Control Systems

控制系統

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- February, 2009

內容

- 簡介
- 頻域模型
- 時域模型
- 時間響應
- 互聯子系統之簡化
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- 穩態誤差
- 根軌跡技巧

Chapter 4　Time Response

建立控制系統之四個主要理由

- 系統的輸出反應 $\begin{cases} \text{強制(forced)響應} \\ \text{自然(natural)響應} \end{cases}$

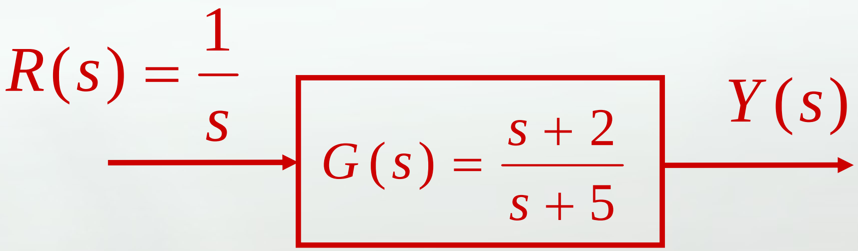
- 評估方法：微分方程→拉氏轉換→反拉氏轉換

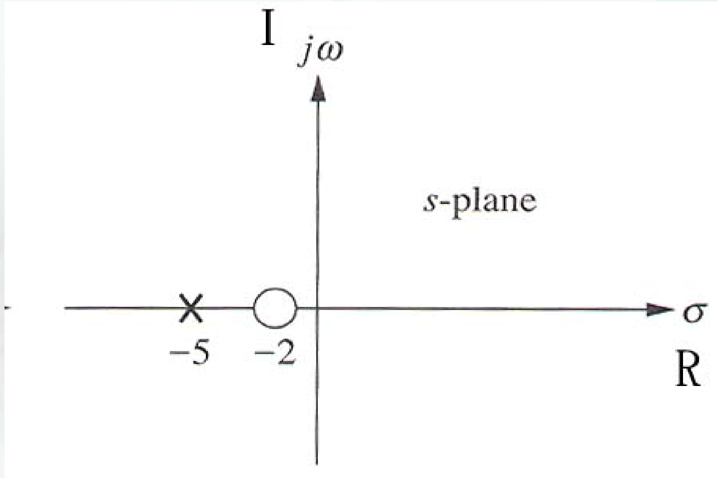
- 定性方法之一：使用極點、零點與系統響應時間之關係來分析。

極點與零點

- 轉移函數G(s)的極點：(1)能使$G(s)\to\infty$之s值(2)轉移函數分母的根。
- 轉移函數的零點：(1)能使 $G(s)\to 0$之s值(2)轉移函數分子的根。

EX:一階系統的極點與零點





Sol:

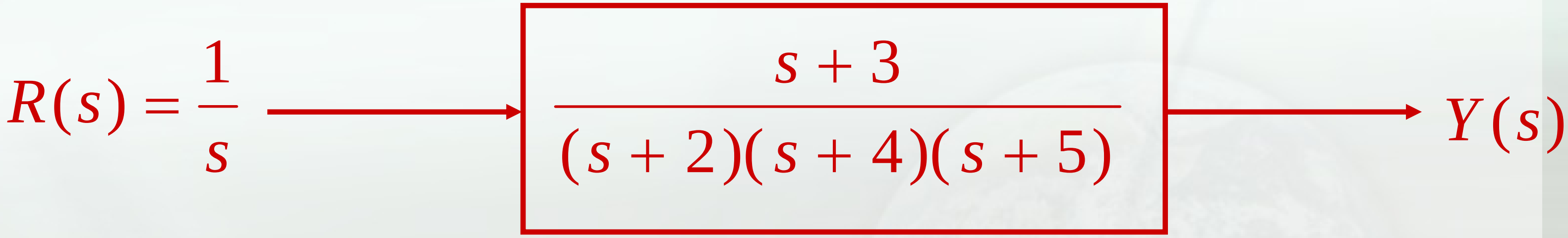
$$Y(s)=G(s)\cdot R(s)=\frac{s+2}{s+5}\cdot\frac{1}{s}=\frac{\frac{2}{5}}{s}+\frac{\frac{3}{5}}{s+5}$$

$$\Rightarrow y(t)=\frac{2}{5}+\frac{3}{5}e^{-5t}$$

討論：

- $R(s)$之極點$(s=0)$ → 強制響應 $\frac{2}{5}$
- $G(s)$之極點$(s=-5)$ → 自然響應 e^{-5t}
- $e^{-\alpha t}$，$-\alpha$為實軸之極點，若α變大 → 越快衰減至0
- 零點與極點→強制與自然響應的振幅

EX：寫出$y(t)$並指出強制與自然響應



Sol:

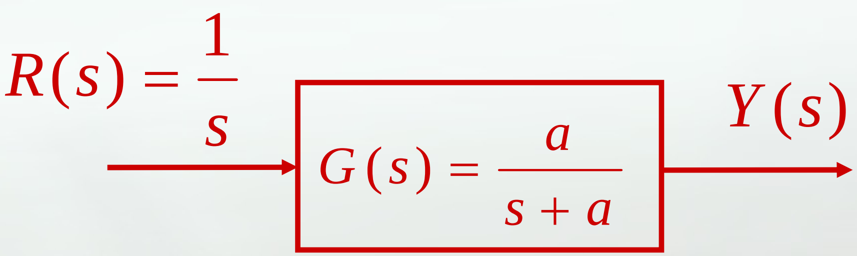
$$Y(s)=G(s)R(s)=\frac{s+3}{s(s+2)(s+4)(s+5)}$$

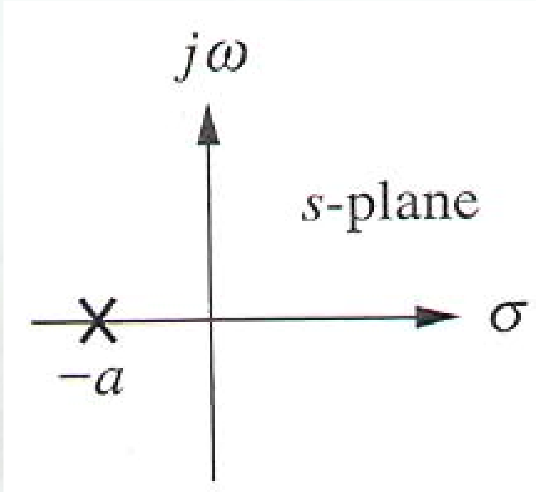
$$=\frac{k_1}{s}+\frac{k_2}{s+2}+\frac{k_3}{s+4}+\frac{k_4}{s+5}$$

$$y(t)=y_{forced}(t)+y_{natural}(t)=k_1+k_2e^{-2t}+k_3e^{-4t}+k_4e^{-5t}$$

$$\Rightarrow y_{forced}(t)=k_1\ ,\quad y_{natural}(t)=k_2e^{-2t}+k_3e^{-4t}+k_4e^{-5t}$$

