Control Systems

控制系統

- 清雲科技大學電機工程系
- Ya-Fu Peng
- February, 2009

內容

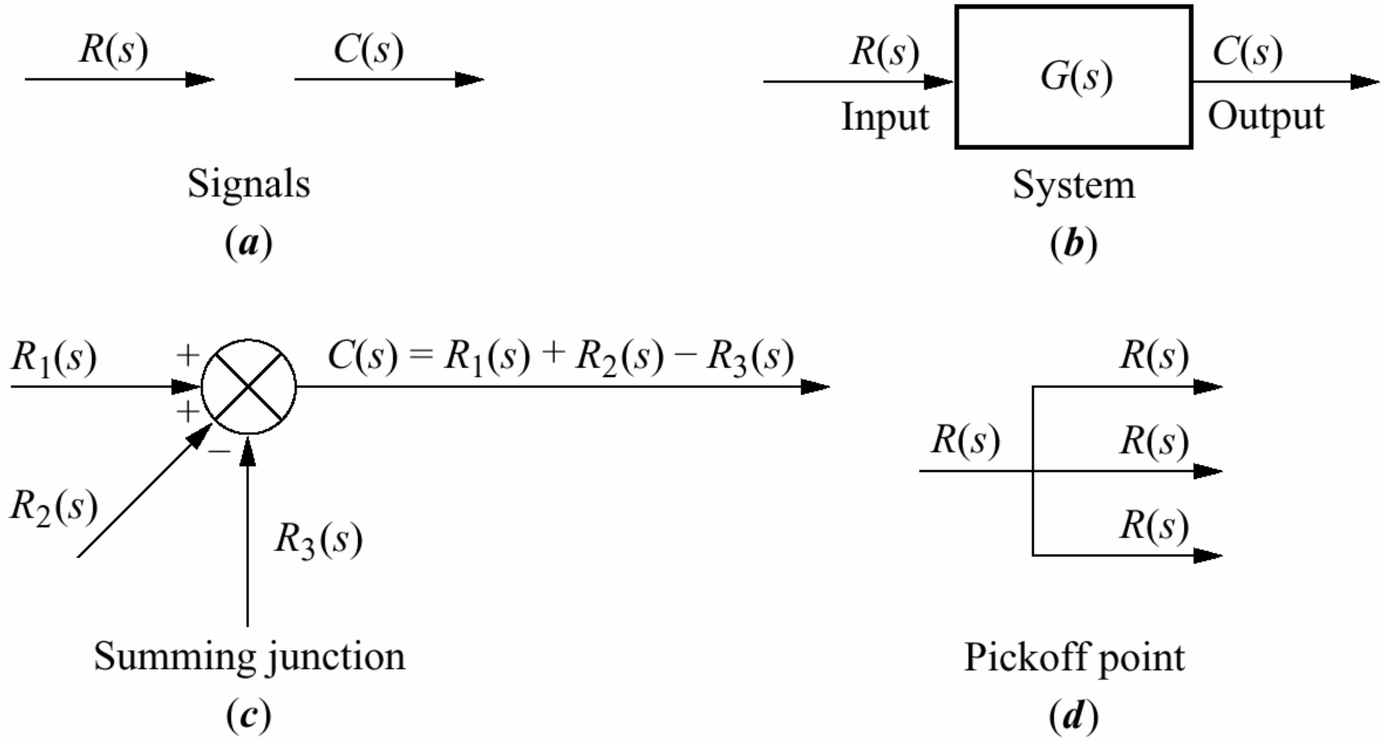
- 簡介
- 頻域模型
- 時域模型
- 時間響應
- 互聯子系統之簡化
- 穩定度
- 穩態誤差
- 根軌跡技巧

Chapter 5 Reduction of Multiple Subsystems（互聯子系統之簡介）

- 互聯子系統以兩種方式表示
  - （1）方 塊 圖 → 頻域分析
  - （2）信 號 流 程 圖 → 狀態空間分析

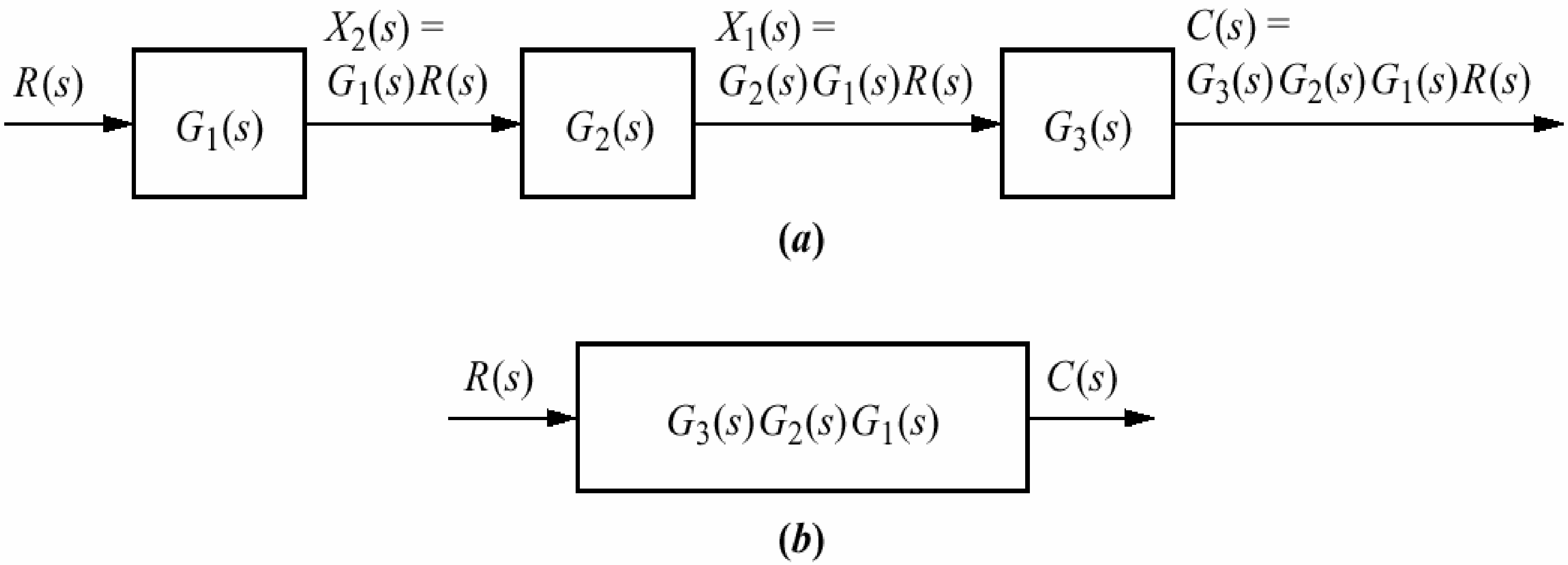
5.2 方塊圖

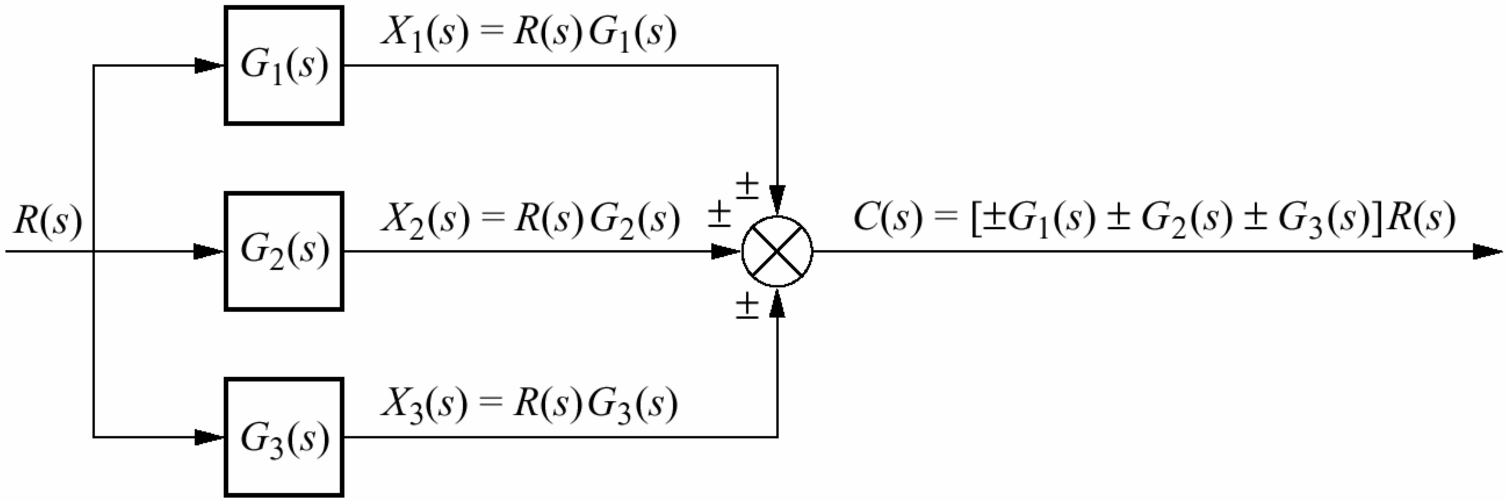
- 元件



R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
R2(s)
R3(s)
C(s) = R1(s) + R2(s) − R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

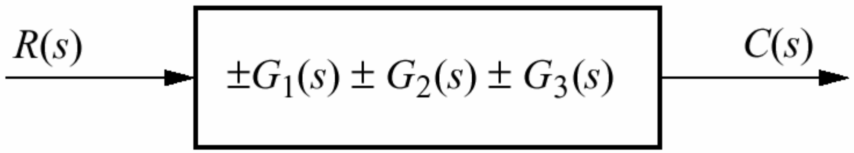
- 串聯形式＋並聯形式


R(s)
G1(s)
X2(s) =
G1(s)R(s)
G2(s)
X1(s) =
G2(s)G1(s)R(s)
G3(s)
C(s) =
G3(s)G2(s)G1(s)R(s)
(a)
R(s)
G3(s)G2(s)G1(s)
C(s)
(b)



$X_1(s) = R(s)G_1(s)$
$G_1(s)$
$R(s)$
$G_2(s)$
$X_2(s) = R(s)G_2(s)$
±
±
±
$C(s) = [\pm G_1(s) \pm G_2(s) \pm G_3(s)]R(s)$
$G_3(s)$
$X_3(s) = R(s)G_3(s)$

