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Selecting Plant Location via a Fuzzy TOPSIS Approach

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A fuzzy TOPSIS approach for selecting plant location is proposed, where the ratings of various alternative locations under various criteria and the weights of various criteria are assessed in linguistic terms represented by fuzzy numbers. In the proposed method, the ratings and weights assigned by decision makers are averaged and normalised into a comparable scale. The membership function of each normalised weighted rating can be developed by interval arithmetic of fuzzy numbers. To avoid complicated aggregation of fuzzy numbers, these normalised weighted ratings are defuzzified into crisp values. A closeness coefficient is defined to determine the ranking order of alternative locations by calculating the distances to both the ideal and negative-ideal solutions. Using the suggested method, the decision makers' fuzzy assessments with different rating viewpoints and the trade-off among different criteria are considered in the aggregation procedure to assure more convincing decision making. A numerical example demonstrates the feasibility of the proposed method.

Keywords: Fuzzy numbers; Fuzzy TOPSIS; Interval arithmetic; Membership function

1. Introduction

Selecting a plant location is very important for a manufacturing company in minimising cost and maximising use of resources. Many potential attributes (criteria) must be considered in selecting a particular plant location, including investment cost, human resources, availability of acquirement material, climate, etc. [1–3]. These attributes can be classified into two categories: subjective and objective. Subjective attributes are qualitatively defined, e.g. climatic, human resources, and objective attributes are quantitatively defined, e.g. investment cost.

Many precision-based plant location methods have been investigated [1,2,4,5]. However, in real life, the evaluation data of plant location suitability for various subjective criteria and the weights of the criteria are usually expressed in linguistic terms [6]. Thus, Liang and Wang [7] proposed a fuzzy multi-criteria decision-making method for facility site selection, where the ratings of various alternative locations under various subjective criteria and the weights of all criteria are assessed in linguistic terms represented by fuzzy numbers. Despite its merits, limitations are found in their method. In their method, the fuzzy ratings and fuzzy weights are not normalised. This cannot ensure the compatibility between them. Moreover, the membership function of each fuzzy weighted rating is not presented. These limitations deter the application of their method. To solve these problems, a fuzzy TOPSIS approach is suggested for plant location selection problems.

The technique for order preference by similarity to an ideal solution (TOPSIS) was initiated by Hwang and Yoon [8]. This technique is based on the concept that the ideal alternative has the best level for all attributes considered, whereas the negative-ideal is the one with all the worst attribute values. A TOPSIS solution is defined as the alternative which is simultaneously farthest from the negative-ideal and closest to the ideal alternative. In fuzzy TOPSIS, attribute values are represented by fuzzy numbers. Using the proposed method, the ratings assigned by decision makers to each alternative for the different criteria and the weights assigned by decision makers to each of the criteria are first averaged [7,9,10]. These averaged ratings and weights are then normalised into a comparable scale. The membership function of each normalised weighted rating of each alternative for each criterion can be developed by using interval arithmetic of fuzzy numbers. To avoid a complicated aggregation of irregular fuzzy numbers, these normalised weighted ratings are defuzzified into crisp values by a fuzzy number ranking method [11–13]. A closeness coefficient is then defined to determine the ranking order of alternatives by calculating the distances of alternatives to both the ideal and negative-ideal solutions.

Using the suggested method, the decision makers' fuzzy assessments with different rating viewpoints and the trade-off among different criteria are considered in the aggregation procedure to ensure more convincing decision making. A numerical example demonstrates the feasibility of the proposed method.