一階系統：





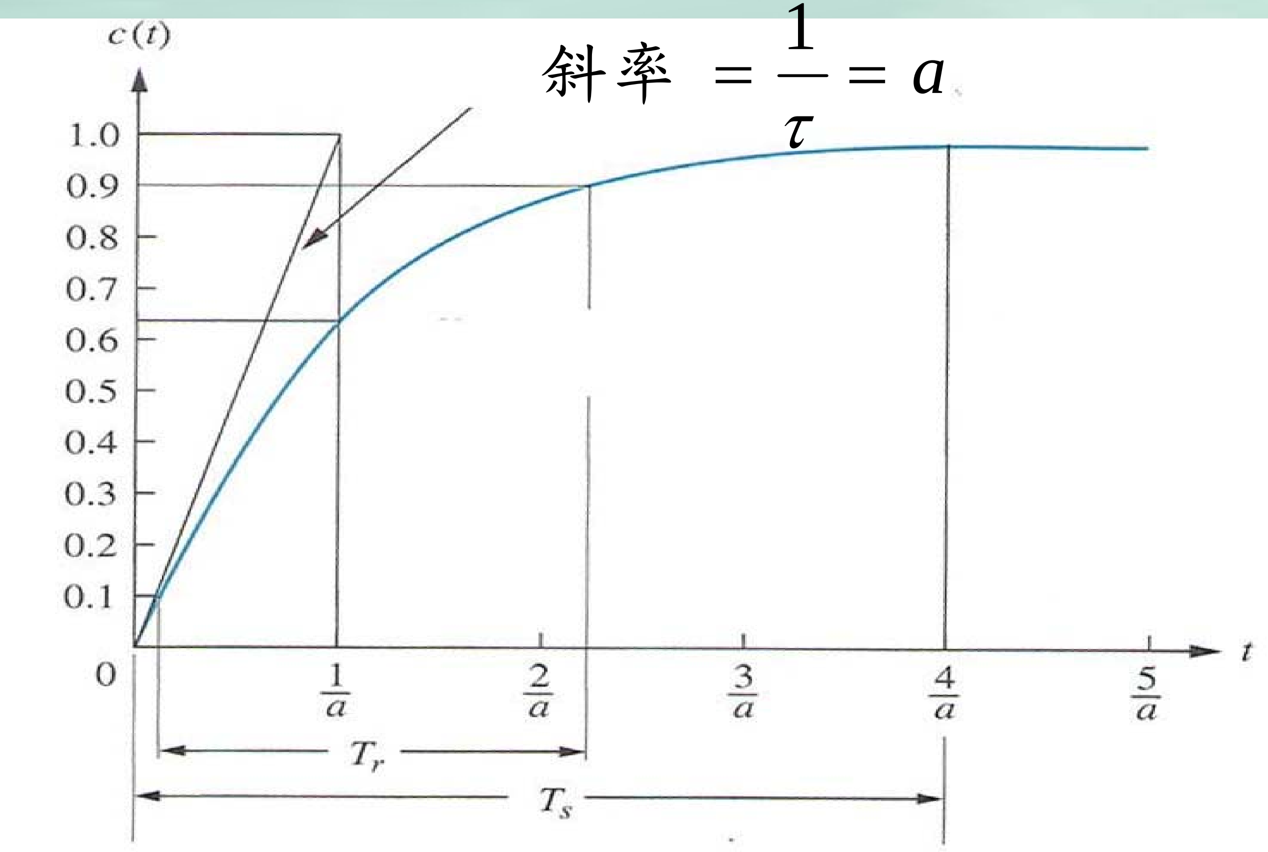
$$Y(s) = G(s) \cdot R(s)$$

$$= \frac{a}{s(s+a)} = \frac{1}{s} + \frac{-1}{s+a}$$

$$y(t) = y_{forced}(t) + y_{natural}(t)$$

$$= 1 - e^{-at}$$

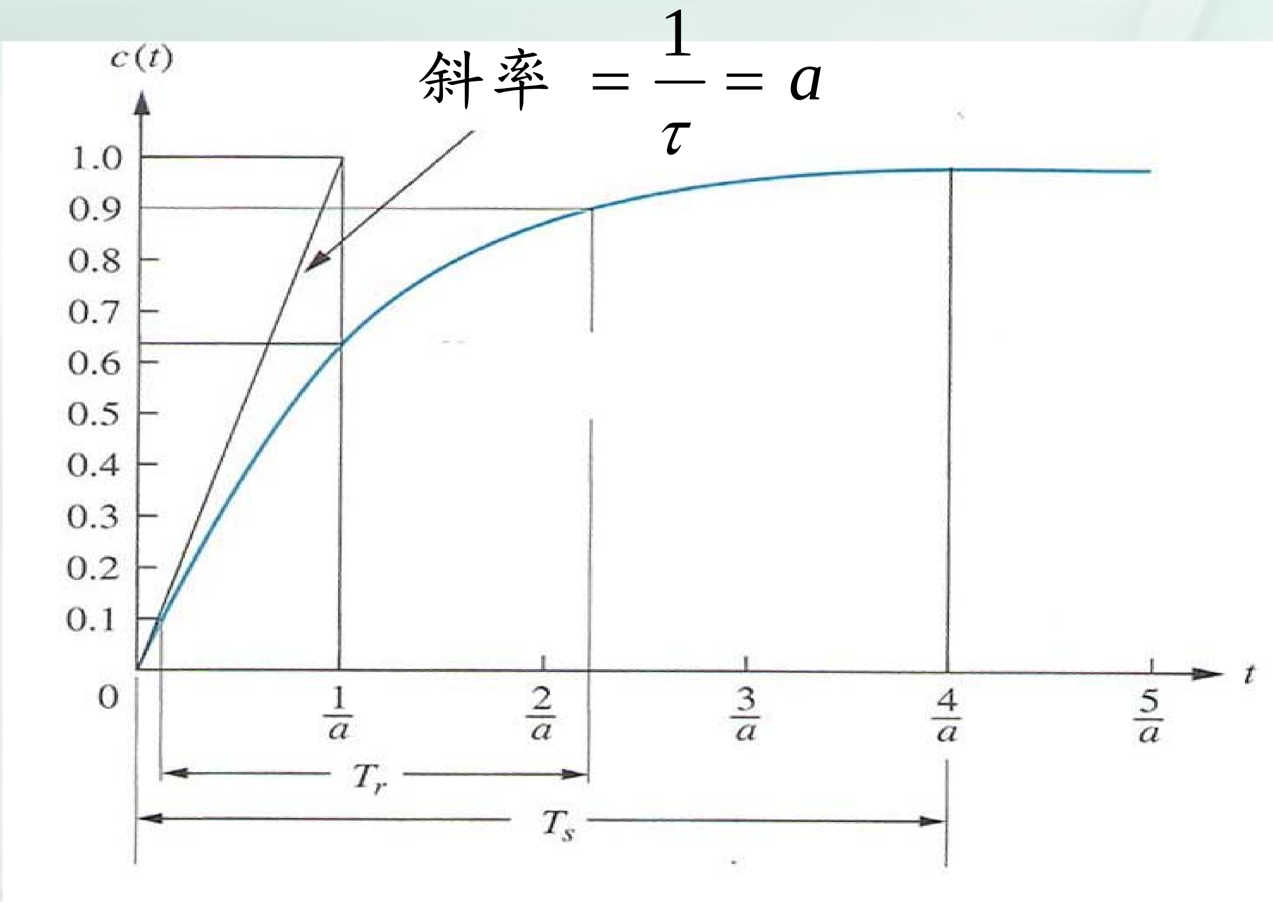
$$\Rightarrow \begin{array}{l} y_{forced}(t) = 1 \\ y_{natural}(t) = -e^{-at} \end{array}$$



$$\text{斜率} = \frac{1}{\tau} = a$$

$$\because e^{-at}\Big|_{t=\frac{1}{a}} = e^{-1} = 0.37$$

$$\therefore y(t)\Big|_{t=\frac{1}{a}} = 1 - e^{-at}\Big|_{t=\frac{1}{a}} = 1 - 0.37 = 0.63$$



- T_S :步階響應上升至終值98%時的時間

- 時間常數 τ

$$\tau = \frac{1}{a}$$

- 上升時間 Tr（Rise Time）

$$c(t) = 1 - e^{-at}$$

$$\Rightarrow \text{Tr} = ? \qquad c(t_1) = 0.1, \quad c(t_2) = 0.9$$

$$\Rightarrow \text{Tr} = t_2 - t_1$$

$$\Rightarrow \quad 0.1 = 1 - e^{-at_1} \quad \Rightarrow \quad e^{-at_1} = 0.9$$

$$\Rightarrow \quad 0.9 = 1 - e^{-at_2} \quad \Rightarrow \quad e^{-at_2} = 0.1$$

$$\Rightarrow \quad \begin{cases} -at_1 = \ln 0.9 = -0.11 \\ -at_2 = \ln 0.1 = -2.30 \end{cases}$$

$$\Rightarrow \begin{cases} t_1 = \dfrac{0.11}{a} \\ t_2 = \dfrac{2.30}{a} \end{cases} \Rightarrow \mathbf{Tr} = t_2 - t_1 = \frac{2.19}{a} \quad \#$$

- 安定時間 Ts（Setting Time）
  - 響應到達終值2%（or 5%）以內的時間

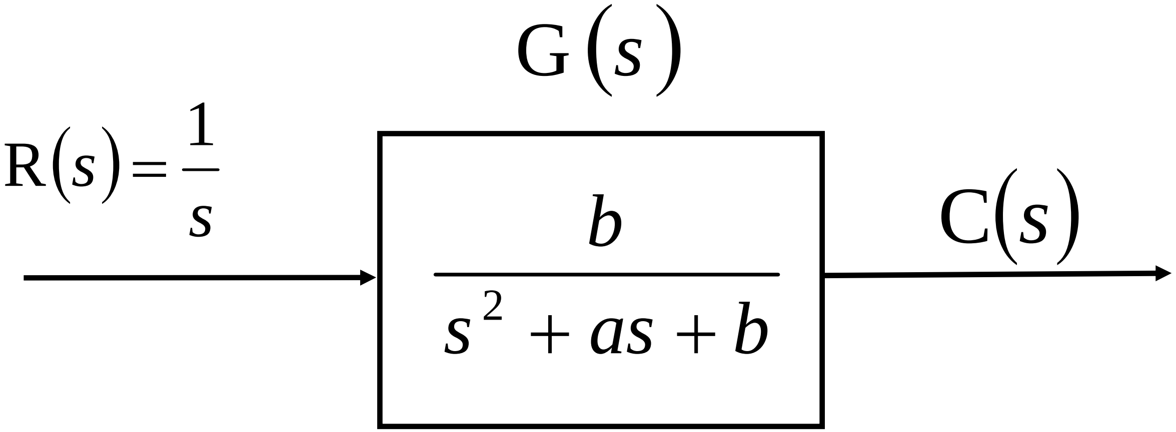
$$c(t) = 1 - e^{-at} = 0.98$$

$$\Rightarrow e^{-at} = 0.02$$

$$\Rightarrow -at = -3.91$$

$$\Rightarrow \quad t = \frac{3.91}{a} \; \# \approx \frac{4}{a}$$

4.4 二階系統簡介



1、過阻尼響應（Overdamped Response）

極點：兩個在 $-\sigma_1, -\sigma_2$ 的實根

自然響應：兩個指數，$\tau=$ 極點之倒數

$$c_n(t)=k_1e^{-\sigma_1 t}+k_2e^{-\sigma_2 t}$$

2、欠阻尼響應（Underdamped Response）

極點：兩個在$-\sigma_d \pm j\omega_d$ 的實根

τ → 時間常數

自然響應：具阻尼的弦式波，指數外包封的時間常數

$$= \text{極點實部之倒數} \left(\tau = \frac{1}{\sigma_d} \right)$$

而弦式振盪的角頻率為極點之虛部

$$c(t) = Ae^{-\sigma_d t} \cos(\omega_d t - \theta)$$

3、無阻尼響應（Undamped Response）

極點：兩個在 $\pm j\omega_1$ 的虛數

自然響應：無阻尼弦式波的角頻率等於極點之虛部

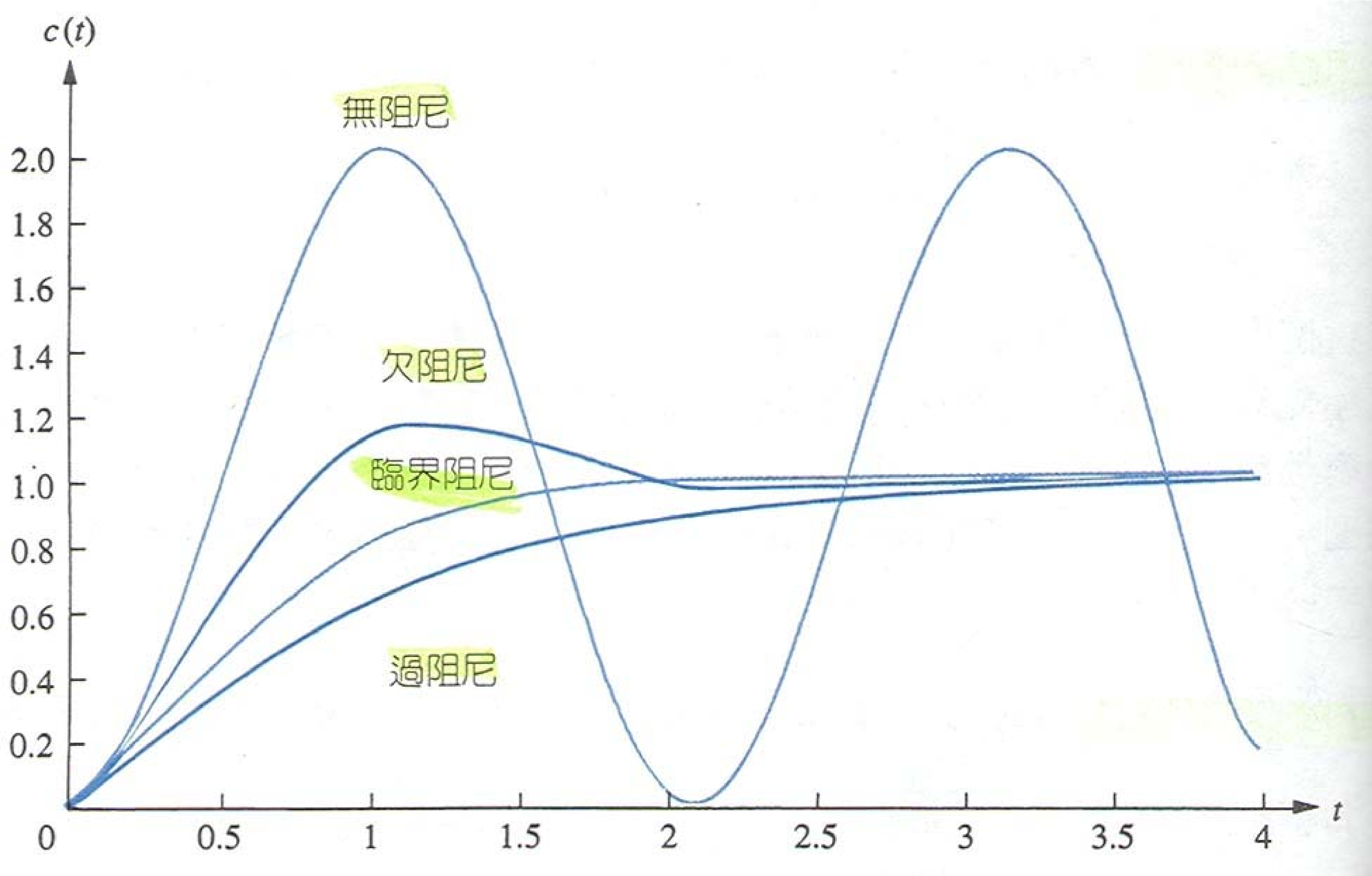
$$c(t) = A\cos(\omega_1 t - \theta)$$

4、臨界阻尼響應（Critically Damped Response）

極點：兩個在 $-\sigma_1$ 的變數

自然響應：重根，時間常數等於極點位置的倒數

$$c(t) = k_1 e^{-\sigma_1 t} + k_2 t e^{-\sigma_1 t}$$



c(t)
無阻尼
欠阻尼
臨界阻尼
過阻尼
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0.5
1
1.5
2
2.5
3
3.5
4
t

4.5 一般二階系統

- 自然頻率 ω_n（Natural Frequency）

二階步階之 ω_n ＝無阻尼時的振盪頻率

$\zeta = 0$

Ex ：

$$G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2} \longrightarrow g(t) = \omega_n \sin \omega_n t$$

