and accuracy levels of GPS positioning is definitely required, and needs to be established.

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