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2. Fuzzy Numbers

Definition 1. A real fuzzy number A is described as any fuzzy subset of the real line R with membership function f_A which possesses the following properties [14]:

1. f_A is a continuous mapping from R to the closed interval [0, 1].
2. $f_A(x) = 0$, for all $x \in (-\infty,a]$.
3. f_A is strictly increasing on $[a,b]$.
4. $f_A(x) = 1$, for all $x \in [b,c]$.
5. f_A is strictly decreasing on $[c,d]$.
6. $f_A(x) = 0$, for all $x \in [d,\infty)$

where a, b, c, and d are real numbers.
We may let $a = -\infty$, or $a = b$, or $b= c$, or $c = d$, or $d = +\infty$. Unless elsewhere specified, it is assumed that A is convex, normal and bounded, i.e. $-\infty < a$, $d < \infty$.

The membership function f_A of the fuzzy number A can also be expressed as:

$$f_A(x) = \begin{cases} f_A^L(x) & (a \le x \le b) \\ 1 & (b \le x \le c) \\ f_A^R(x) & (c \le x \le d) \\ 0 & otherwise \end{cases} \tag{1}$$

where $f_A^L(x)$ and $f_A^R(x)$ are the left and right membership functions of fuzzy number A, respectively.

In this paper, triangular fuzzy numbers will be used in our model. The fuzzy number A is a triangular fuzzy number if its membership function f_A is given by [15]:

$$f_A(x) = \begin{cases} (x-a)/(b-a) & (a\le x\le b) \\ (x-c)/(b-c) & (b\le x\le c) \\ 0 & otherwise \end{cases} \tag{2}$$

where a, b and c are real numbers.
Definition 2. The α-cut of fuzzy number A can be defined as [16]:

$$A^\alpha=\{x|f_A(x) \ge \alpha\},$$

where $x \in R$, $\alpha \in [0,1]$.

A^α is a non-empty bounded closed interval contained in R and it can be denoted by $A^\alpha=[A_l^\alpha,A_u^\alpha]$, where A_l^α and A_u^α are the lower and upper bounds of the closed interval, respectively. For example, if a triangular fuzzy number $A = (a, b, c)$, then the α-cut of A can be expressed as:

$$A^\alpha=[A_l^\alpha,A_u^\alpha] = [(b-a)\alpha + a,(b-c)\alpha + c] \tag{3}$$

Given fuzzy numbers A and B, $A,B \in R^+$, the α-cuts of A and B are $A^\alpha=[A_l^\alpha,A_u^\alpha]$ and $B^\alpha=[B_l^\alpha,B_u^\alpha]$, respectively. By interval arithmetic, some main operations of A and B can be expressed as follows [16]:

$$(A\oplus B)^\alpha= [A_l^\alpha+B_l^\alpha, A_u^\alpha+B_u^\alpha] \tag{4}$$

$$(A \ominus B)^\alpha= [A_l^\alpha-B_u^\alpha,A_u^\alpha-B_l^\alpha] \tag{5}$$

$$(A\otimes B)^\alpha= [A_l^\alpha\, B_l^\alpha,A_u^\alpha\, B_u^\alpha] \tag{6}$$

$$(A\oslash B)^\alpha= \left[\frac{A_l^\alpha}{B_u^\alpha}, \frac{A_u^\alpha}{B_l^\alpha}\right] \tag{7}$$

$$(A\otimes r)^\alpha= [A_l^\alpha\, r, A_u^\alpha\, r],\ r \in R^+. \tag{8}$$

3. A fuzzy TOPSIS Approach for Plant Location Selection

Assume that a committee of k decision makers (i.e. D_1, D_2, …, D_k) is responsible for evaluating m alternative locations (i.e., A_1, A_2, …, A_m) under n selection criteria (i.e. C_1, C_2, …, C_n), where the suitability ratings of alternatives under each of the criteria, as well as the weights of the criteria, are assessed in linguistic terms [6] represented by triangular fuzzy numbers. Criteria are classified into benefit (B) and cost (C).