- 另一種形式

$$G(s)=\frac{b}{s^2+\boxed{a}s+b}=\frac{\omega_n^2}{s^2+\boxed{2\zeta\omega_n}s+\omega_n^2}$$

$$\left(a=2\zeta\omega_n\right)$$

- 阻尼比 ζ（Damping Ratio）

$$\zeta=\frac{\text{指數衰減頻率}}{\text{自然頻率}}=\frac{|\sigma|}{\omega_n}=\boxed{\frac{a/2}{\omega_n}}$$

（σ：實部）

Ex：求 ζ 和 ω_n $\quad G(s) = \dfrac{36}{s^2 + 4.2s + 36}$

$$\omega_n^2 = 36 \quad \Rightarrow \quad \omega_n = 6$$

$$4.2 = 2 \cdot \zeta \cdot 6 \quad \Rightarrow \quad \zeta = \frac{4.2}{12} = 0.35$$

#

- $G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

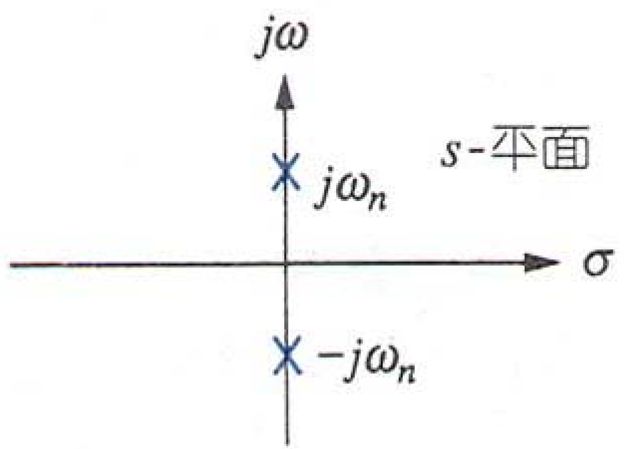
$$\Rightarrow s = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

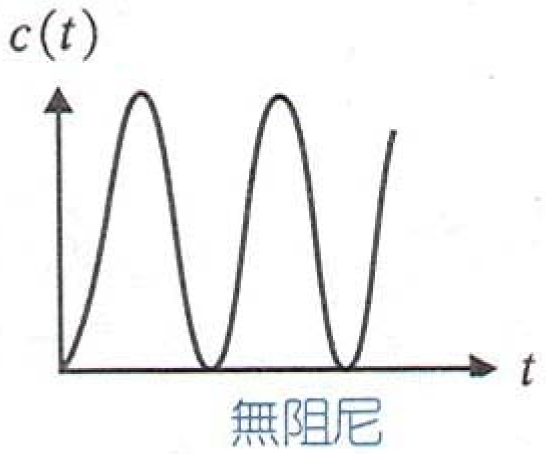
#

＊

1、無阻尼響應

$$\zeta = 0 \quad , \quad G(s) = \frac{\omega_n^2}{s^2 + \omega_n^2} \Rightarrow s = \pm j\omega_n$$

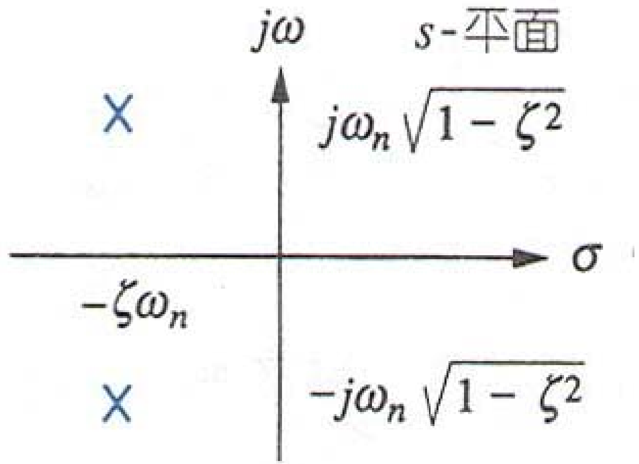


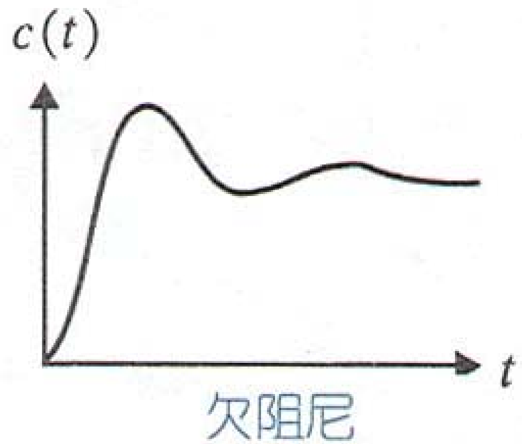


2、欠阻尼響應

$$0<\zeta<1\ ，\ G(s)=\frac{\omega_n^{\ 2}}{s^2+2\zeta\omega_n s+\omega_n^{\ 2}}$$

$$\Rightarrow s=-\zeta\omega_n\pm j\omega_n\sqrt{\zeta^2-1}=-\zeta\omega_n\pm j\omega_n\sqrt{1-\zeta^2}$$

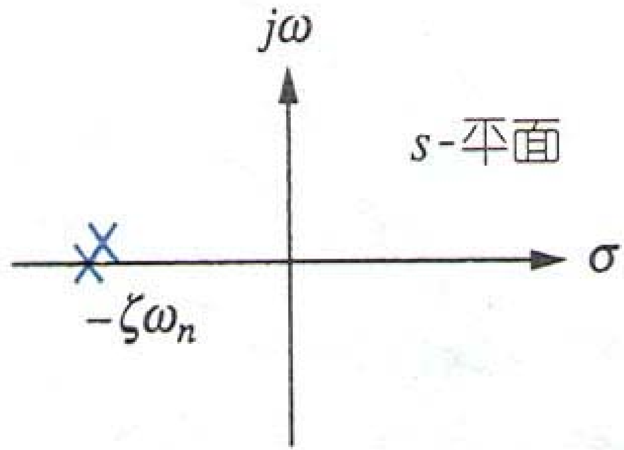


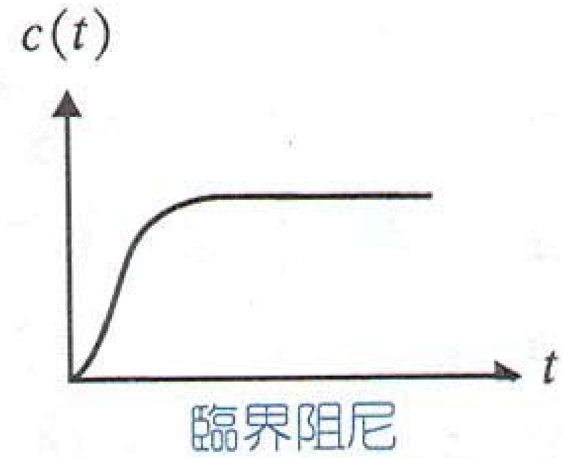


3、臨界阻尼響應

$$\zeta = 1 \quad , \quad G(s) = \frac{\omega_n^{\ 2}}{s^2 + 2\omega_n s + \omega_n^{\ 2}}$$

$$\Rightarrow s = -\zeta\omega_n$$

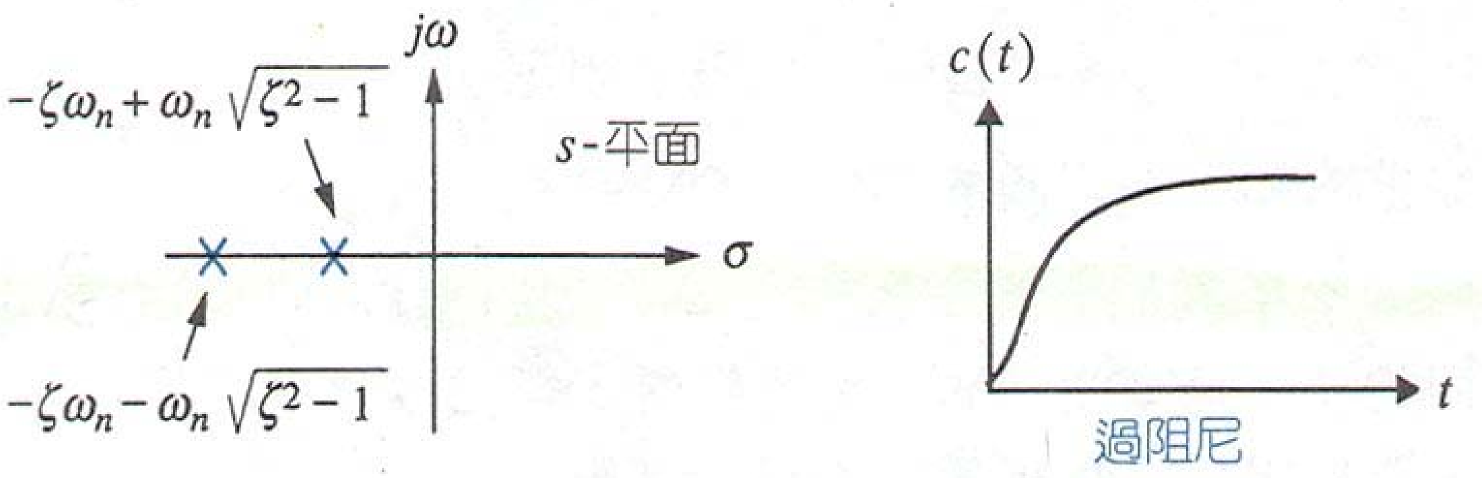




4、過阻尼響應

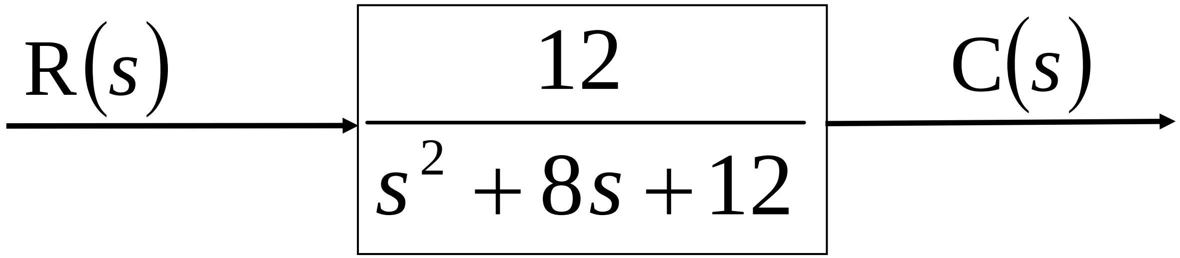
$$\zeta > 1 \quad , \quad G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\Rightarrow s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$