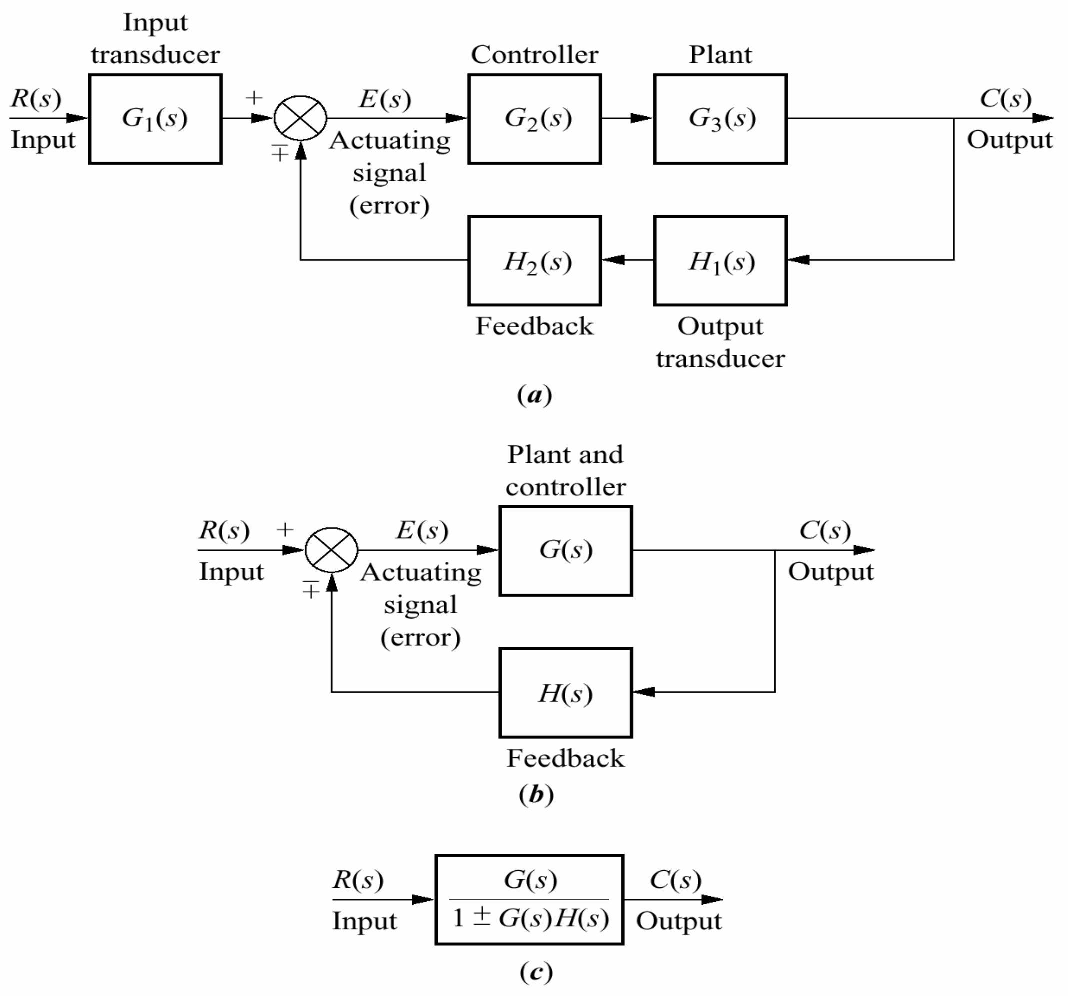
(*a*)



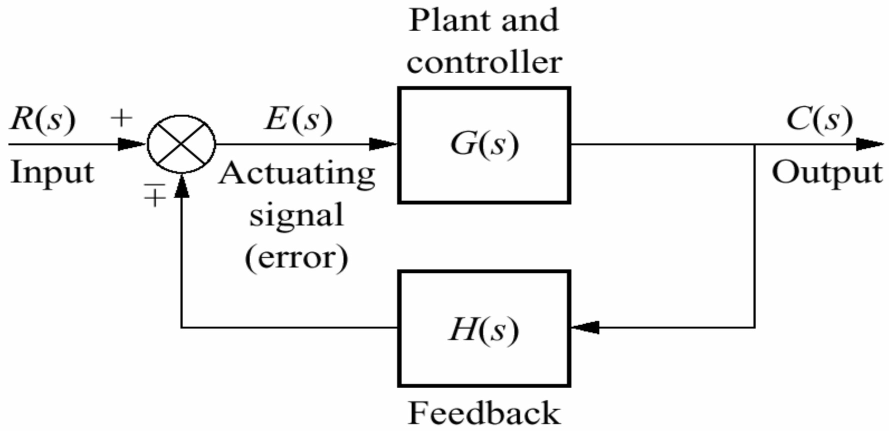
$R(s)$
$\pm G_1(s) \pm G_2(s) \pm G_3(s)$
$C(s)$

(*b*)

■ 回授形式



Input
transducer
R(s)
Input
G1(s)
+
∓
E(s)
Actuating
signal
(error)
Controller
G2(s)
Plant
G3(s)
C(s)
Output
H2(s)
Feedback
H1(s)
Output
transducer
(a)
Plant and
controller
R(s)
Input
+
∓
E(s)
Actuating
signal
(error)
G(s)
C(s)
Output
H(s)
Feedback
(b)
R(s)
Input
G(s)
1 ± G(s)H(s)
C(s)
Output
(c)



$$E(s) = R(s) \mp C(s)H(s)$$

$$C(s) = E(s)\cdot G(s) = \left(R(s) \mp C(s)H(s)\right)\cdot G(s)$$

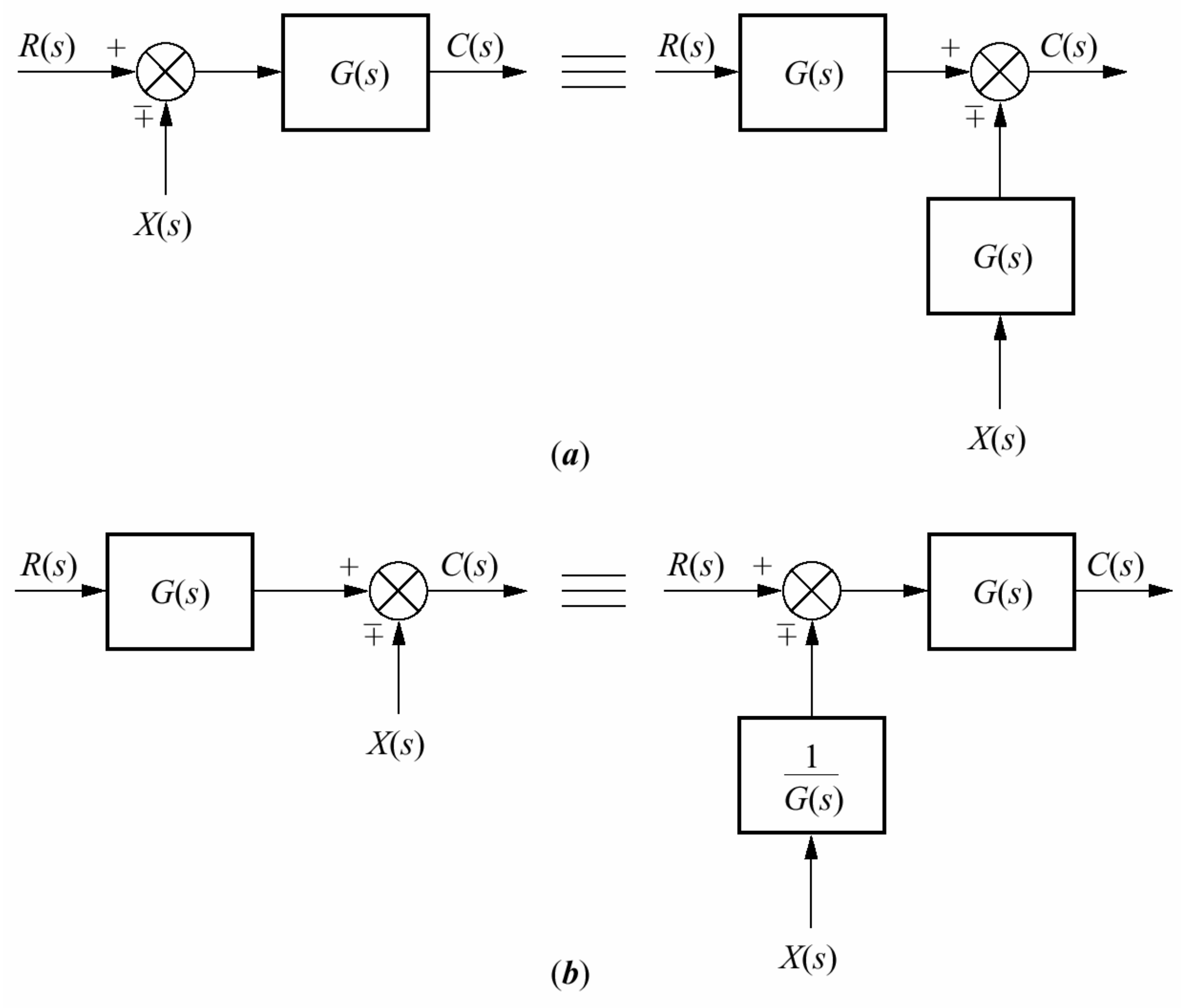
$$= R(s)G(s) \mp C(s)G(s)H(s)$$

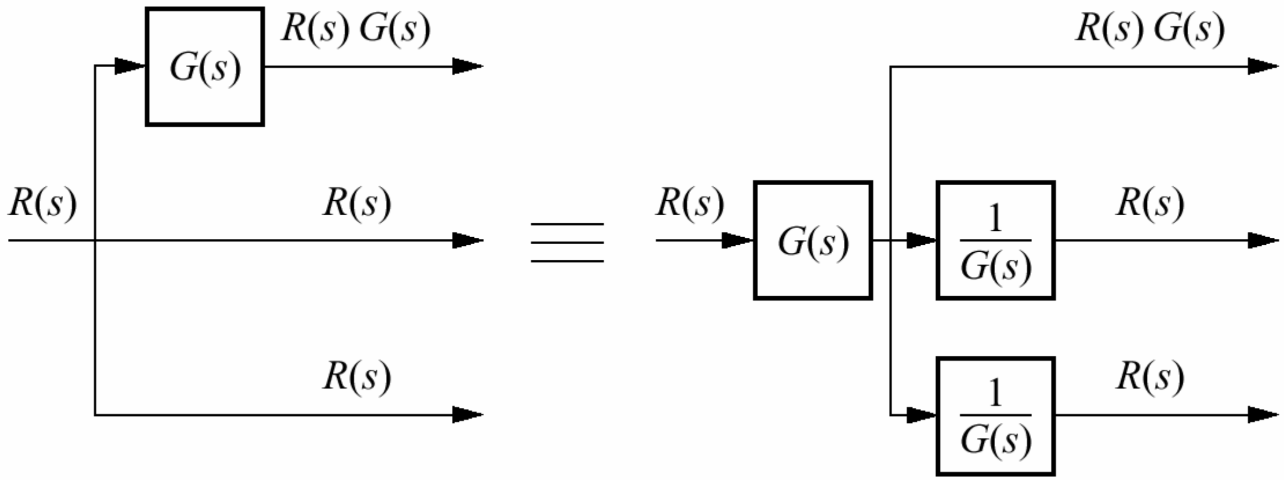
$$C(s)\left[1 \pm G(s)H(s)\right] = R(s)G(s)$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$