3.1 Average Suitability Ratings and Perform Normalisation

Let $x_{ijt}=(o_{ijt},p_{ijt},q_{ijt})$, $x_{ijt} \in R^+$, $i=1, 2, \dots, m$, $j=1, 2, \dots, n$, $t=1, 2, \dots, k$, be the suitability rating assigned to alternative A_i by decision maker D_t for criterion C_j. The averaged suitability rating, $x_{ij}=(o_{ij},p_{ij},q_{ij})$, of alternative A_i for criterion C_j assessed by the committee of k decision makers can be evaluated as [7,9,10]:

$$x_{ij}=(1/k)\otimes(x_{ij1}\oplus x_{ij2}\oplus \dots \oplus x_{ijk}) \tag{9}$$

where,

$$o_{ij}=\sum_{t=1}^{k} o_{ijt}/k,\ p_{ij}=\sum_{t=1}^{k} p_{ijt}/k,\ q_{ij}=\sum_{t=1}^{k} q_{ijt}/k$$

To ensure compatibility between averaged ratings and averaged weights, the averaged ratings are normalised into a comparable scale as follows [11]:

$$\bar{x}_{ij} = x_{ij}/\sum_{i=1}^{m} x_{ij}, \text{ for each } j \in B$$

$$\bar{x}_{ij} = (x_{ij}^{+}+x_{ij}^{-}-x_{ij})/\sum_{i=1}^{m} x_{ij}, \text{ for each } j \in C \tag{10}$$

$$x_{ij}^{+} = \max_i x_{ij},\ x_{ij}^{-}=\min_i x_{ij}$$

where $\bar{x}_{ij}$ denotes the normalised value of x_{ij}, x_{ij}^{+} and x_{ij}^{-} can be determined by a ranking method.

3.2 Average Weights and Perform Normalisation

Let $w_{jt}=(a_{jt},b_{jt},c_{jt})$, $w_{jt} \in R^+$, $j=1, 2, \dots, n$, $t=1, 2, \dots, k$, be the weight assigned by decision maker D_t to criterion C_j. The averaged weight, $w_j=(a_j,b_j,c_j)$, of criterion C_j assessed by the committee of k decision makers can be evaluated as [7,9,10]:

$$w_j=(1/k)\otimes(w_{j1}\oplus w_{j2}\oplus\dots\oplus w_{jk}) \tag{11}$$

where,

$$a_j=\sum_{t=1}^{k} a_{jt}/k,\ b_j=\sum_{t=1}^{k} b_{jt}/k,\ c_j=\sum_{t=1}^{k} c_{jt}/k$$

To ensure compatibility between the averaged ratings and averaged weights, the averaged weights are normalised into a comparable scale as follows:

$$\overline{w}_j=w_j/\sum_{j=1}^{n} w_j \tag{12}$$

where $\overline{w}_j$ denotes the normalised value of w_j.

3.3 Develop Membership Function of each Normalised Weighted Rating

The membership function of each normalised weighted rating, i.e. $U_{ij}=\overline{x}_{ij}\otimes\overline{w}_j$, can be developed by interval arithmetic of fuzzy numbers. By Eqs (3), (6), and (7), the α-cuts of $U_{ij}=\overline{x}_{ij}\otimes\overline{w}_j$ can be presented as follows [10–11]:

Let,

$$\overline{x}_{ij}=s_{ij}/\sum_{i=1}^{m} x_{ij},\ s_{ij}=(e_{ij},f_{ij},g_{ij})$$

where,

$$s_{ij}=\begin{cases} x_{ij} & \text{for each } j\in B \\ x_{ij}^{+}+x_{ij}^{-}-x_{ij} & \text{for each } j\in C \end{cases}$$

$$U_{ij}^{\alpha}=\overline{x}_{ij}^{\alpha}\otimes\overline{w}_j^{\alpha}=\frac{[\left(f_{ij}-e_{ij}\right)\alpha+e_{ij},\ \left(f_{ij}-g_{ij}\right)\alpha+g_{ij}]}{\left[\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)\alpha+\sum_{i=1}^{m}o_{ij},\ \left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)\alpha+\sum_{i=1}^{m}q_{ij}\right]}\otimes\frac{[\left(b_j-a_j\right)\alpha+a_j,\ \left(b_j-c_j\right)\alpha+c_j]}{\left[\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right)\alpha+\sum_{j=1}^{n}a_j,\ \left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right)\alpha+\sum_{j=1}^{n}c_j\right]}$$