Ex ：判斷下圖之阻尼特性 並求出 ζ 及 ω_n

（a）

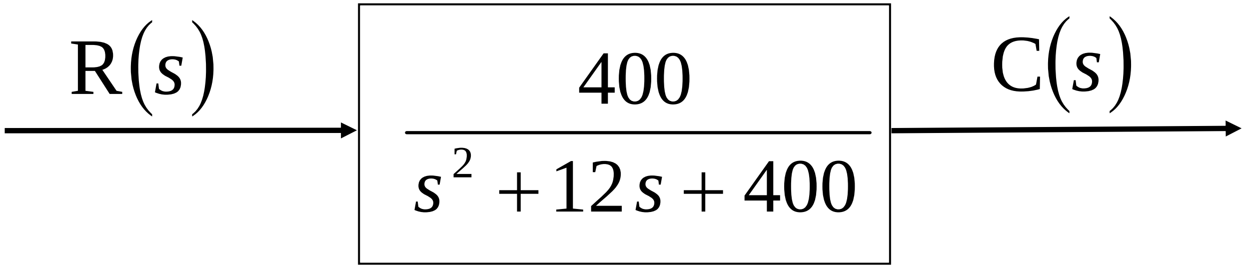


$$\omega_n^2 = 12 \Rightarrow \omega_n = 3.46$$

$$2 \cdot \zeta \cdot 3.46 = 8 \Rightarrow \zeta = \frac{8}{2 \times 3.46} = 1.156$$

$$\because \zeta > 1 \Rightarrow$$ 過阻尼響應 #

（b）

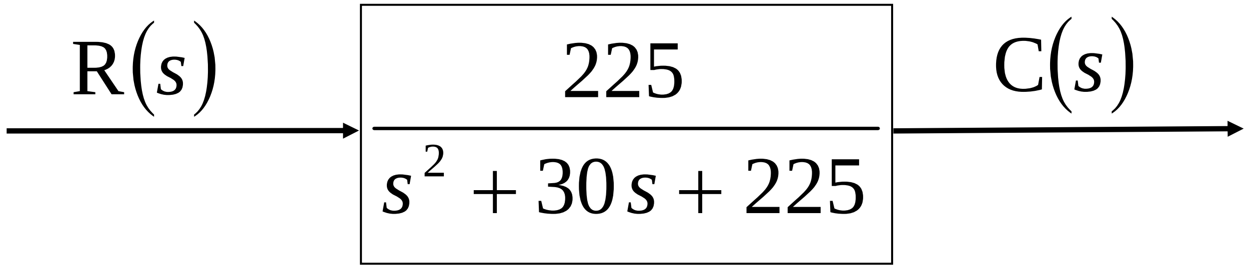


$$\omega_n^2 = 400 \Rightarrow \omega_n = 20$$

$$2 \cdot \zeta \cdot 20 = 12 \Rightarrow \zeta = \frac{12}{40} = 0.3$$

$$\because 0 < \zeta < 1 \Rightarrow$$ <u>欠阻尼響應</u> #

(c)

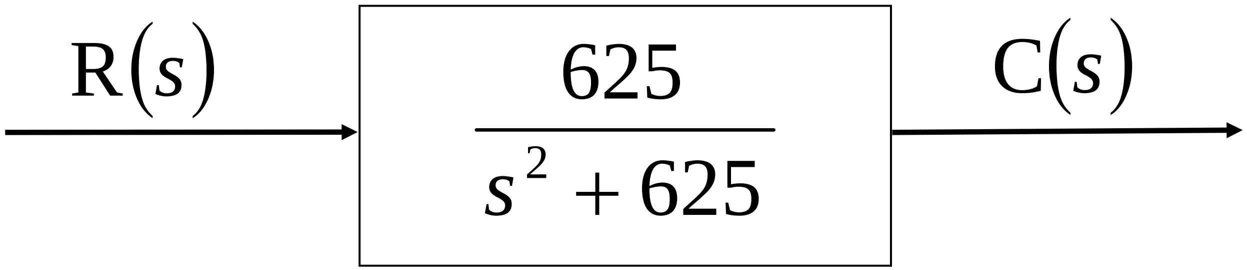


$$\omega_n^{\,2} = 225 \quad \Rightarrow \quad \omega_n = 15$$

$$2 \cdot \zeta \cdot 15 = 30 \quad \Rightarrow \quad \zeta = 1$$

$$\because \zeta = 1 \quad \Rightarrow$$ <u>臨界阻尼響應</u> #

（d）



$$\omega_n^{\,2} = 625 \Rightarrow \omega_n = 25$$

$$2 \cdot \zeta \cdot 25 = 0 \Rightarrow \zeta = 0$$

$$\because \zeta = 0 \Rightarrow$$ 無阻尼響應 #

4.6 欠阻尼二階系統－實際問題普遍的模型

$$R(s) \longrightarrow \boxed{G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}} \longrightarrow C(s)$$

- 步階響應 $\longrightarrow R(s) = \dfrac{1}{s}$

$$\Rightarrow C(s) = R(s) \cdot G(s) = \frac{\omega_n^2}{s\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}$$

$$= \frac{k_1}{s} + \frac{k_2 s + k_3}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

∵欠阻尼 $\zeta < 1 \Rightarrow$

$$C(s) = \frac{1}{s} + \frac{-s - 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} + \frac{-(s+\zeta\omega_n) - \zeta\omega_n}{(s+\omega_n)^2 + \omega_n^2 - \zeta^2\omega_n^2}$$

$$= \frac{1}{s} + \frac{-(s+\zeta\omega_n) - \frac{\zeta}{\sqrt{1-\zeta^2}}\omega_n\sqrt{1-\zeta^2}}{(s+\omega_n)^2 + \omega_n^2\left(1-\zeta^2\right)}$$

$$c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t\right)$$

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos\left(\omega_n\sqrt{1-\zeta^2}\,t - \phi\right)$$

$$\phi = \tan^{-1}\left(\zeta \Big/ \sqrt{1-\zeta^2}\right)$$

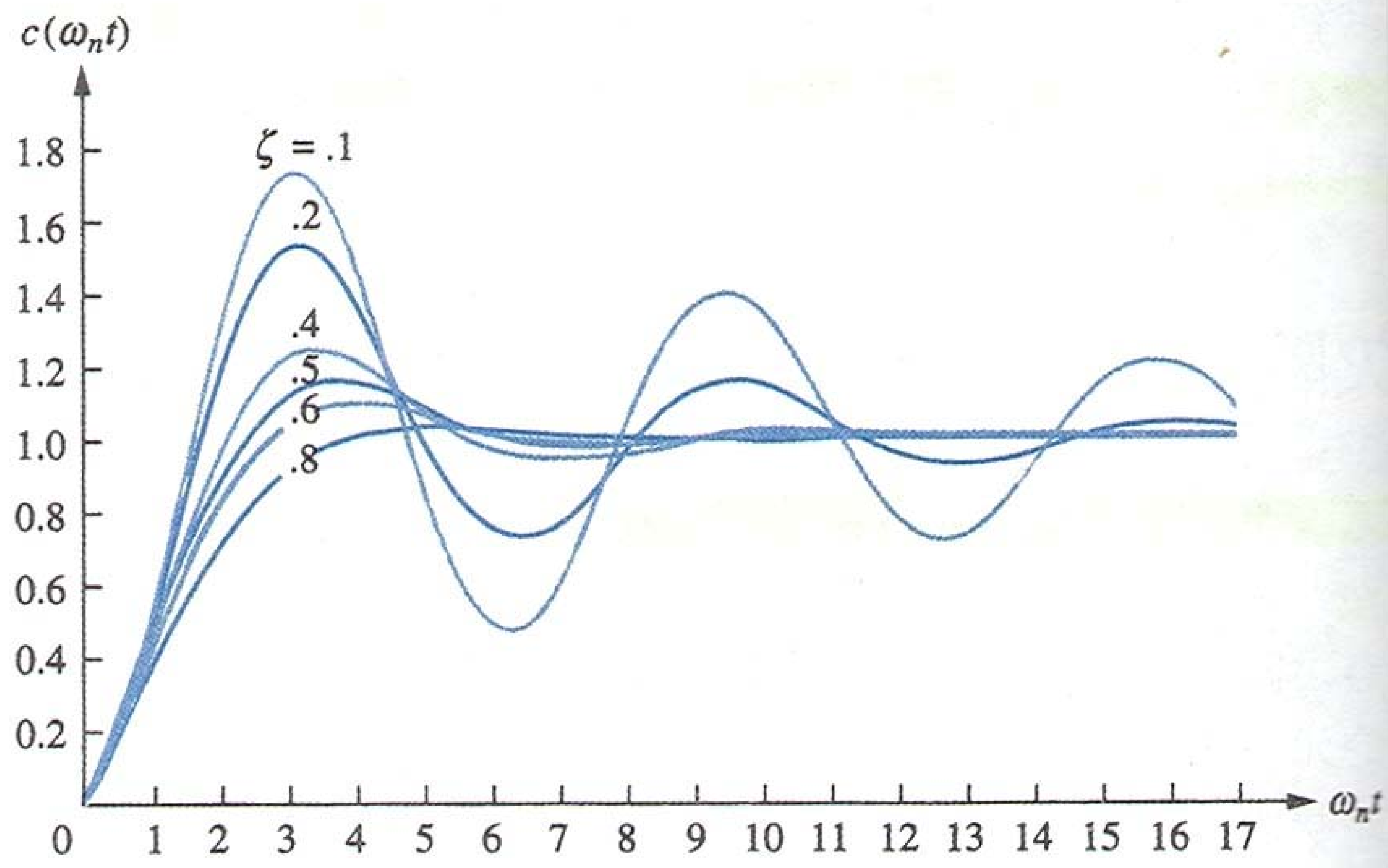


圖 4.13　不同阻尼下，二階欠阻尼系統的響應

• 欠阻尼響應的參數（二階或二階以上適用）

1、峰值時間（peak time），T_p：響應到達第一個最大值的時間

2、超越量百分比，%OS：在峰值時間之超越量與穩態值之百分比（or最後值）

3、安定時間（setting time），T_s：暫態的阻尼振盪達到±2%穩定值所需的時間

4、上升時間（rise time），T_r：波形從穩定值0.1制0.9所需的時間

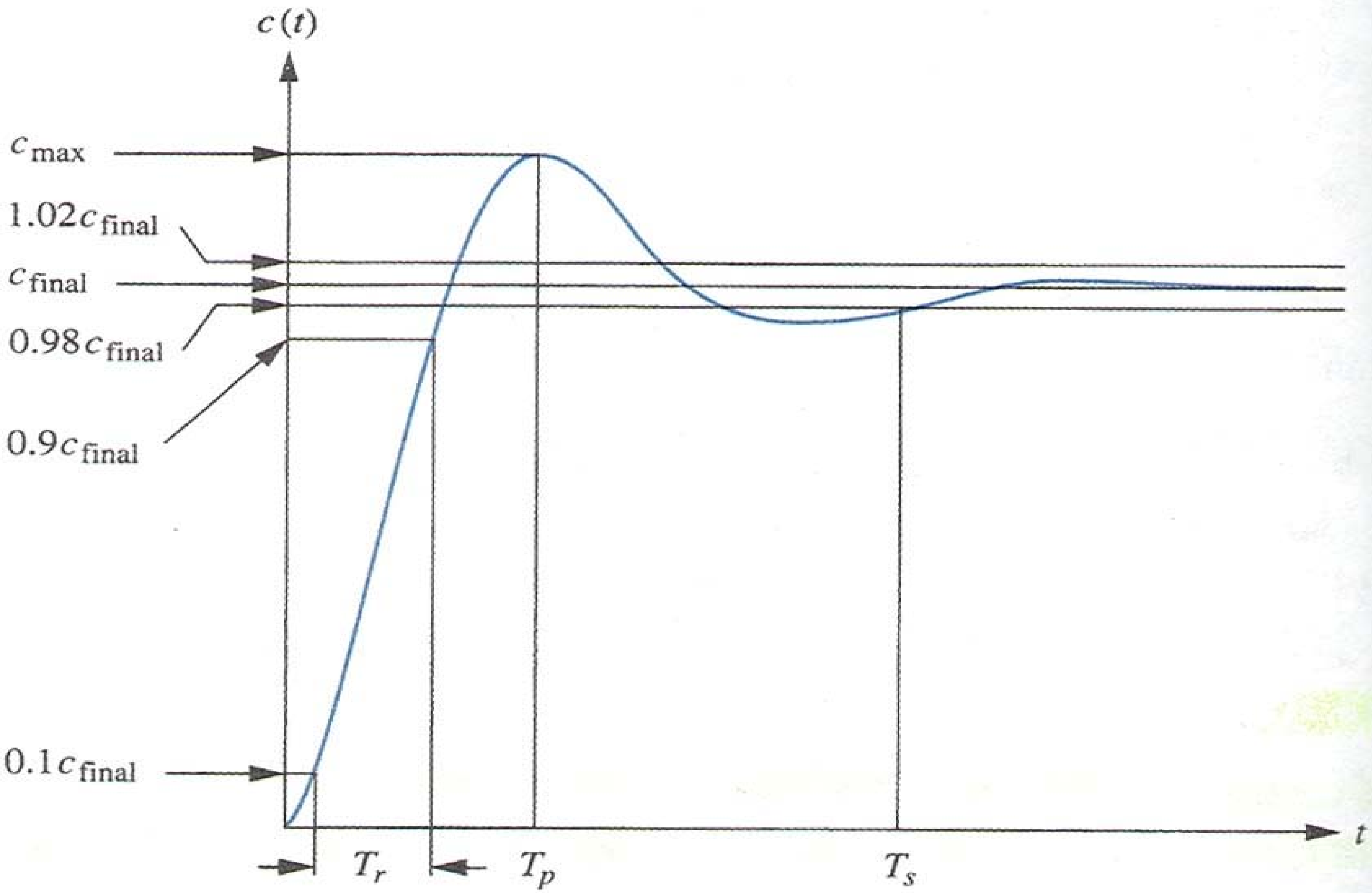


圖 4.14　二階欠阻尼響應的規格說明

- T_p 的計算

將 $c(t)$ 微分令其為0之第一時間即為Tp

$$\dot{c}(t)=0 \quad 求 \quad t$$

亦可

$$L\left[\dot{c}(t)\right]=sC(s)=s\cdot\frac{\omega_n^{\,2}}{s\left(s^2+2\zeta\omega_n s+\omega_n^{\,2}\right)}$$