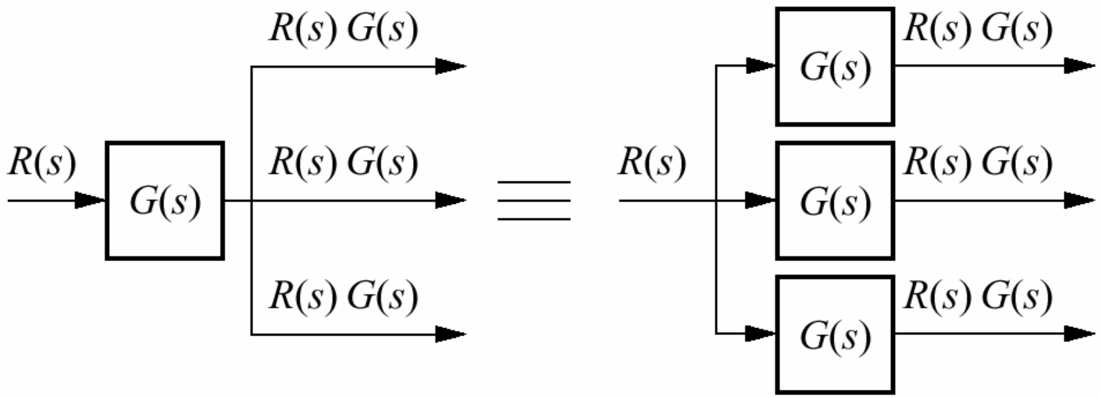
#

■ 方塊圖簡化



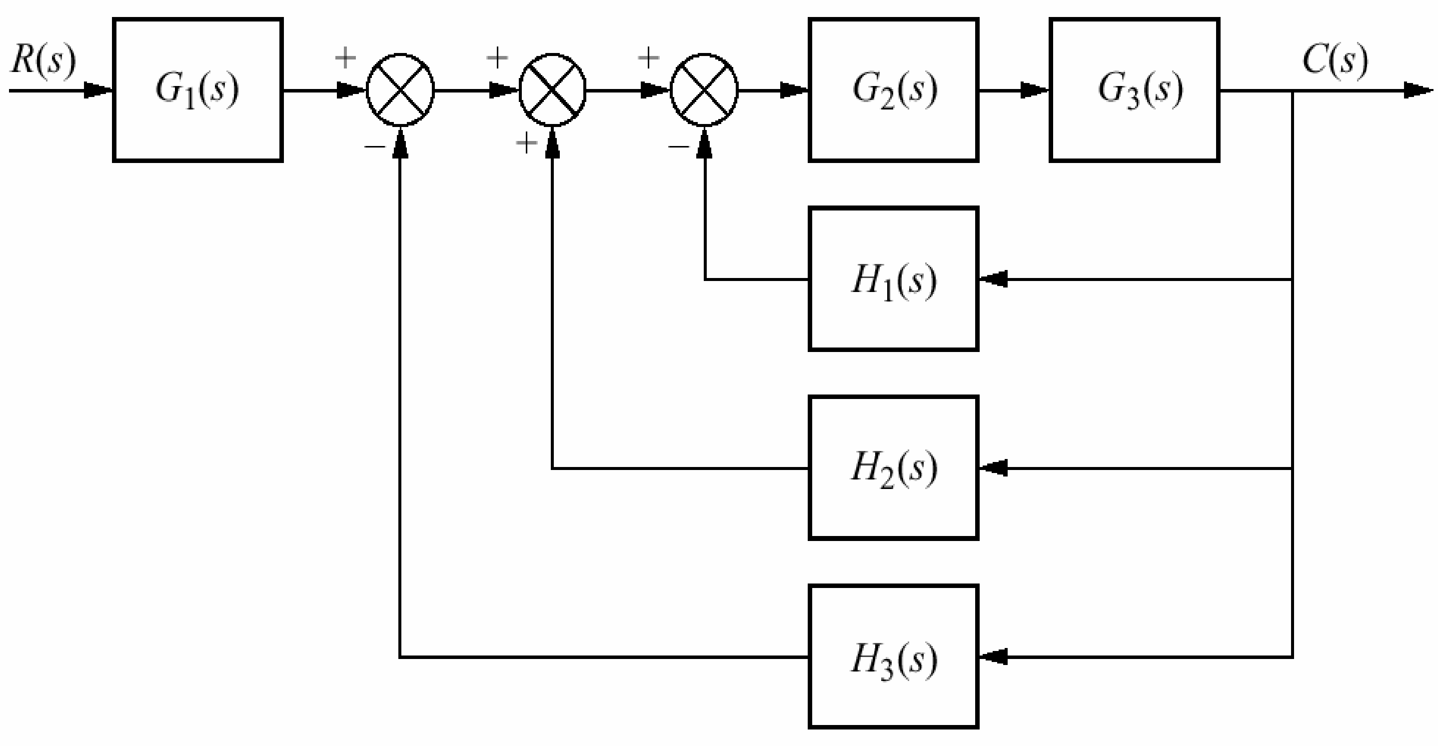

G(s)
R(s) G(s)
R(s)
R(s)
R(s)
R(s)
G(s)
R(s) G(s)
1
G(s)
R(s)
1
G(s)
R(s)

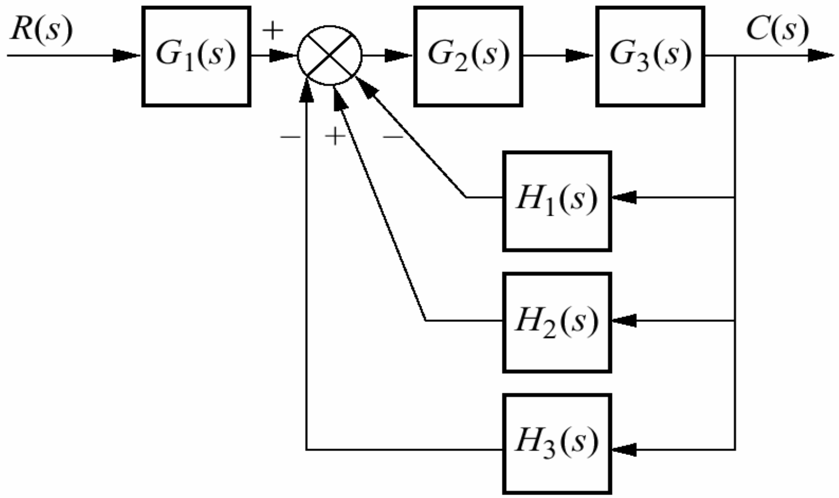
(*a*)


R(s) G(s)
R(s)
G(s)
R(s) G(s)
R(s) G(s)
R(s)
G(s)
R(s) G(s)
G(s)
R(s) G(s)
G(s)
R(s) G(s)

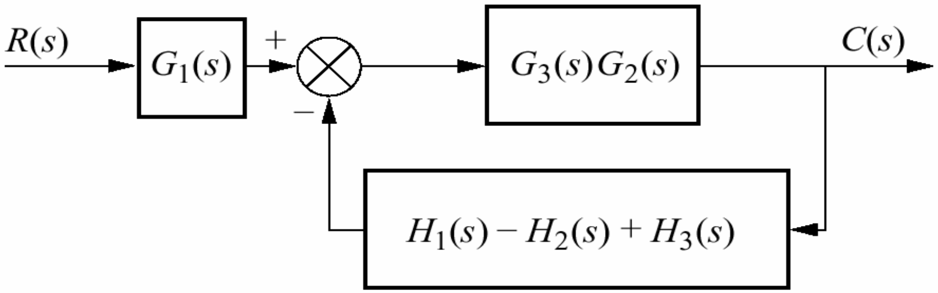
(*b*)

- EX5.1

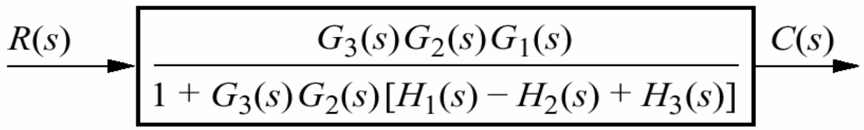



R(s)
G1(s)
+
G2(s)
G3(s)
C(s)
−
+
−
H1(s)
H2(s)
H3(s)

(a)

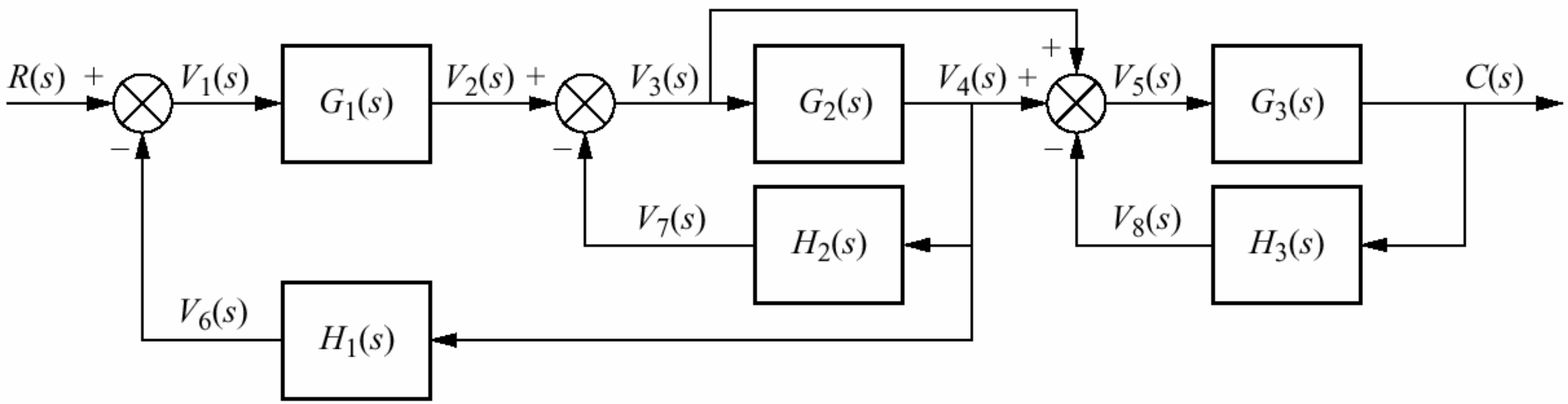

R(s)
G1(s)
+
−
G3(s)G2(s)
C(s)
H1(s) − H2(s) + H3(s)