$$=\left[\frac{\left(f_{ij}-e_{ij}\right)\alpha+e_{ij}}{\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)\alpha+\sum_{i=1}^{m}q_{ij}},\frac{\left(f_{ij}-g_{ij}\right)\alpha+g_{ij}}{\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)\alpha+\sum_{i=1}^{m}o_{ij}}\right]\otimes\left[\frac{\left(b_j-a_j\right)\alpha+a_j}{\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right)\alpha+\sum_{j=1}^{n}c_j},\frac{\left(b_j-c_j\right)\alpha+c_j}{\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right)\alpha+\sum_{j=1}^{n}a_j}\right]$$

$$=\left[\frac{\left(f_{ij}-e_{ij}\right)\left(b_j-a_j\right)\alpha^2+\left[e_{ij}\left(b_j-a_j\right)+a_j\left(f_{ij}-e_{ij}\right)\right]\alpha+e_{ij}a_j}{\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right)\alpha^2+\left[\sum_{i=1}^{m}q_{ij}\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right)+\sum_{j=1}^{n}c_j\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)\right]\alpha+\sum_{i=1}^{m}q_{ij}\sum_{j=1}^{n}c_j},\right.$$
$$\left.\frac{\left(f_{ij}-g_{ij}\right)\left(b_j-c_j\right)\alpha^2+\left[g_{ij}\left(b_j-c_j\right)+c_j\left(f_{ij}-g_{ij}\right)\right]\alpha+g_{ij}c_j}{\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right)\alpha^2+\left[\sum_{i=1}^{m}o_{ij}\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right)+\sum_{j=1}^{n}a_j\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)\right]\alpha+\sum_{i=1}^{m}o_{ij}\sum_{j=1}^{n}a_j}\right] \tag{13}$$

Let

$$I_{ij1}=\left(f_{ij}-e_{ij}\right)\left(b_j-a_j\right),\ J_{ij1}=e_{ij}\left(b_j-a_j\right)+a_j\left(f_{ij}-e_{ij}\right)$$

$$K_{j1}=\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right),\ L_{j1}=\sum_{i=1}^{m}q_{ij}\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}c_j\right)+\sum_{j=1}^{n}c_j\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}q_{ij}\right)$$

$$I_{ij2}=\left(f_{ij}-g_{ij}\right)\left(b_j-c_j\right),\ J_{ij2}=g_{ij}\left(b_j-c_j\right)+c_j\left(f_{ij}-g_{ij}\right)$$

$$K_{j2}=\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right),\ L_{j2}=\sum_{i=1}^{m}o_{ij}\left(\sum_{j=1}^{n}b_j-\sum_{j=1}^{n}a_j\right)+\sum_{j=1}^{n}a_j\left(\sum_{i=1}^{m}p_{ij}-\sum_{i=1}^{m}o_{ij}\right)$$

$$V_{ij}=e_{ij}a_j,\ V_j=\sum_{i=1}^{m}q_{ij}\sum_{j=1}^{n}c_j,\ Y_{ij}=f_{ij}b_j,\ Y_j=\sum_{i=1}^{m}p_{ij}\sum_{j=1}^{n}b_j,$$
$$Z_{ij}=g_{ij}c_j,\ Z_j=\sum_{i=1}^{m}o_{ij}\sum_{j=1}^{n}a_j$$

Applying the above assumption to Eq. (13), we have two shorter equations to solve, namely:

$$\frac{I_{ij1}\alpha^2+J_{ij1}\alpha+V_{ij}}{K_{j1}\alpha^2+L_{j1}\alpha+V_j}-x=0 \tag{14}$$

$$\frac{I_{ij2}\alpha^2+J_{ij2}\alpha+Z_{ij}}{K_{j2}\alpha^2+L_{j2}\alpha+Z_j}-x=0 \tag{15}$$