$$=\frac{\omega_n^{\,2}}{s^2+2\zeta\omega_n s+\omega_n^{\,2}}$$

$$L\left[\dot{c}(t)\right]=\frac{\omega_n^{\ 2}}{\left(s^2+2\zeta\omega_n s+\omega_n^{\ 2}\right)}=\frac{\dfrac{\omega_n}{\sqrt{1-\zeta^2}}\,\omega_n\sqrt{1-\zeta^2}}{\left(s+\zeta\omega_n\right)^2+\omega_n^{\ 2}\left(1-\zeta^2\right)}$$

$$\Rightarrow\ \dot{c}(t)=\frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\sin\omega_n\sqrt{1-\zeta^2}\,t=0$$

$$\Rightarrow\ \omega_n\sqrt{1-\zeta^2}\,t=n\pi$$

$\because$ 第一個 $t=Tp$

$$t=\frac{n\pi}{\omega_n\sqrt{1-\zeta^2}}$$

$$\Rightarrow\ Tp=\frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

- **%OS 的計算**（From Fig 4.14）

$$\%OS = \frac{C_{\max} - C_{\text{final}}}{C_{\text{final}}} \times 100$$

$$\because c(t) = 1 - e^{-\zeta\omega_n t}\left(\cos\omega_n\sqrt{1-\zeta^2}\,t + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\omega_n\sqrt{1-\zeta^2}\,t\right)$$

$$C_{\max} = C(Tp) = 1 - e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)}\left(\underset{1}{\cos\pi} + \frac{\zeta}{\sqrt{1-\zeta^2}}\underset{0}{\sin\pi}\right)$$

$$= 1 - e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)}$$

$C_{\text{final}} = 1$（**from** $c(t)$ **or 終值定理**）

$$\Rightarrow \%OS = \frac{e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)}}{1} \times 100$$

$$\%OS = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)} \times 100$$ #

反之亦可由 $\%OS$ 求 ζ

$$\Rightarrow \zeta = \frac{-\ln\left(\%OS / 100\right)}{\sqrt{\pi^2 + \ln^2\left(\%OS / 100\right)}}$$

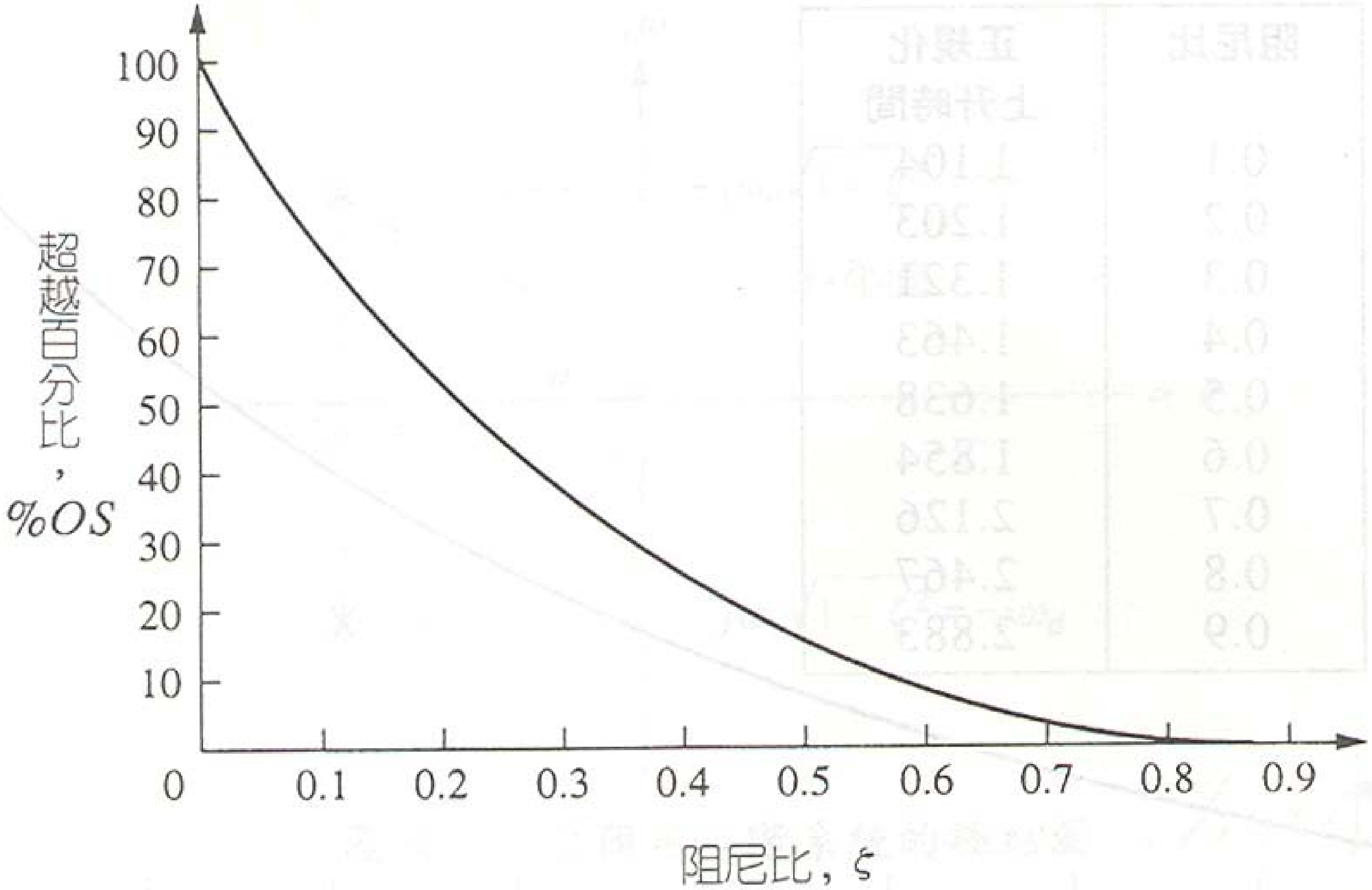


圖 4.15　超越百分比和阻尼比之關係

- T_s 的計算

$C_{\text{final}} \pm 2\%$

or

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos\left(\omega_n\sqrt{1-\zeta^2}\, t - \phi\right) = 0.98$$

$$\Rightarrow \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos\left(\omega_n\sqrt{1-\zeta^2}\, t - \phi\right) = 0.02$$

假設 $\cos\left(\omega_n\sqrt{1-\zeta^2}\,t-\phi\right)\overset{(\max)}{=}1$ 即保守估計θ

$$\Rightarrow \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}=0.02$$

$$\Rightarrow e^{-\zeta\omega_n t}=0.02\sqrt{1-\zeta^2}$$

$$\Rightarrow -\zeta\omega_n t=\ln\left(0.02\sqrt{1-\zeta^2}\right)$$

$$\Rightarrow t = \frac{-\ln\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega_n}$$