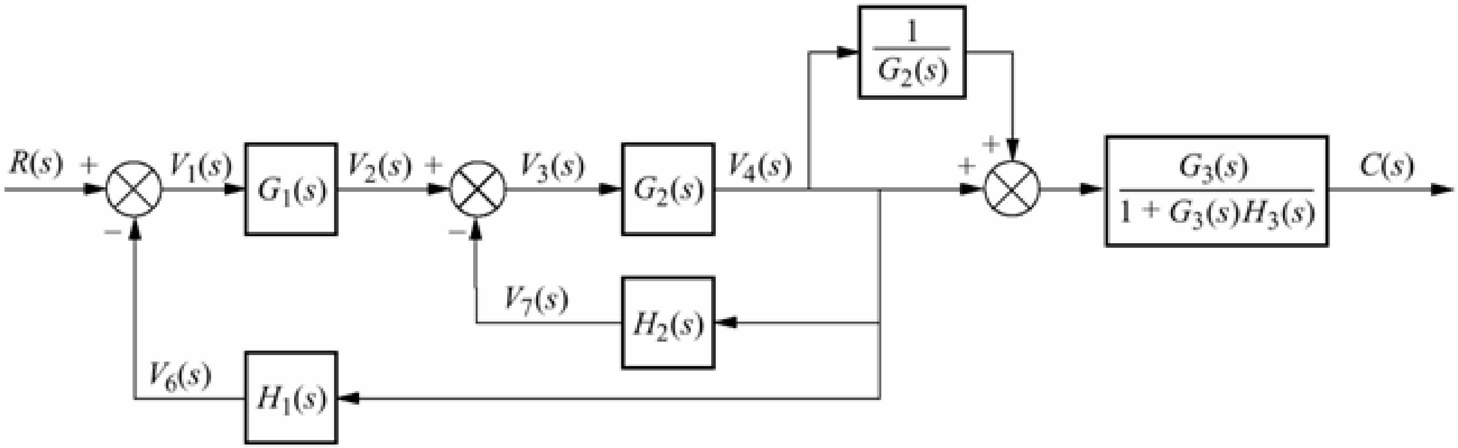
(b)


R(s)
G3(s)G2(s)G1(s)
1 + G3(s)G2(s)[H1(s) − H2(s) + H3(s)]
C(s)

(c)

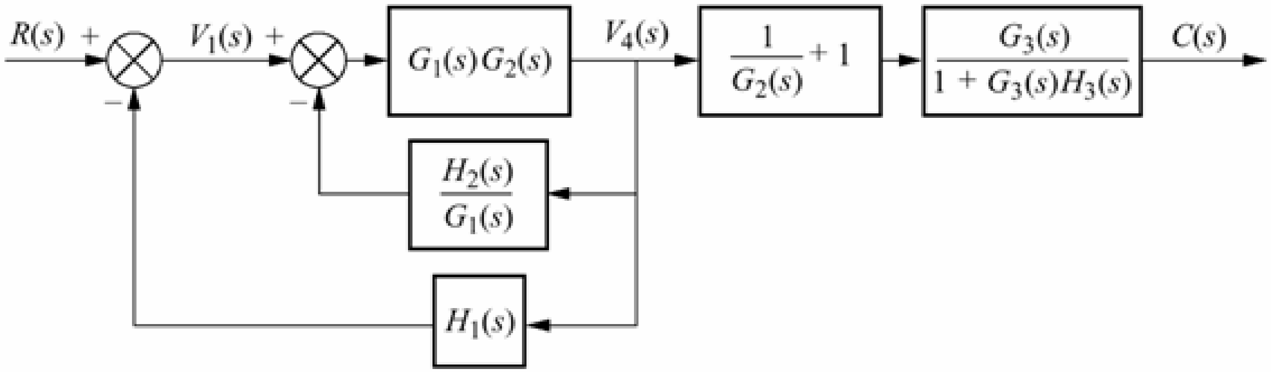
- EX5.2





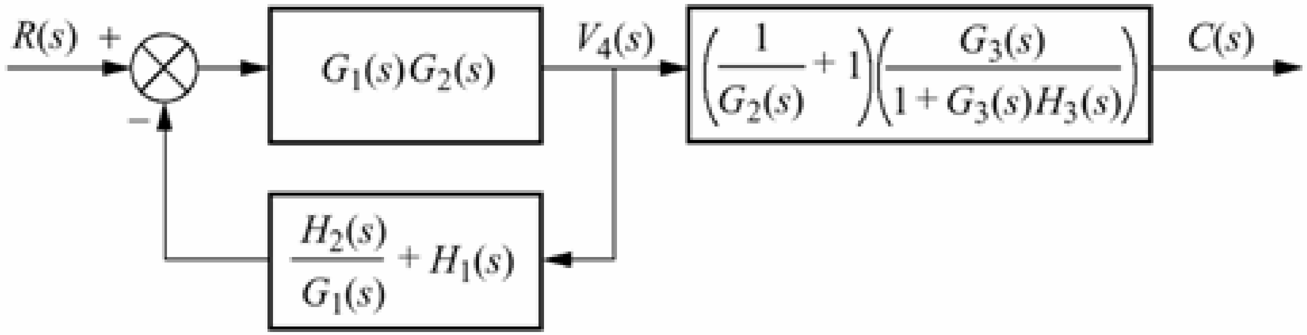
R(s)
+
V1(s)
G1(s)
V2(s)
+
V3(s)
G2(s)
V4(s)
1
G2(s)
+
+
G3(s)
1 + G3(s)H3(s)
C(s)
−
−
V7(s)
H2(s)
V6(s)
H1(s)

(*a*)

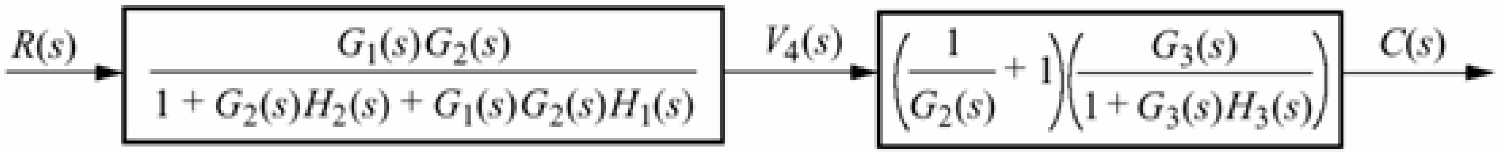


R(s)
+
V1(s)
+
G1(s)G2(s)
V4(s)
1
G2(s)
+ 1
G3(s)
1 + G3(s)H3(s)
C(s)
−
−
H2(s)
G1(s)
H1(s)

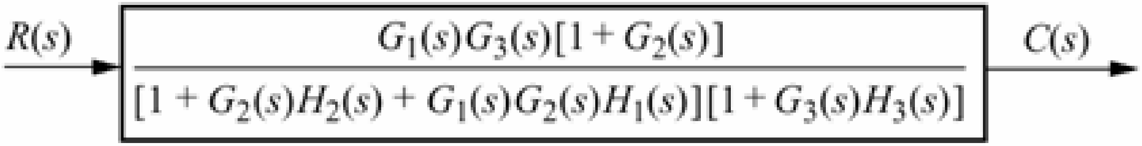
(*b*)


R(s)
+
−
G1(s)G2(s)
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)
H2(s)/G1(s) + H1(s)

(*c*)


R(s)
G1(s)G2(s) / (1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)

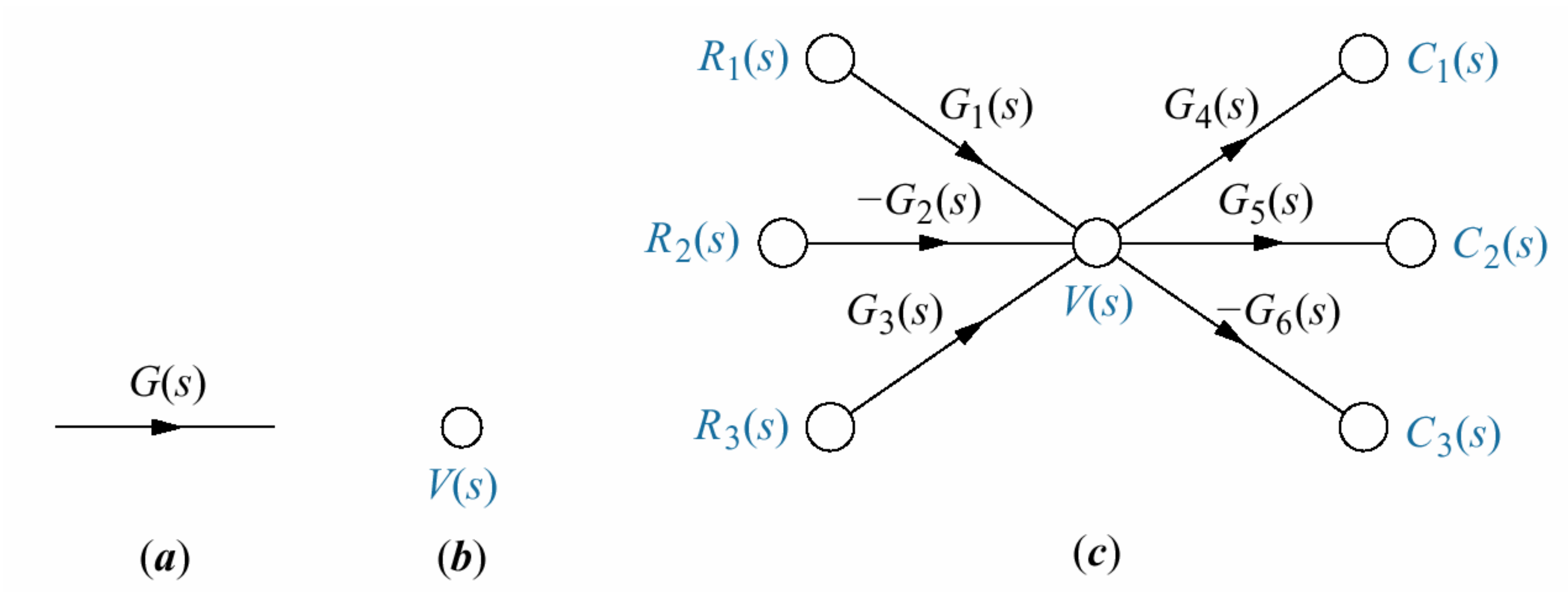
(*d*)

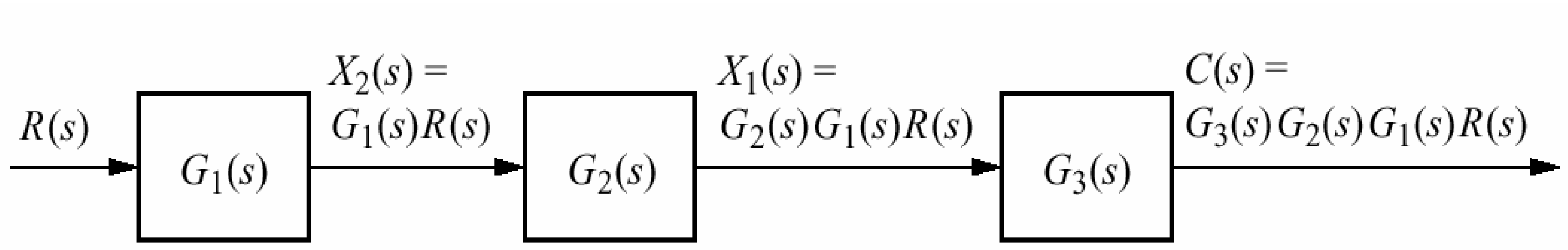

R(s)
G1(s)G3(s)[1 + G2(s)] / [1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)]
C(s)