Only roots in [0,1] will be retained in (14) and (15). The left membership function, i.e. $f^L_{U_{ij}}(x)$, and the right membership function, i.e. $f^R_{U_{ij}}(x)$, of U_{ij} can then be produced as:

$$f^L_{U_{ij}}(x)=\frac{(L_{j1}x-J_{ij1})+[(L_{j1}^2-4K_{j1}V_j)x^2+4(K_{j1}V_{ij}+I_{ij1}V_j-\frac{1}{2}J_{ij1}L_{j1})x+J_{ij1}^2-4I_{ij1}V_{ij}]^{1/2}}{2(I_{ij1}-K_{j1}x)}$$
$$\left(\frac{V_{ij}}{V_j}\le x\le\frac{Y_{ij}}{Y_j}\right) \tag{16}$$

$$f^R_{U_{ij}}(x)=\frac{(L_{j2}x-J_{ij2})-[(L_{j2}^2-4K_{j2}Z_j)x^2+4(K_{j2}Z_{ij}+I_{ij2}Z_j-\frac{1}{2}J_{ij2}L_{j2})x+J_{ij2}^2-4I_{ij2}Z_{ij}]^{1/2}}{2(I_{ij2}-K_{j2}x)}$$
$$\left(\frac{Y_{ij}}{Y_j}\le x\le\frac{Z_{ij}}{Z_j}\right) \tag{17}$$

For convenience, U_{ij} can be expressed as:

$$U_{ij}=\left(\frac{V_{ij}}{V_j},\frac{Y_{ij}}{Y_j},\frac{Z_{ij}}{Z_j};\ I_{ij1},J_{ij1},K_{j1},L_{j1};\ I_{ij2},J_{ij2},K_{j2},L_{j2}\right)\ (i=1\sim m,\quad j=1\sim n) \tag{18}$$

3.4 Determine Ideal and Negative-Ideal Solutions

To avoid complicated calculation of irregular fuzzy numbers, U_{ij}, i=1~m, j=1~n, are defuzzified into crisp values u_{ij}, i=1~m, j=1~n. Then we can define the ideal (A^+) and negative-ideal (A^-) solutions as:

$$A^+ = \left(u_1^+,\ldots,u_j^+,\ldots,u_n^+\right), \tag{19}$$

$$A^- = \left(u_1^-,\ldots,u_j^-,\ldots,u_n^-\right) \tag{20}$$

where $u_j^+ = \max_i u_{ij}$, $u_j^- = \min_i u_{ij}$, for each j.

3.5 Calculate the Distance of Each Alternative from A$^+$ and A$^-$

$$d_i^+ = \sum_{j=1}^{n} |u_{ij} - u_j^+| \quad (i=1\sim m) \tag{21}$$

$$d_i^- = \sum_{j=1}^{n} |u_{ij} - u_j^-| \quad i=1\sim m \tag{22}$$

where d_i^+ denotes the distance between each alternative and ideal solution, d_i^- denotes the distance between each alternative and negative-ideal solution.

3.5 Calculate Closeness Coefficient

The closeness coefficient of alternative A_i with respect to ideal solution A^+ can be defined as:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (0 < C_i < 1, \quad i=1\sim m) \tag{23}$$

A_i is closer to A^+ than to A^- as C_i approaches 1. A preference order can be determined by the descending order of C_i, i=1~m.

The fuzzy TOPSIS approach for the plant location selection is as follows:

Step 1. Form a committee of decision makers and identify the evaluation criteria.

Step 2. Determine the appropriate linguistic terms represented fuzzy numbers for the ratings of alternatives versus various criteria as well as the weights of criteria. Choose a fuzzy number ranking method.