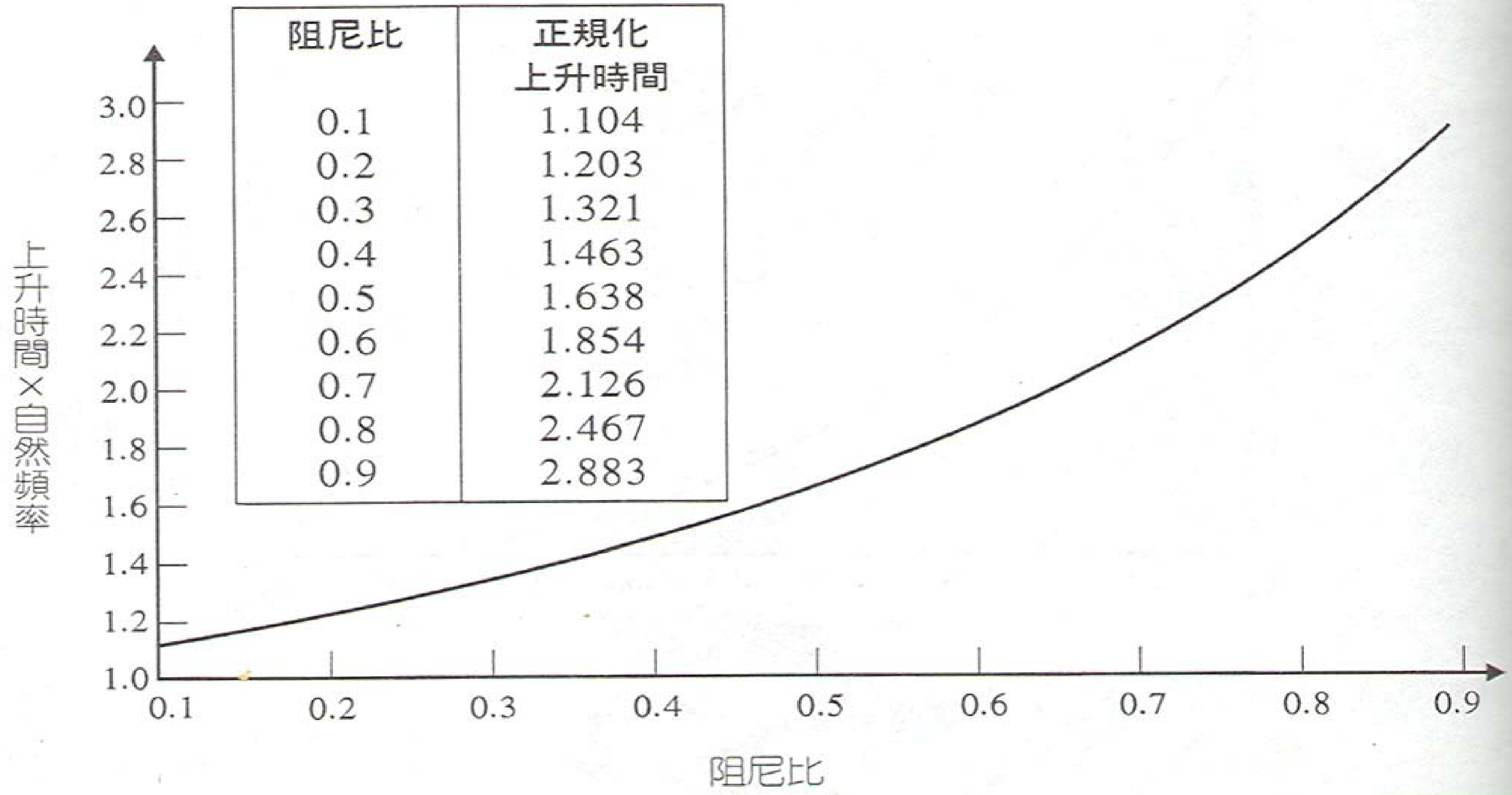
又 $\zeta = 0 \sim 0.9$

$$\Rightarrow -\ln\left(0.02\sqrt{1-\zeta^2}\right) = 3.91 \sim 4.74 \approx 4$$

$$\Rightarrow \quad T_s \approx \frac{4}{\zeta\omega_n} \quad \#$$

• T_r 的計算：

無法精確分析上升時間與阻尼比的關係，但可求出 ω_n 、 T_r **VS** ζ 之關係



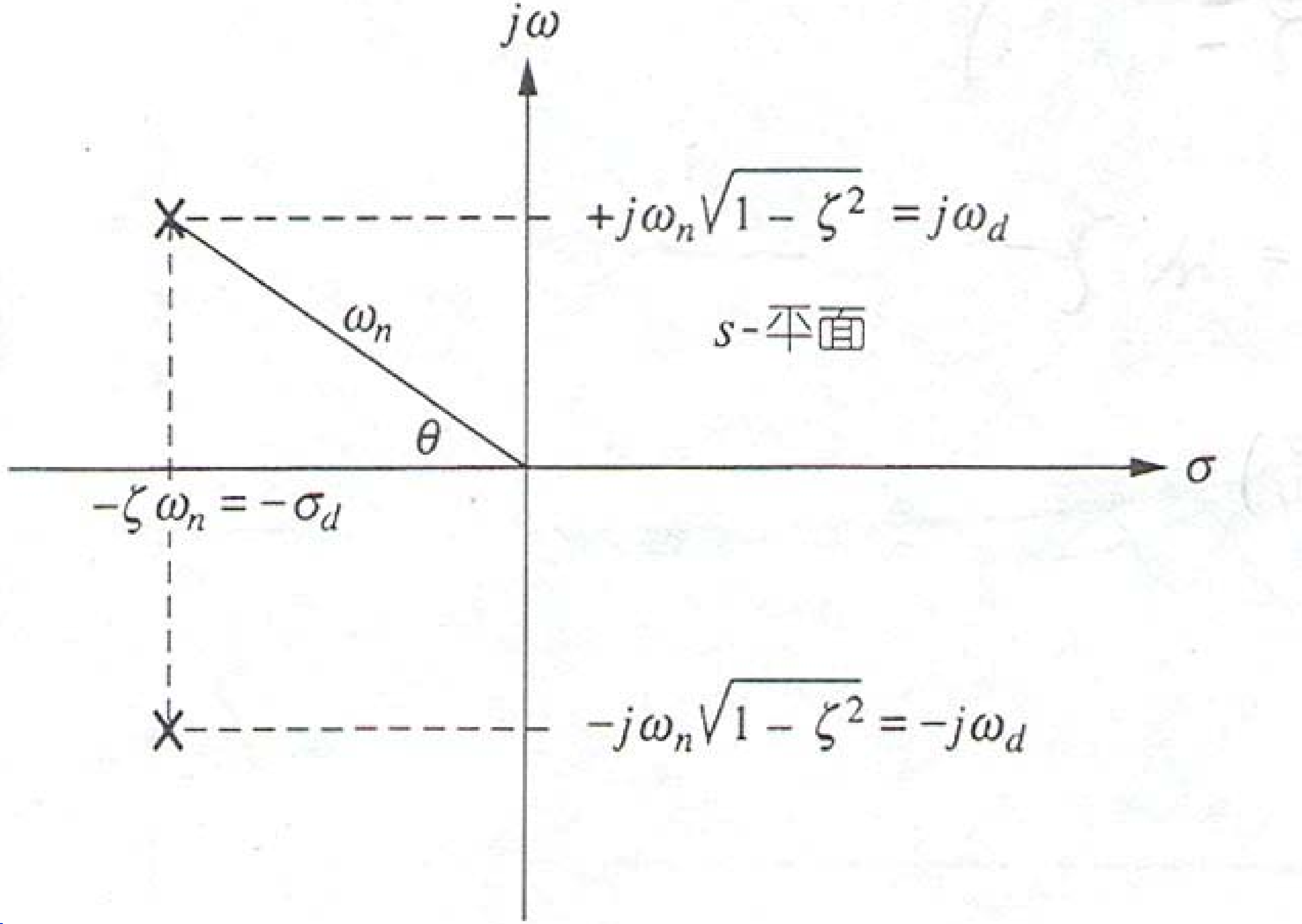
阻尼比	正規化上升時間
0.1	1.104
0.2	1.203
0.3	1.321
0.4	1.463
0.5	1.638
0.6	1.854
0.7	2.126
0.8	2.467
0.9	2.883

- ω_d：極點虛部，稱為振盪的阻尼頻率

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \quad \Rightarrow \quad T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{\pi}{\omega_d}$$

- σ_d：極點實部，稱為指數阻尼頻率

$$\sigma_d = \zeta \omega_n$$



$\zeta = \cos\theta$

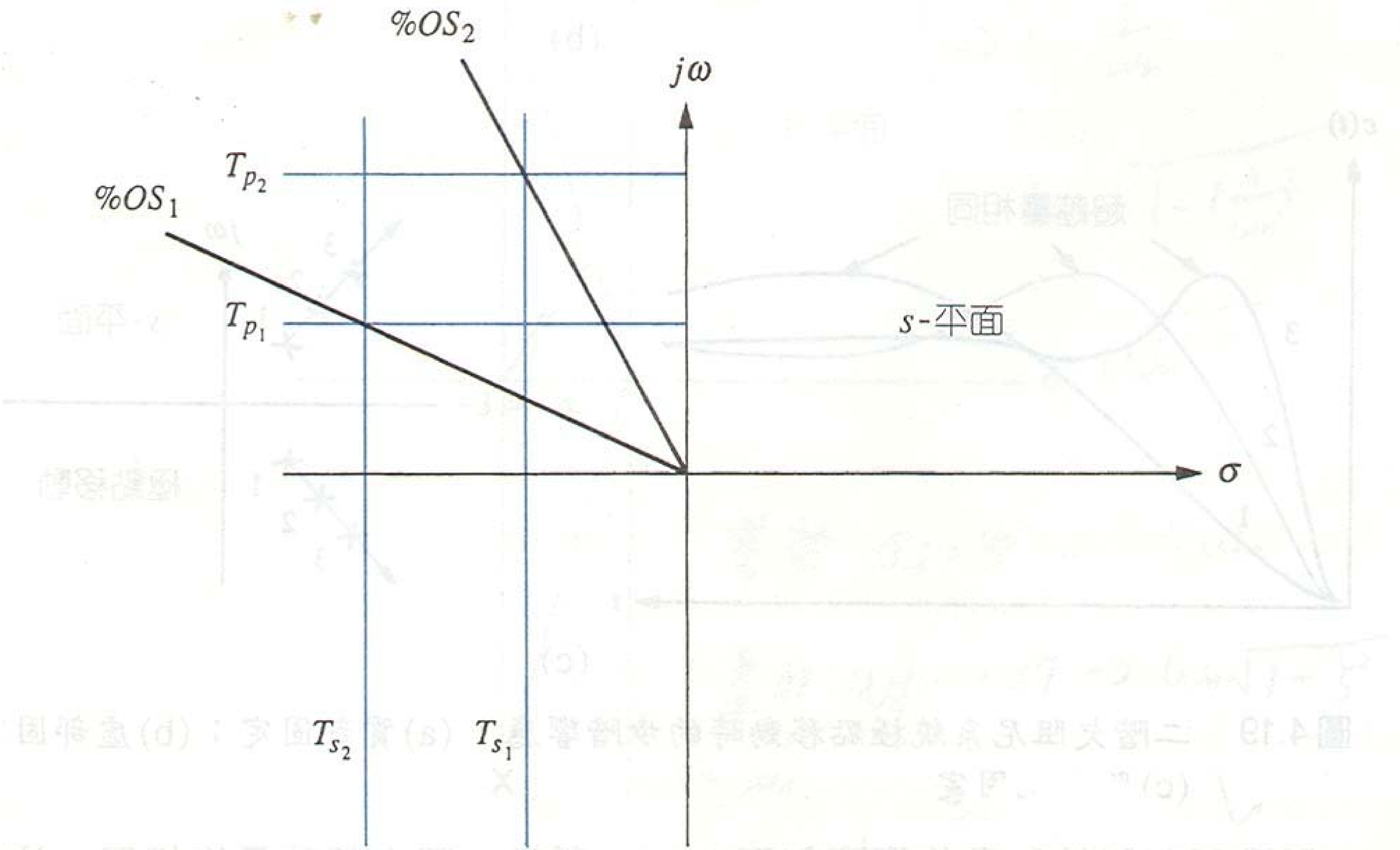


圖 4.18

$$T_{s_2} < T_{s_1}\ ;\ T_{p_2} < T_{p_1}\ ;\ \%OS_1 < \%OS_2$$

Ex：求 ζ，ω_n，T_p，$\%OS$ 和 T_s

$$G(s)=\frac{100}{s^2+15s+100}$$

$$\omega_n=10$$

$$2\cdot\zeta\cdot 10=15 \quad \Rightarrow \quad \zeta=0.75$$

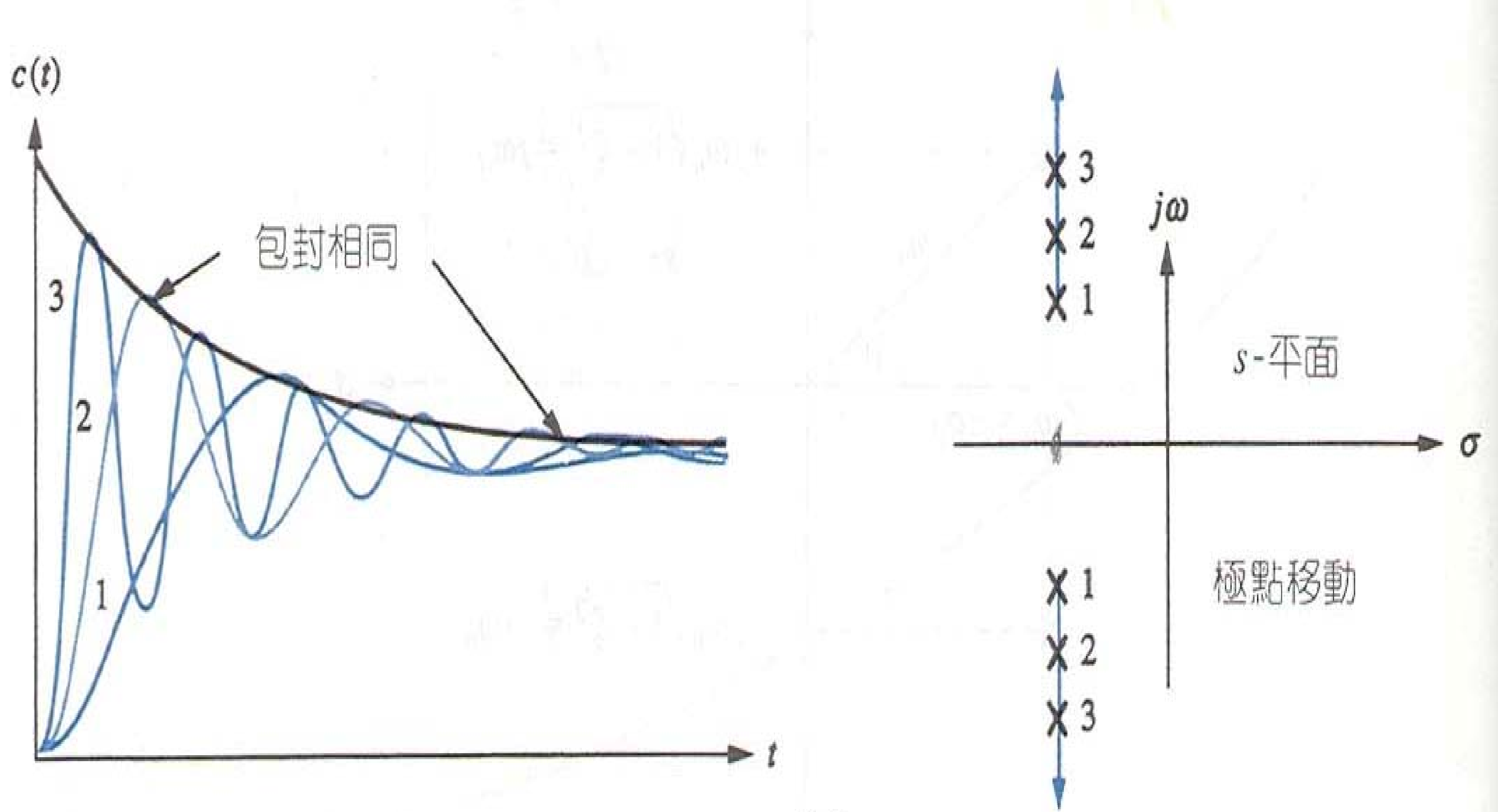
又

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{3.14}{10\sqrt{1-0.75^2}} = 0.475 \text{ sec}$$