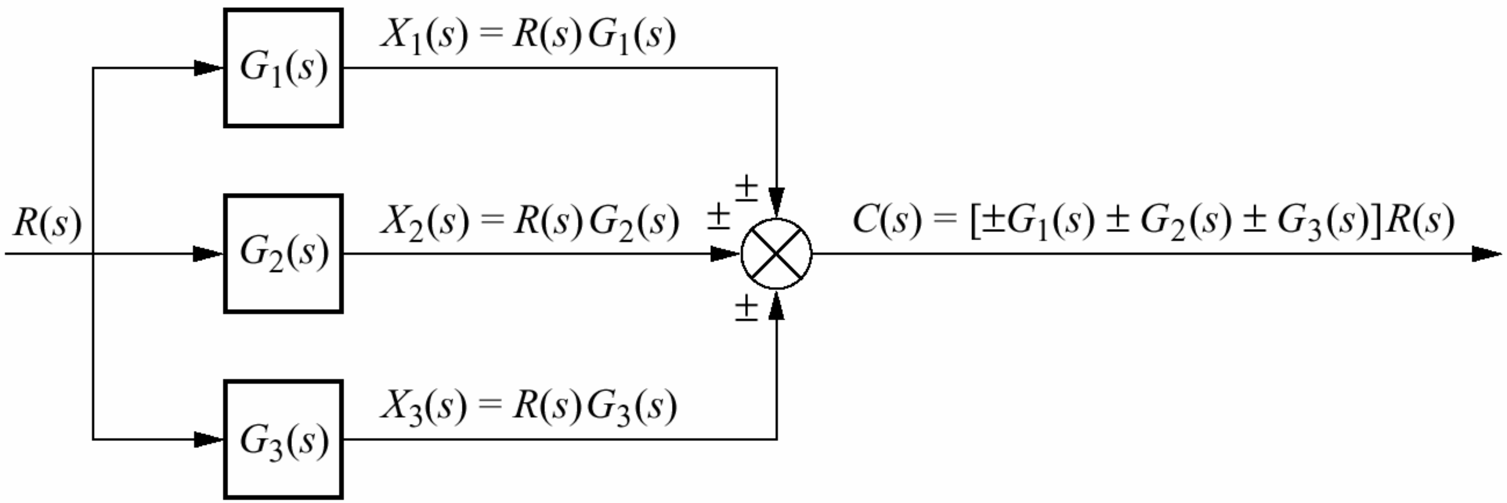
(*e*)

5.4 信號流程圖

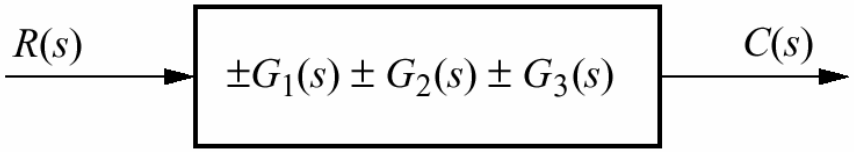
- 信號流程圖用分枝代表系統，用節點代表信號




R(s)
G1(s)
X2(s) =
G1(s)R(s)
G2(s)
X1(s) =
G2(s)G1(s)R(s)
G3(s)
C(s) =
G3(s)G2(s)G1(s)R(s)

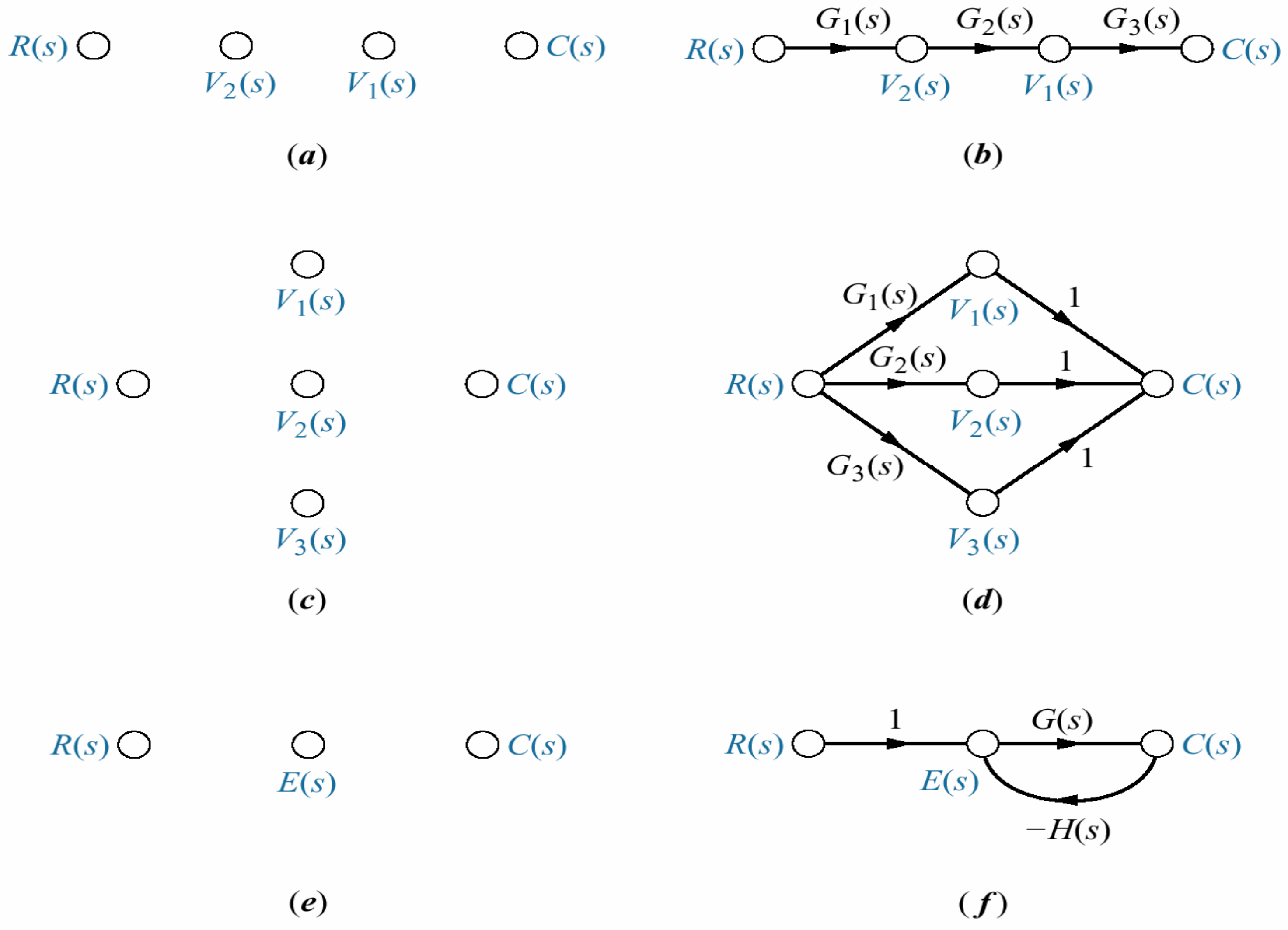

R(s)
G1(s)
G2(s)
G3(s)
X1(s) = R(s)G1(s)
X2(s) = R(s)G2(s)
X3(s) = R(s)G3(s)
±
±
±
C(s) = [±G1(s) ± G2(s) ± G3(s)]R(s)

(*a*)

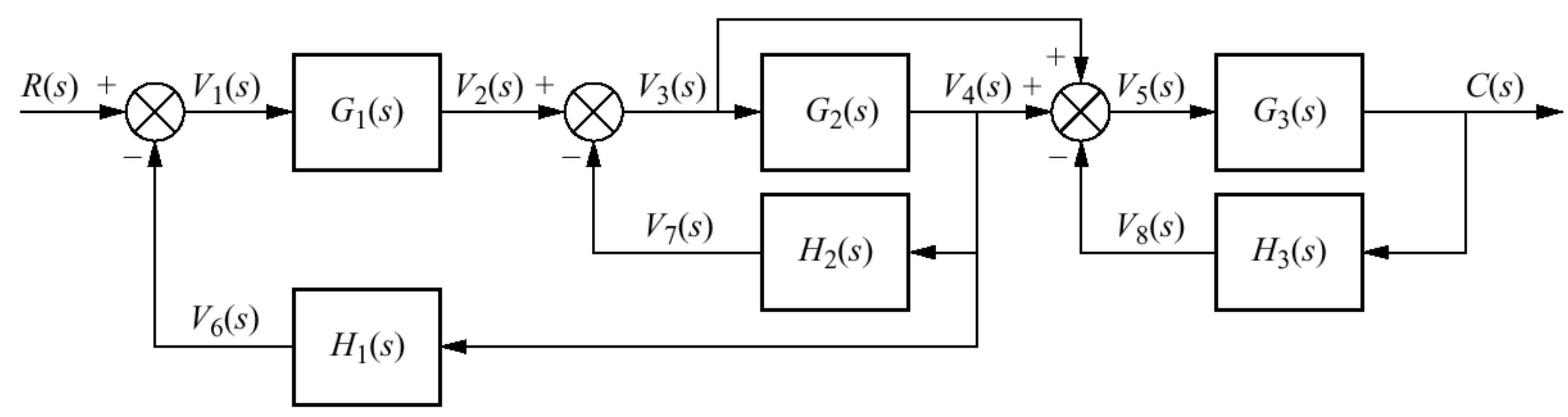

R(s)
±G1(s) ± G2(s) ± G3(s)
C(s)

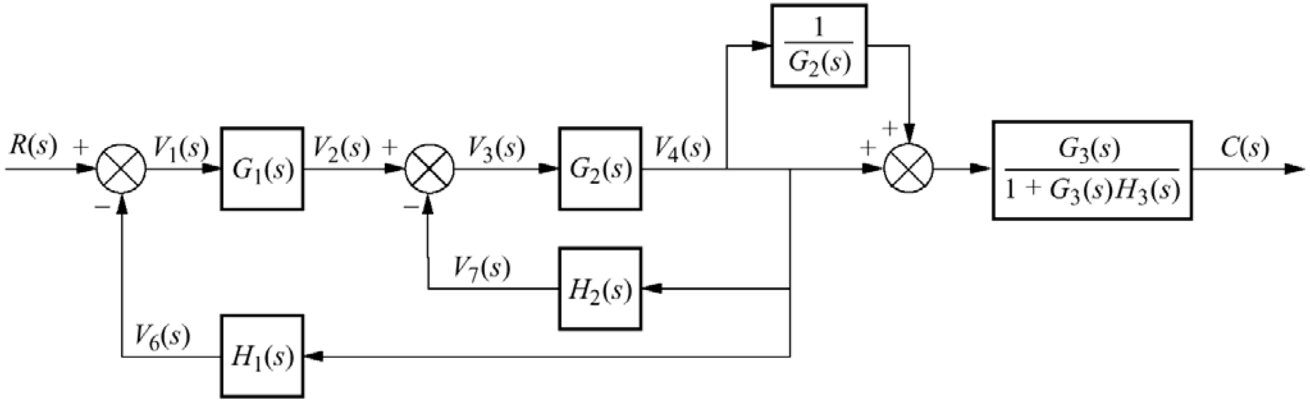
(*b*)