Step 3. Pool the decision makers' opinions to obtain the averaged ratings and weights and perform normalisation.

Step 4. Develop the membership function of each normalised weighted rating and perform defuzzification.

Step 5. Determine the ideal and negative-ideal solutions.

Step 6. Calculate the distance of each alternative from ideal and negative-ideal solutions.

Step 7. Calculate the closeness coefficient of each alternative.

Step 8. Determine the ranking order of alternatives by closeness coefficients.

4. Numerical Example

In this section, a hypothetical plant location selection problem is designed to demonstrate the feasibility of the proposed method.

Step 1. Assume that a manufacturing company must select a location to build a new plant. After preliminary screening, three locations A_1, A_2, and A_3 are chosen for further evaluation. A committee of three decision makers D_1, D_2 and D_3 is formed to conduct the evaluation and selection of the three locations. Three benefit criteria, availability of skilled workers (C_1), expansion possibility (C_2), availability of acquirement material (C_3), and one cost criterion, investment cost (C_4), are considered.

Step 2. Assume that the decision makers use the linguistic rating set S = {VP, P, F, G, VG}, where VP = very poor = (0, 0, 3), P = poor = (0, 3, 5), F = fair = (2, 5, 8), G = good = (5, 7, 10), and VG = very good = (7, 10, 10) to evaluate the suitability of each alternative under each of the benefit criteria. Also assume that the decision makers employ a linguistic weighting set W = {VL, L, M, H, VH}, where VL = very low = (0, 0.1, 0.3), L = low = (0.1, 0.3, 0.5), M = medium = (0.3, 0.5, 0.7), H = high = (0.5, 0.7, 0.9), and VH = very high = (0.7, 0.9, 1), to assess the importance of all the criteria. The ranking method of total integral value with $\alpha=\frac{1}{2}$ (degree of optimism) from [12] is used in this example. Moreover, assume that the suitability ratings of alternatives versus benefit criteria are given in Table 1 and the importance weights of the criteria are given in Table 2. The investment costs (million $) assessed by decision makers for the three alternatives are A_1: (77, 82, 87), A_2: (78, 82, 89), A_3: (82, 87, 92).

Step 3. By Eq. (9), the averaged suitability rating of each alternative A_i under benefit criterion C_j from the decision-making committee can be obtained, as shown in Table 1. By Eq. (11), the averaged weights of the criteria from the decision-making committee can be obtained, as shown in Table 2.

Step 4. By Eq. (18), the normalised weighted rating for each alternative can be obtained as follows:

U_{11} = (0.0315, 0.0898, 0.2733; 0.4667, 2.4556, 3.3333, −37.0000; 0.3333, −3.5333, 5.8667, 28.3111)

U_{12} = (0.0140, 0.0716, 0.3396; 0.5333, 1.7556, 6.0000, −48.8222; 0.6000, −4.2333, 5.8667, 22.9778)

U_{13} = (0.0085, 0.0490, 0.2409; 0.4667, 1.2111, 5.7778, −47.4667; 0.5333, −3.3556, 5.6000, 22.6000)

U_{14} = (0.0630, 0.1141, 0.2008; 1.0000, 19.9000, 9.3333, −236.4667; 0.5000, −14.2000, 11.2000, 216.6667)

U_{21} = (0.0315, 0.0898, 0.2733; 0.4667, 2.4556, 3.3333, −37.0000; 0.3333, −3.5333, 5.8667, 28.3111)

U_{22} = (0.0187, 0.0800, 0.3657; 0.4667, 1.8111, 6.0000, −48.8222; 0.6000, −4.3667, 5.8667, 22.9778)

U_{23} = (0.0102, 0.0548, 0.2698; 0.4667, 1.3444, 5.7778, −47.4667; 0.6000, −3.7667, 5.6000, 22.6000)

U_{24} = (0.0622, 0.1141, 0.1964; 1.2000, 20.4000, 9.3333, −236.4667; 0.3000, −12.0000, 11.2000, 216.6667)

U_{31} = (0.0352, 0.1010, 0.2733; 0.5333, 2.7778, 3.3333, −37.0000; 0.1667, −2.6000, 5.8667, 28.3111)

Table 1. Ratings of alternatives under benefit criteria and averaged ratings.