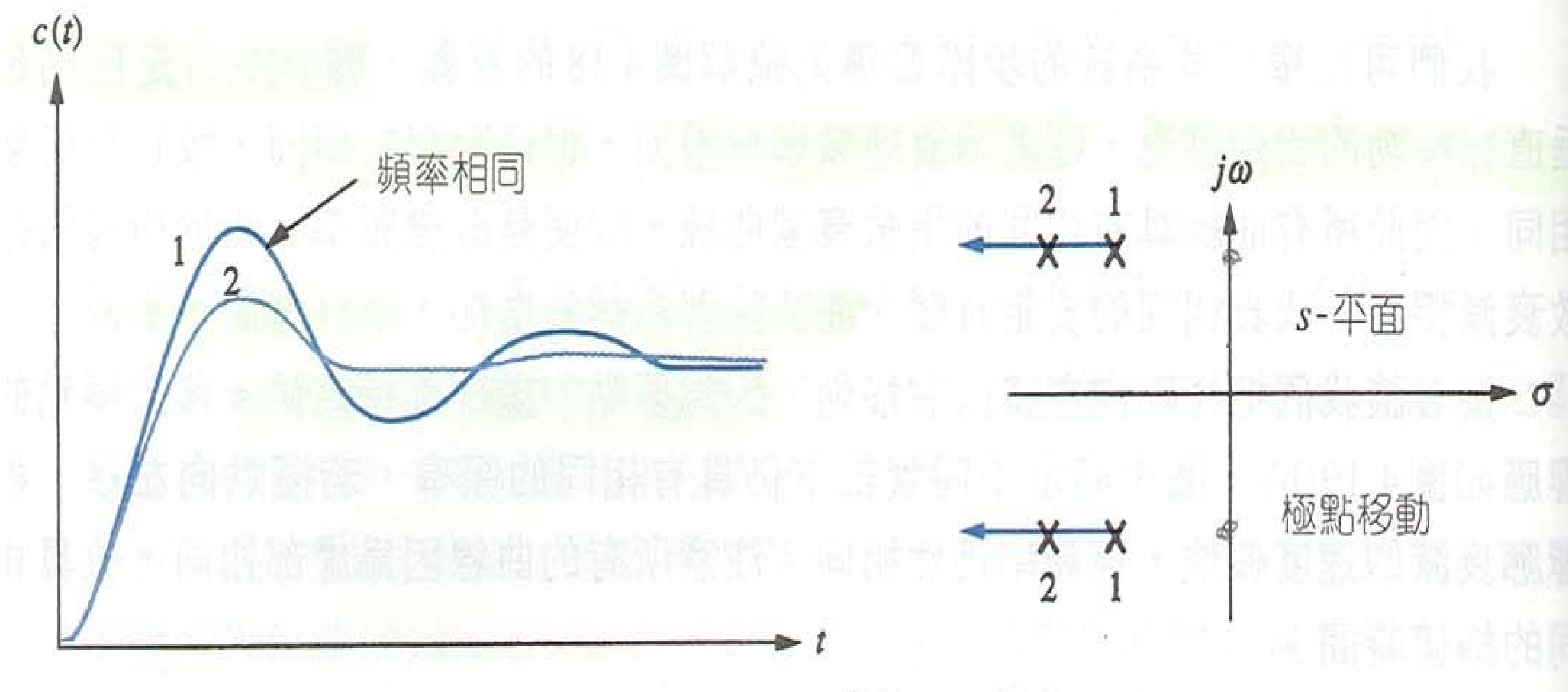
$$\%OS = e^{-\left(\zeta\pi / \sqrt{1-\zeta^2}\right)} \times 100 = e^{-\left(0.75\pi / \sqrt{1-0.75^2}\right)} \times 100$$

$$= 2.838\%$$

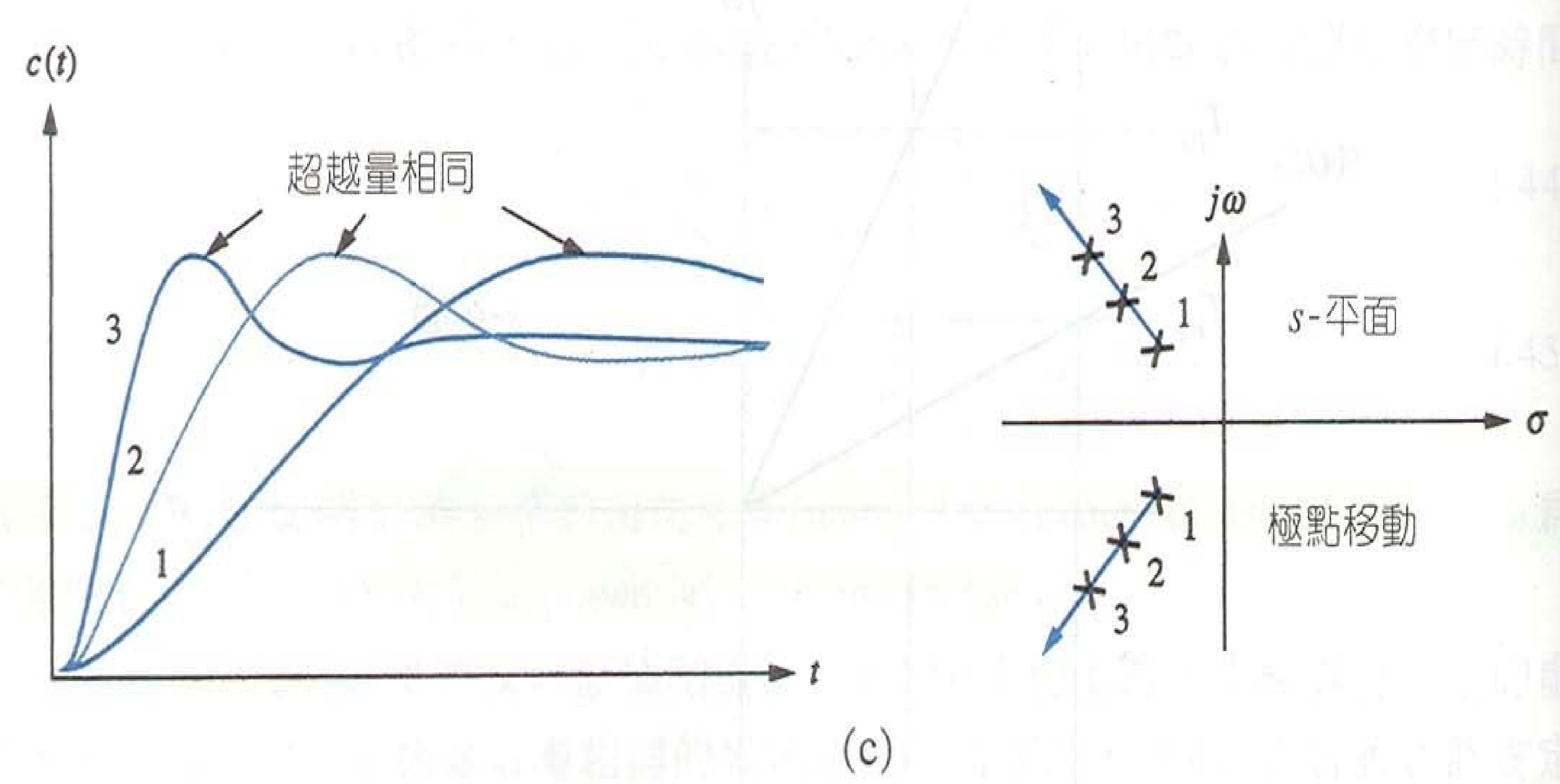
$$T_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{0.75 \times 10} = 0.533 \text{ sec}$$ #



(a)



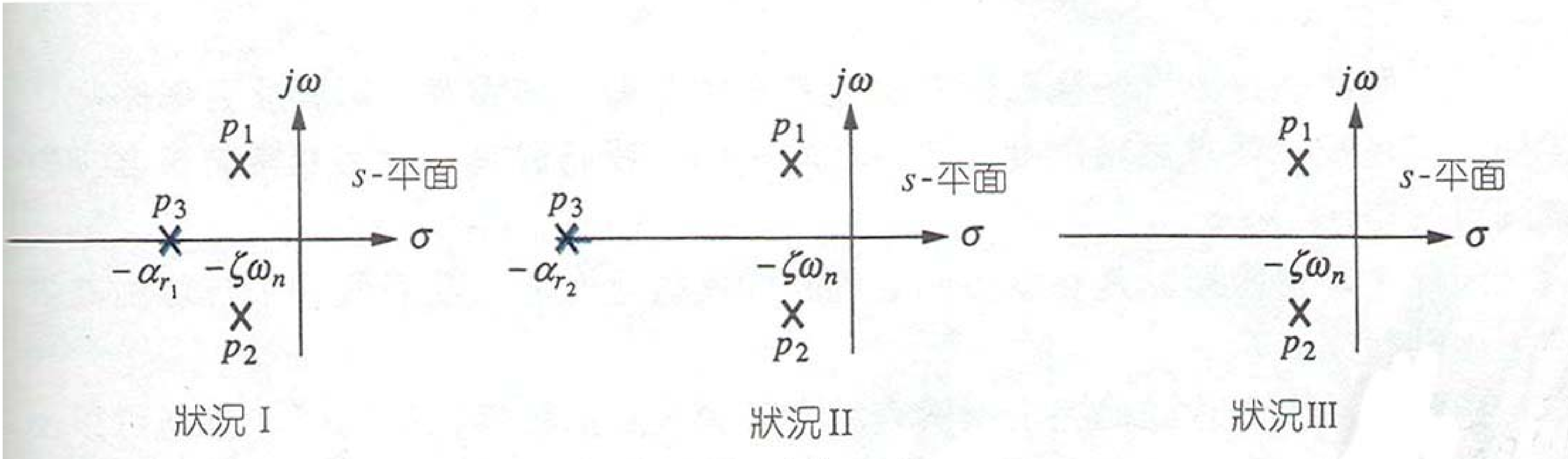
(b)