■ 例5.5

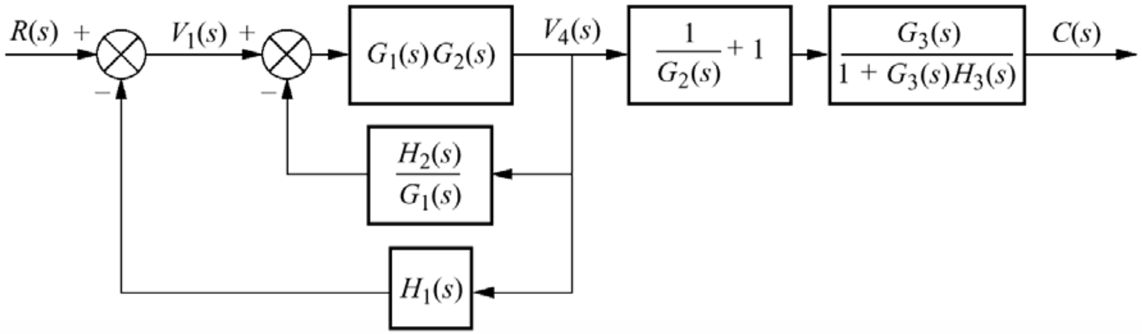


- EX5.2

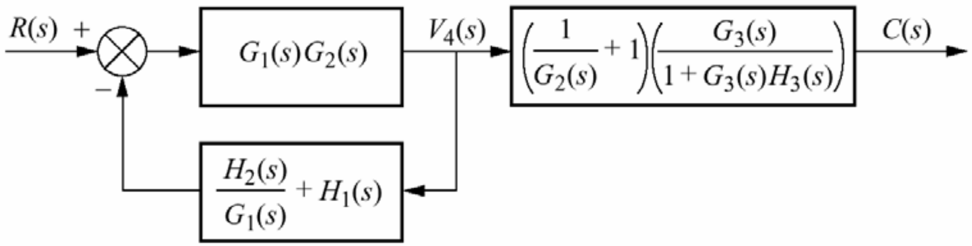



R(s)
V1(s)
G1(s)
V2(s)
V3(s)
G2(s)
V4(s)
1/G2(s)
G3(s)/(1 + G3(s)H3(s))
C(s)
V7(s)
H2(s)
V6(s)
H1(s)

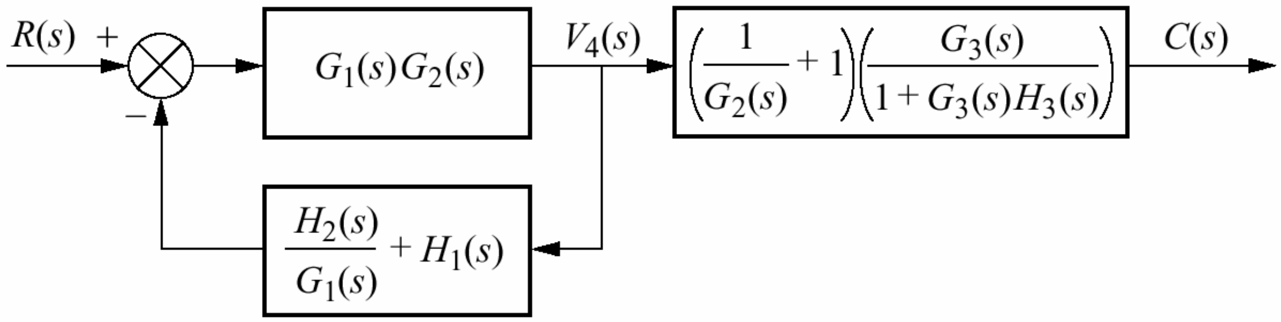
(*a*)


R(s)
V1(s)
G1(s)G2(s)
V4(s)
1/G2(s) + 1
G3(s)/(1 + G3(s)H3(s))
C(s)
H2(s)/G1(s)
H1(s)

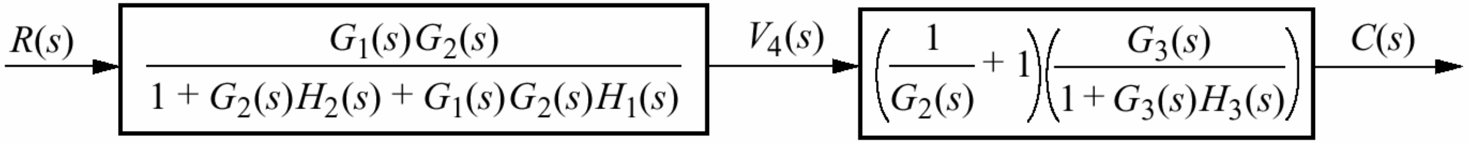
(*b*)


R(s)
G1(s)G2(s)
V4(s)
(1/G2(s) + 1)(G3(s)/(1+G3(s)H3(s)))
C(s)
H2(s)/G1(s) + H1(s)

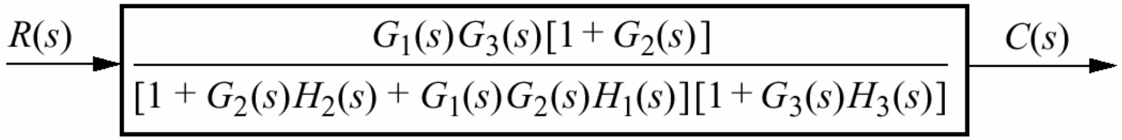
(*c*)


R(s)
+
−
G1(s)G2(s)
V4(s)
C(s)
H2(s)/G1(s) + H1(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))

(c)

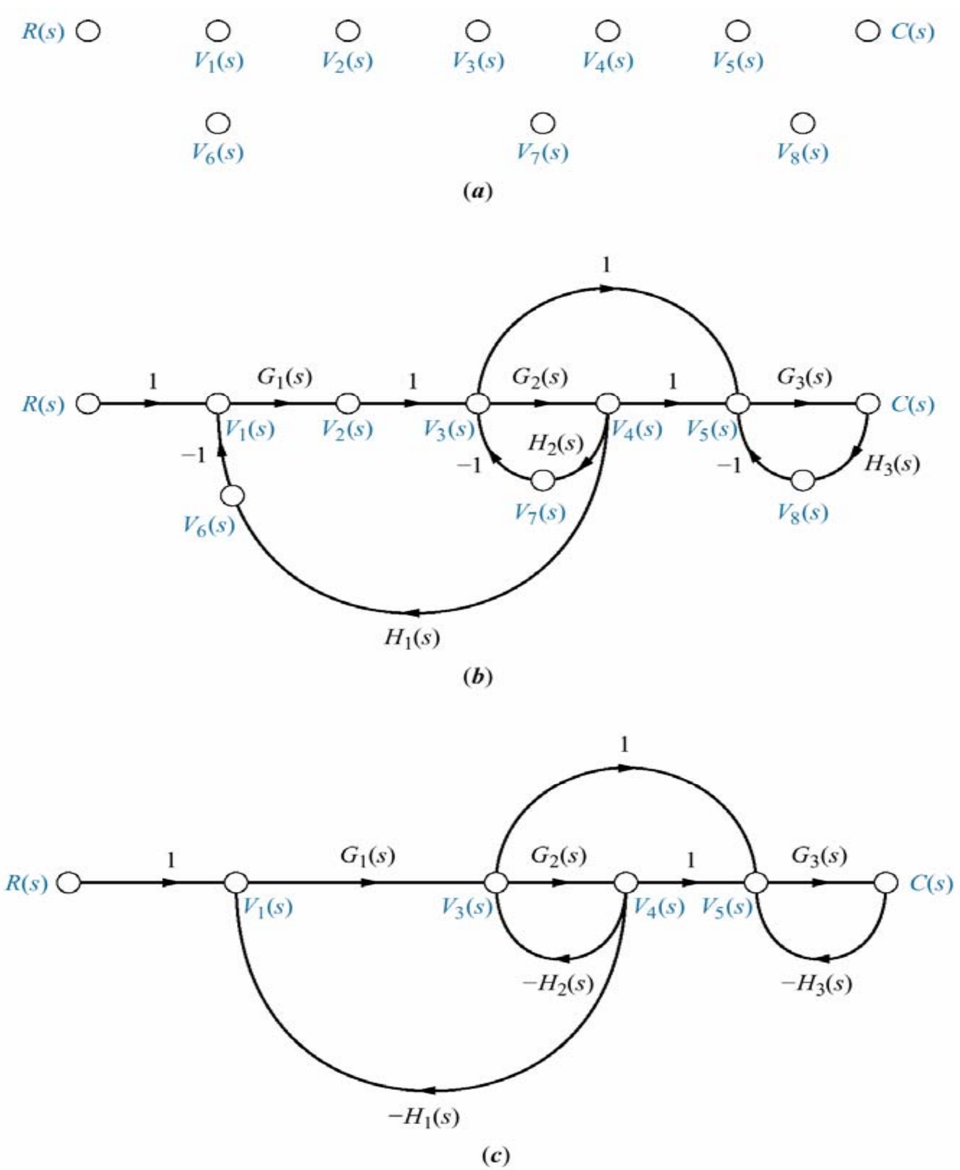

R(s)
G1(s)G2(s) / (1 + G2(s)H2(s) + G1(s)G2(s)H1(s))
V4(s)
(1/G2(s) + 1)(G3(s)/(1 + G3(s)H3(s)))
C(s)

(d)


R(s)
G1(s)G3(s)[1 + G2(s)] / [1 + G2(s)H2(s) + G1(s)G2(s)H1(s)][1 + G3(s)H3(s)]
C(s)

(e)