Criteria	Candidates	Decision-makers			Averaged ratings x_{ij}
		D_1	D_2	D_3	
C_1	A_1	G	G	VG	(5.6667, 8.0000, 10.0000)
	A_2	VG	G	G	(5.6667, 8.0000, 10.0000)
	A_3	VG	VG	G	(6.3333, 9.0000, 10.0000)
C_2	A_1	F	G	F	(3.0000, 5.6667, 8.6667)
	A_2	G	F	G	(4.0000, 6.3333, 9.3333)
	A_3	G	G	F	(4.0000, 6.3333, 9.3333)
C_3	A_1	P	G	G	(3.3333, 5.6667, 8.3333)
	A_2	G	F	G	(4.0000, 6.3333, 9.3333)
	A_3	G	G	F	(4.0000, 6.3333, 9.3333)

Table 2. The weights of criteria and averaged weights.

Criteria	Decision-makers			Averaged weights w_j
	D_1	D_2	D_3	
C_1	VH	H	H	(0.5667, 0.7667, 0.9333)
C_2	H	M	H	(0.4333, 0.6333, 0.8333)
C_3	L	M	M	(0.2333, 0.4333, 0.6333)
C_4	VH	VH	VH	(0.7000, 0.9000, 1.0000)

U_{32} = (0.0187, 0.0800, 0.3657; 0.4667, 1.8111, 6.0000, −48.8222; 0.6000, −4.3667, 5.8667, 22.9778)

U_{33} = (0.0102, 0.0548, 0.2698; 0.4667, 1.3444, 5.7778, −47.4667; 0.6000, −3.7667, 5.6000, 22.6000)

U_{34} = (0.0592, 0.1076, 0.1899; 1.0000, 18.9000, 9.3333, −236.4667; 0.5000, 13.7000, 11.2000, 216.6667)

By the ranking method of total integral value with $\alpha=\frac{1}{2}$ from [12], u_{ij} can be produced as: u_{11} = 0.1095, u_{12} = 0.1033, u_{13} = 0.0719, u_{14} = 0.1191, u_{21} = 0.1095, u_{22} = 0.1141, u_{23} = 0.0807, u_{24} = 0.1180, u_{31} = 0.1167, u_{32} = 0.1141, u_{33} = 0.0807, u_{34} = 0.1123.

Step 5. By Eqs (19) and (20), the ideal and negative-ideal solutions can be obtained as:

$$A^{+}=(u_{31},u_{32},u_{33},u_{14}), \quad A^{-}=(u_{11},u_{12},u_{13},u_{34})$$

Steps 6 and 7. By Eqs (21) and (22), the distance of each alternative from ideal and negative-ideal solutions can be easily obtained; and by Eq. (23), the closeness coefficient of each alternative can be produced as: C_1 = 0.2024, C_2 = 0.7530, and C_3 = 0.7976.

Step 8. According to the closeness coefficient, the ranking order of the three alternative locations is A_3, A_2, and A_1. Thus, the best selection is location A_3.

5. Conclusions

A fuzzy TOPSIS approach for the plant location selection is proposed. Using the proposed method, the ratings and weights assigned by decision makers are averaged and normalised into a comparable scale. The membership function of each normalised weighted rating of each alternative location for each criterion is clearly developed. To avoid a complicated calculation of fuzzy numbers, these normalised weighted ratings are defuzzified into crisp values to help calculate the distances of each alternative location to both the ideal and negative-ideal solutions. A closeness coefficient is then defined to determine the ranking order of alternatives. A numerical example has demonstrated the computational process of the proposed method. The proposed method can also be applied to other fuzzy management problems.

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