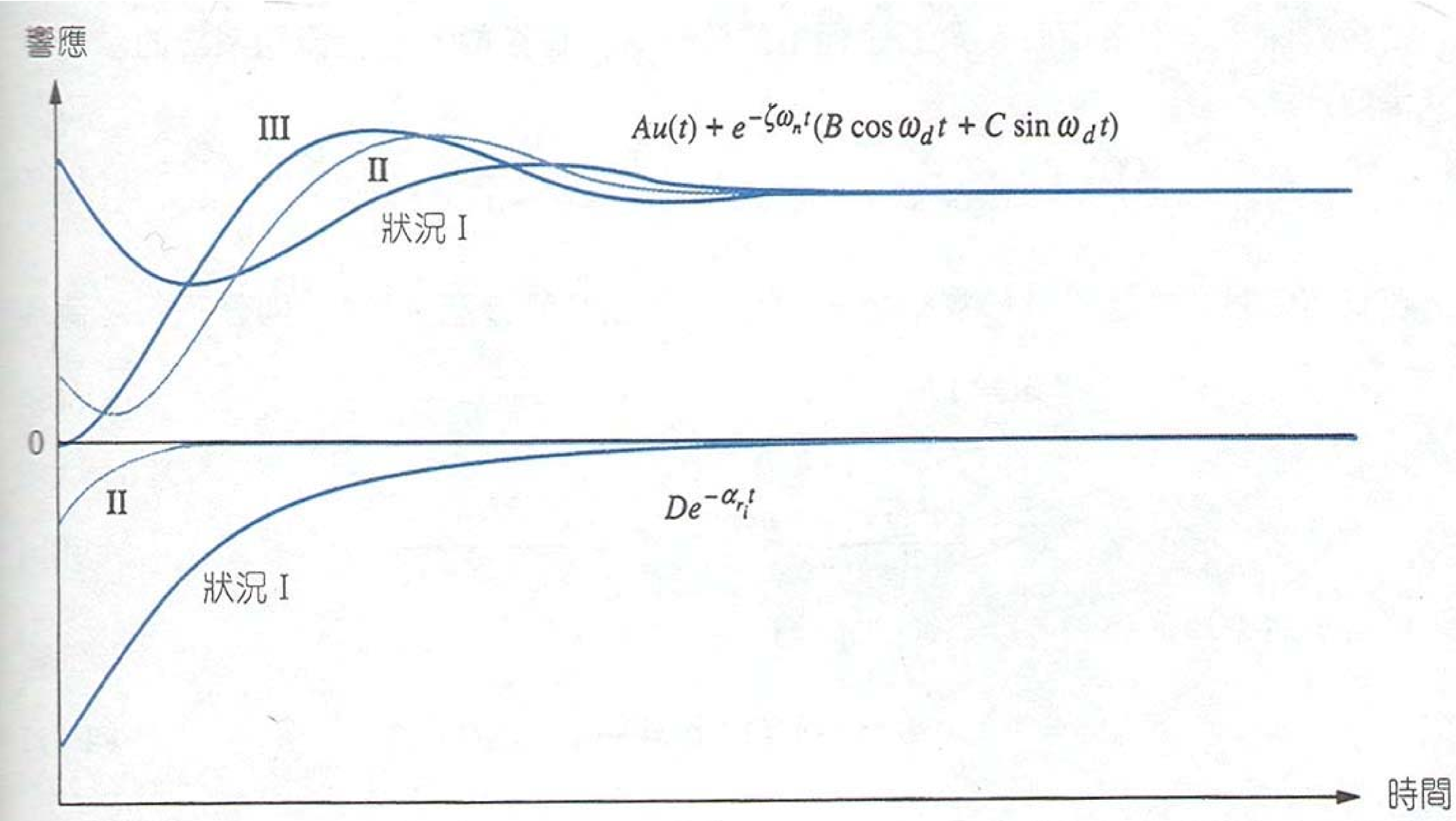
極點 y軸, x 軸 及 x , y 軸移動 ς 固定

4.7 增加極點之系統響應

- 上一節有關 $\%OS$, T_s, T_p 等的公式僅適合應用於兩個複數極點無零點的系統：



(a)



響應
III
$Au(t) + e^{-\zeta\omega_n t}(B\cos\omega_d t + C\sin\omega_d t)$
II
狀況 I
0
II
$De^{-\alpha_{r_i} t}$
狀況 I
時間

(b)

4.8 含零點之系統響應

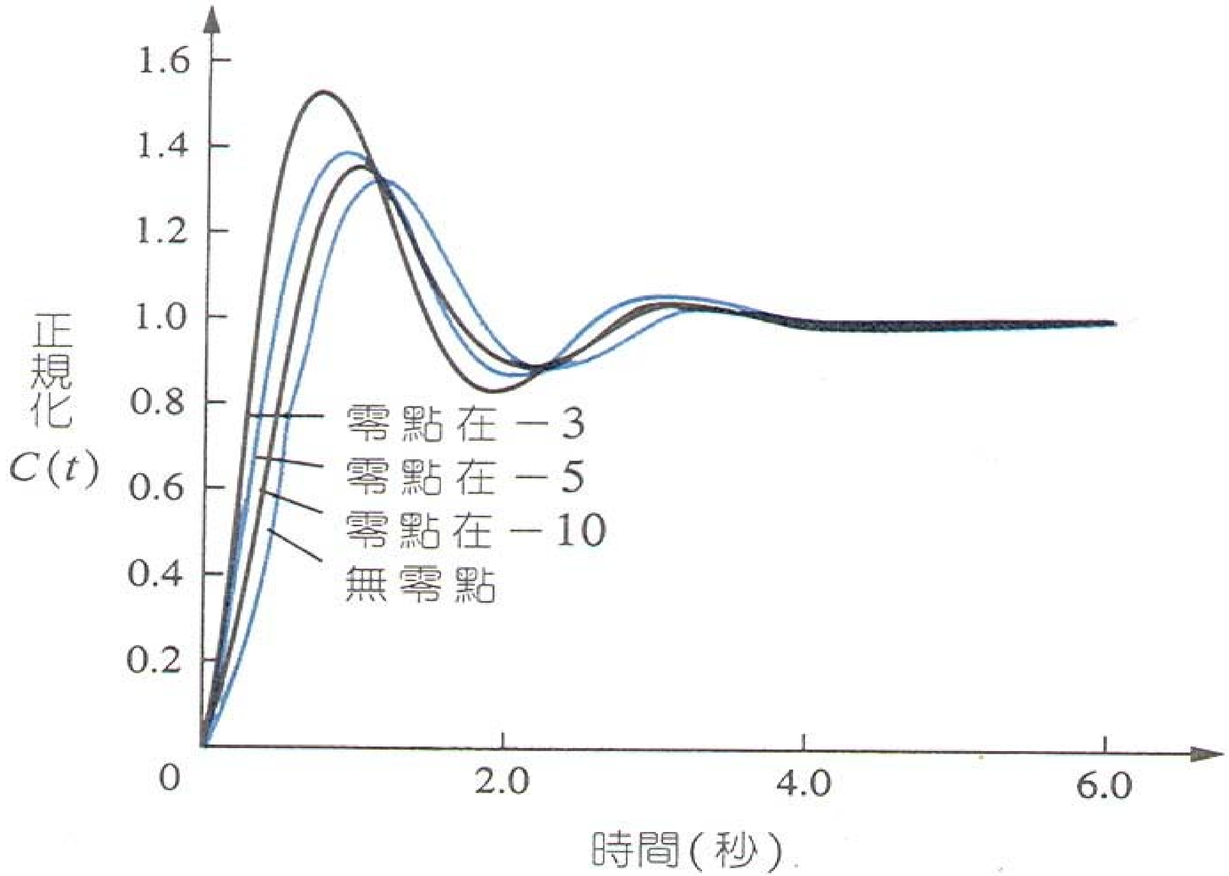


圖 4.25　增加一個零點到兩極點系統的效應

4.10 狀態方程式之拉氏轉換解

$$\dot{X} = Ax + Bu$$

$$y = Cx + Du$$

$\mathcal{L}$

$$s\mathbf{X}(s) - x(0) = AX(s) + BU(s)$$

$$(sI - A)\mathbf{X}(s) = x(0) + BU(s)$$

$$\mathbf{X}(s) = (sI - A)^{-1}[x(0) + BU(s)]$$

$$Y(s) = CX(s) + DU(s)$$

令 $x(0)=0$ 求轉移函數 $\dfrac{Y(s)}{U(s)}$

$$Y(s)=C(sI-A)^{-1}BU(s)+DU(s)$$

$$=\left[C(sI-A)^{-1}B+D\right]U(s)$$

$$G(s)=\frac{Y(s)}{U(s)}=C(sI-A)^{-1}B+D$$

Ex：求（1）$X(s)$ and $y(t)$（2）特徵值

$$\Downarrow$$

$$\det(sI-A)=0$$

$$\dot{x}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & 9 \end{bmatrix}x+\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}e^{-t}$$

$$y=\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}x \quad \text{and} \quad x(0)=\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$$

Sol：

$$\mathbf{X}(s) = (s\mathrm{I} - \mathrm{A})^{-1}[\mathrm{x}(0) + \mathrm{BU}(s)]$$

$$\mathrm{Y}(s) = \mathrm{C}(s\mathrm{I} - \mathrm{A})^{-1}[\mathrm{x}(0) + \mathrm{BU}(s)] + \mathrm{DU}(s)$$

$$\Rightarrow (s\mathrm{I} - \mathrm{A}) = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & 9 \end{bmatrix} = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 24 & 26 & s+9 \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{\begin{bmatrix} (S^2+9S+26) & (s+9) & -1 \\ -24 & s^2+9s & s \\ -24s & -(26s+24) & s^2 \end{bmatrix}}{\begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 24 & 26 & s+9 \end{vmatrix}} = \frac{}{s^3+9s^2+26s+24}$$

$$\Rightarrow X(s) = \frac{\begin{bmatrix} (S^2+9S+26) & (s+9) & -1 \\ -24 & s^2+9s & s \\ -24s & -(26s+24) & s^2 \end{bmatrix}}{(s+2)(s+3)(s+4)} \cdot \left(\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{s+1} \end{bmatrix} \right)$$

$$= \frac{\begin{bmatrix} (s^2+9s+26) & (s+9) & -1 \\ -24 & s^2+9s & s \\ -24s & -(26s+24) & s^2 \end{bmatrix}}{(s+2)(s+3)(s+4)} \cdot \begin{bmatrix} 1 \\ 0 \\ \dfrac{2s+3}{s+1} \end{bmatrix}$$

$$\Rightarrow = \frac{\Delta}{(s+1)(s+2)(s+3)(s+4)} \cdot \begin{bmatrix} s+1 \\ 0 \\ 2s+3 \end{bmatrix}$$

$$=\frac{1}{(s+1)(s+2)(s+3)(s+4)}\cdot\begin{bmatrix}(s^2+9s+26)(s+1)+2s+3\\-24(s+1)+2s^2+3s\\-24s(s+1)+2s^3+3s^2\end{bmatrix}$$

$$=\frac{1}{(s+1)(s+2)(s+3)(s+4)}\cdot\begin{bmatrix}s^3+10s^2+37s+29\\2s^2-21s-24\\2s^3-21s^2-24s\end{bmatrix}$$

$$\Rightarrow Y(s) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x \; \frac{\begin{bmatrix} s^3 + 10s^2 + 37s + 29 \\ 2s^2 - 21s - 24 \\ 2s^3 - 21s^2 - 24s \end{bmatrix}}{(s+1)(s+2)(s+3)(s+4)}$$

$$= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \cdot \frac{\begin{bmatrix} s^3 + 10s^2 + 37s + 29 \\ 2s^2 - 21s - 24 \\ 2s^3 - 21s^2 - 24s \end{bmatrix}}{(s+1)(s+2)(s+3)(s+4)} \cdot$$

$$= \frac{s^3 + 12s^2 + 16s + 5}{(s+1)(s+2)(s+3)(s+4)}$$

$$= \frac{-6.5}{s+2} + \frac{19}{s+3} + \frac{11.5}{s+4}$$

$$\therefore y(t) = \left(-6.5e^{-2t} + 19e^{-3t} + 11.5e^{-4t}\right)u(t)$$

#