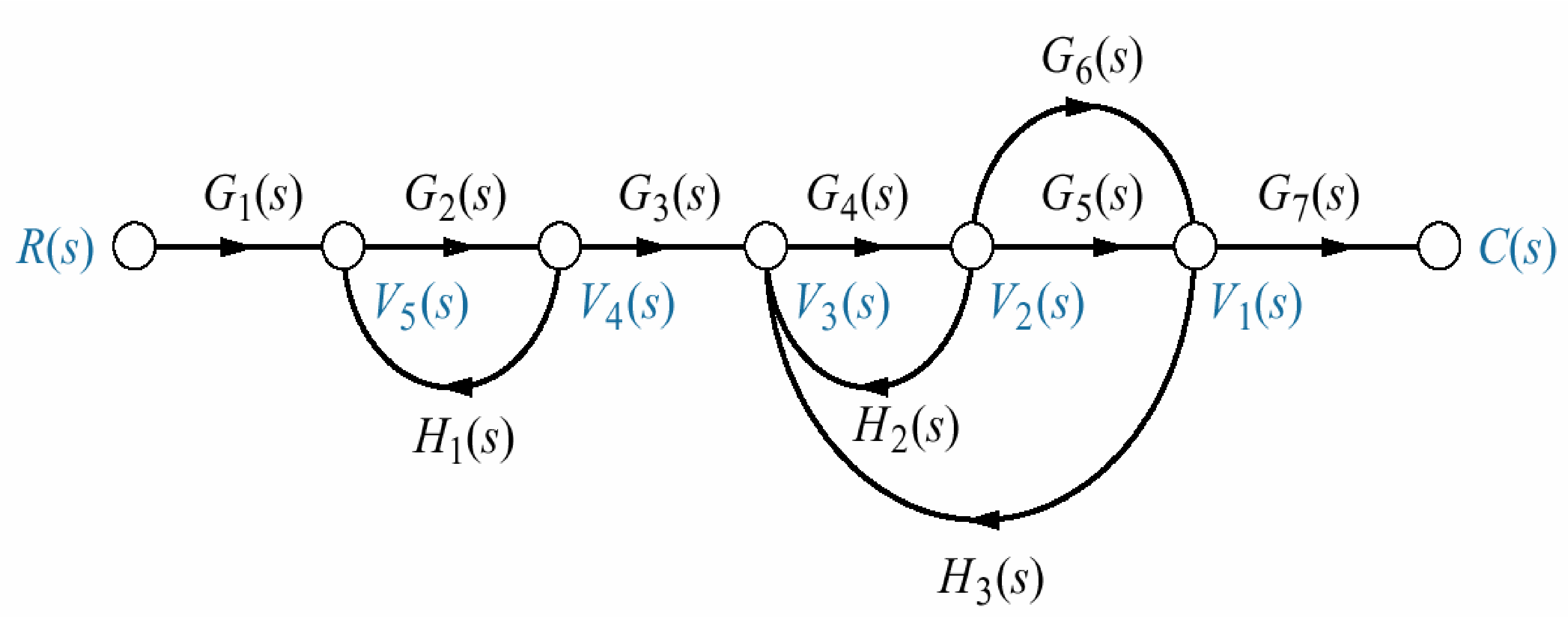
例5.6



R(s)
V1(s)
V2(s)
V3(s)
V4(s)
V5(s)
C(s)
V6(s)
V7(s)
V8(s)
(a)
1
G1(s)
G2(s)
G3(s)
H1(s)
H2(s)
H3(s)
−1
(b)
−H1(s)
−H2(s)
−H3(s)
(c)

5.5 梅生法則（ Mason’s rule ）1953

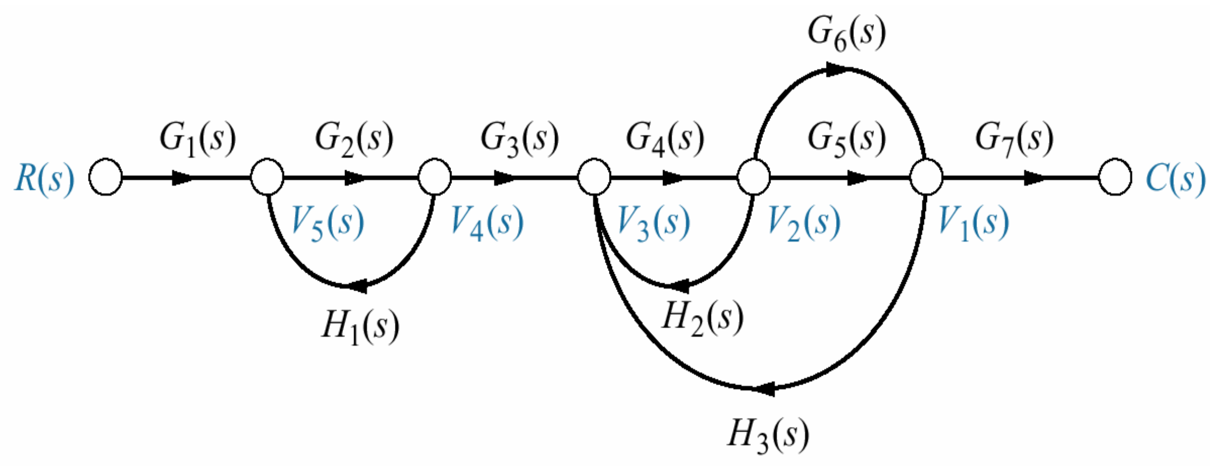
- 簡化流程圖的技巧



定義

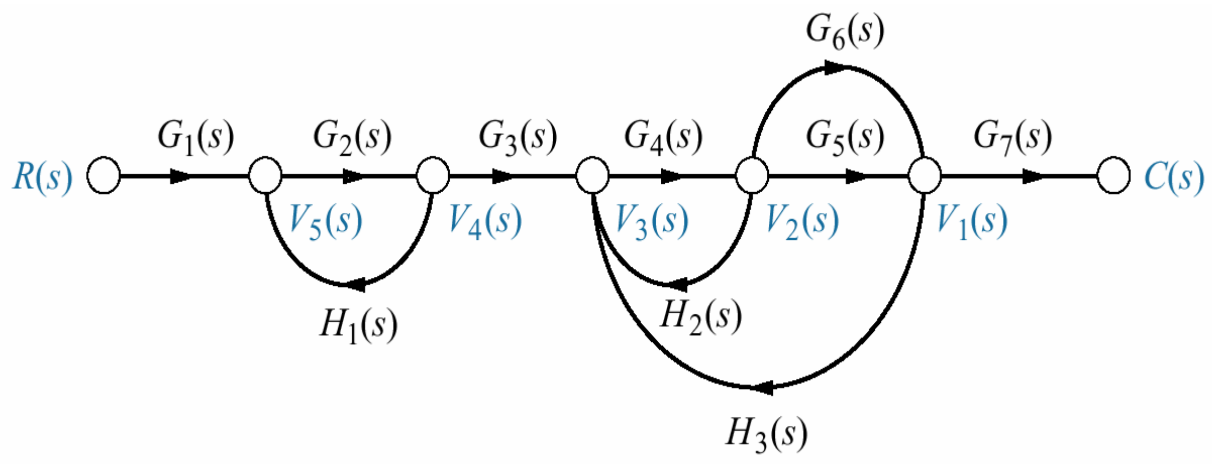
- 迴路增益（Loop gain）：此分枝增益的乘積可沿著依信號流動方向進行，由一個節點開始並終止在同一個節點而不超越任何其他節點一次以上的路徑而得知。

$$\ell_1 = G_2 H_1$$
$$\ell_2 = G_2 H_2$$
$$\ell_3 = G_4 G_5 H_3$$
$$\ell_4 = G_4 G_6 H_3$$



- 順向路徑增益（Forward-path gain）：
  依信號流動的方向進行，每一節點僅通過一次，由輸入至輸出各信號增益之乘機。

$$P_1 = G_1 G_2 G_3 G_4 G_5 G_7$$
$$P_2 = G_1 G_2 G_3 G_4 G_6 G_7$$



- 未接觸迴路：未和其他迴路共用節點

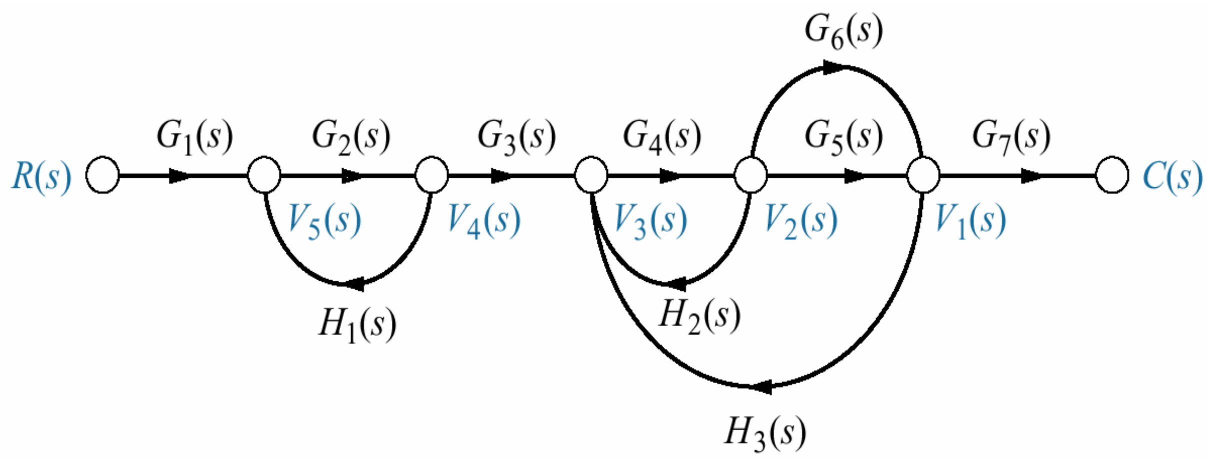
  ℓ_1 與 ℓ_2, ℓ_3 ,ℓ_4 未接觸

- 未接觸迴路增益：

$$\ell_1\ell_2$$

$$\ell_1\ell_3$$

$$\ell_1\ell_4$$



＊梅生法則：一個系統的轉移函數 $C(s)/R(s)$ 可用信號流程圖表示為

$$G(s)=\frac{C(s)}{R(s)}=\frac{\sum_k P_k\Delta_k}{\Delta}$$

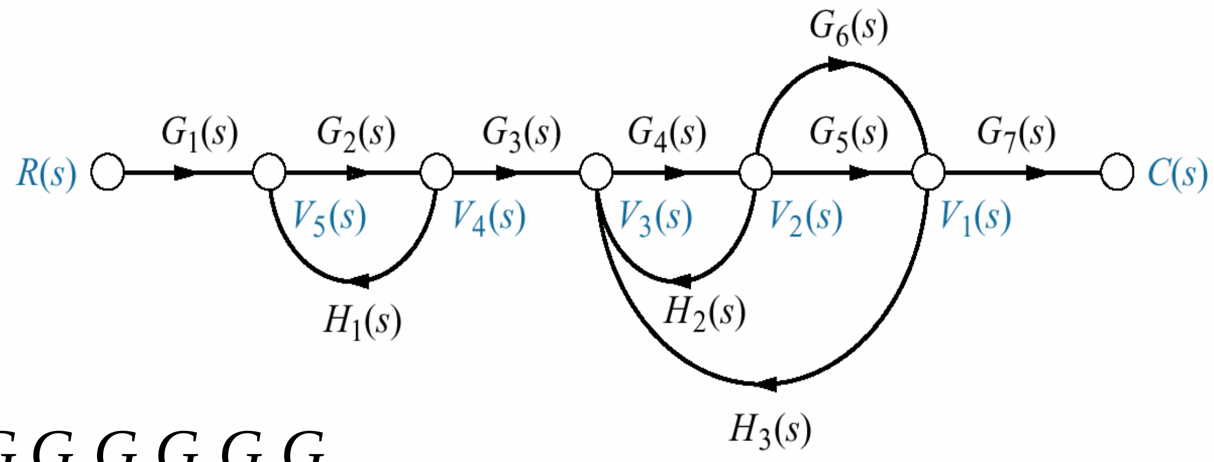
其中

$k=$ 順向路徑的數量

$P_k=$ 第 k 條順向路徑的增益

$\Delta=1-\sum$ 迴路增益 $+\sum$ 每兩個未接觸迴路增益 $-\sum$ 每三個未接觸迴路增益 $+\sum$ 每四個未接觸迴路增益

$\Delta_k=\Delta-\sum$ 接觸到第 k 條順向路徑的增益 與 P_k 接觸者為0



$$P_1 = G_1G_2G_3G_4G_5G_7$$

$$P_2 = G_1G_2G_3G_4G_6G_7$$

$\Delta = 1 - \sum$迴路增益$+ \sum$每兩個未接觸迴路增益

$- \sum$每三個未接觸迴路增益

$+ \sum$每四個未接觸迴路增益

$$\ell_1 = G_2 H_1$$

$$\ell_2 = G_2 H_2$$

$$\ell_3 = G_4 G_5 H_3$$

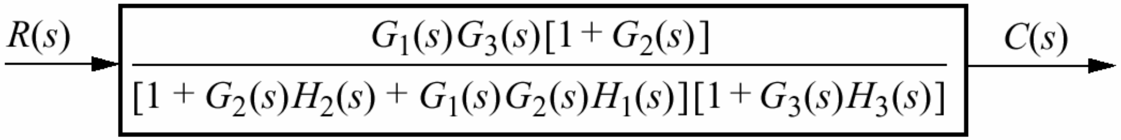
$$\ell_4 = G_4 G_6 H_3$$

$$\ell_1 \ell_2$$

$$\ell_1 \ell_3$$

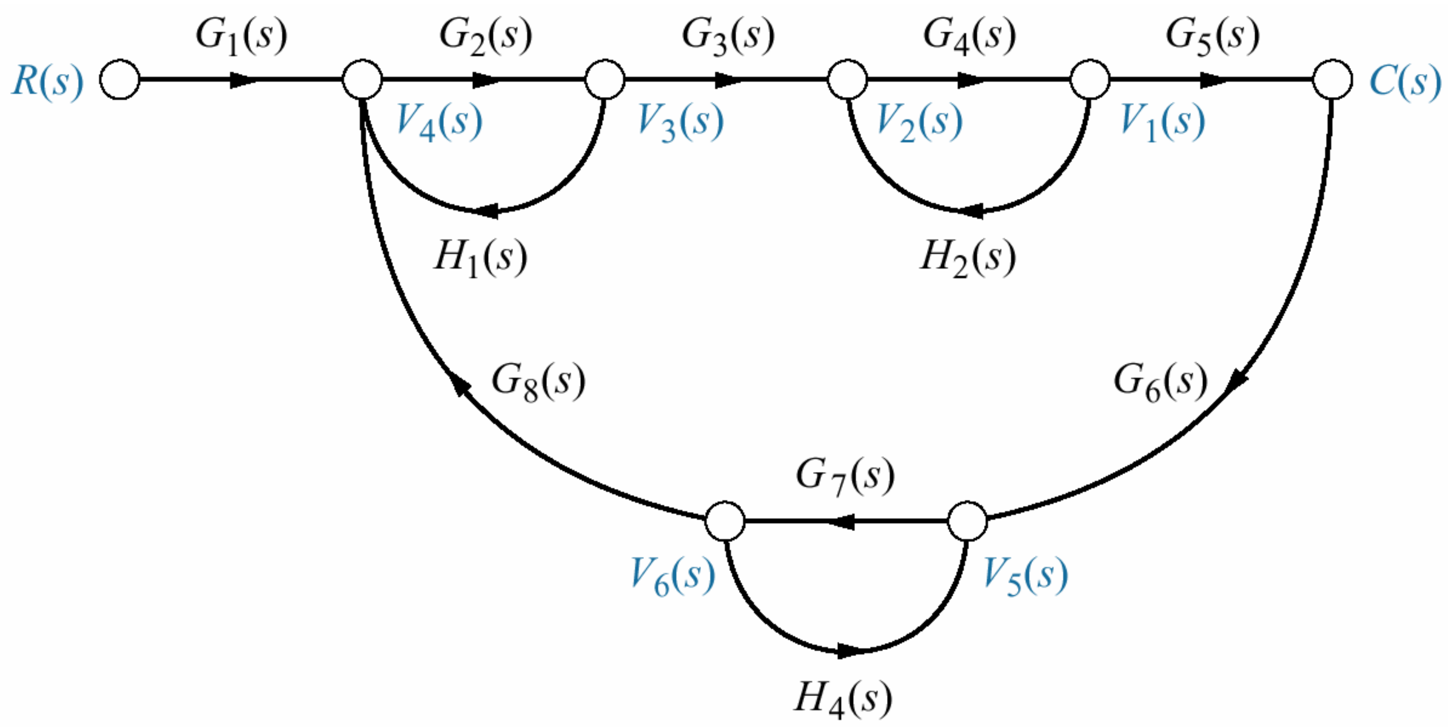
$$\ell_1 \ell_4$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$



$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta}$$

- Ex：求轉移函數 $C(s)/R(s)$



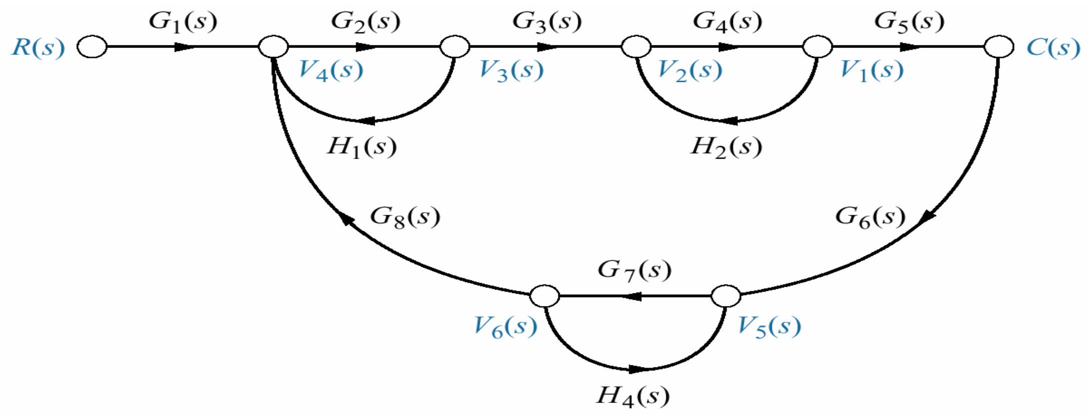
$$P_1 = G_1 G_2 G_3 G_4 G_5$$

$$\ell_1 = G_2 H_1$$

$$\ell_2 = G_4 H_2$$

$$\ell_3 = G_7 H_4$$

$$\ell_4 = G_2 G_3 G_4 G_5 G_6 G_7 G_8$$

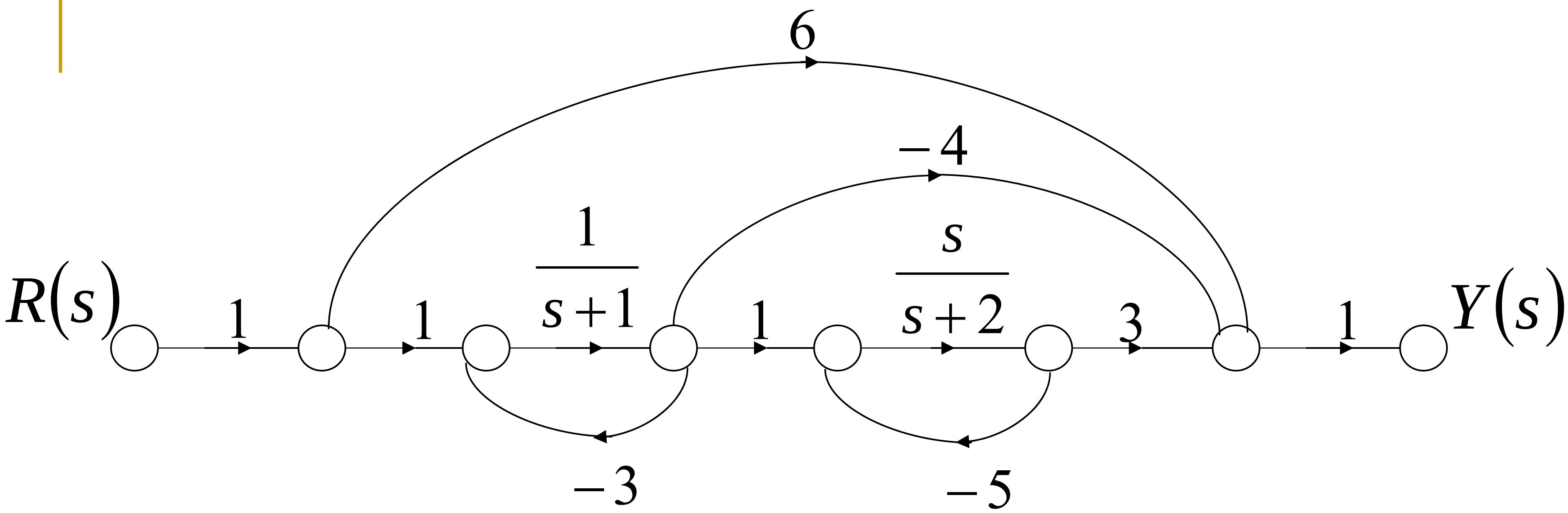


$$\Rightarrow \Delta = 1 - (\ell_1 + \ell_2 + \ell_3 + \ell_4) + (\ell_1 \ell_2 + \ell_1 \ell_3 + \ell_2 \ell_3) - (\ell_1 \ell_2 \ell_3)$$

$\Delta_1 \Rightarrow \ell_1, \ell_2, \ell_4$ 接觸 $\Rightarrow 0$

$$\Rightarrow \Delta_1 \Rightarrow 1 - \ell_3 \quad \Rightarrow \quad T(s) = \frac{\sum_k P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1}{\Delta}$$

#



$$P_1 = 6$$

$$P_2 = \frac{-4}{s+1} \qquad \ell_1 = \frac{-3}{s+1}$$

$$P_3 = \frac{3s}{(s+1)(s+2)} \qquad \ell_2 = \frac{-5s}{s+2}$$

$$\Rightarrow \Delta = 1 - (\ell_1 + \ell_2) + \ell_1 \ell_2$$

$$= 1 - \left(\frac{-3}{s+1} + \frac{-5s}{s+2} \right) + \left(\frac{15s}{(s+1)(s+2)} \right)$$

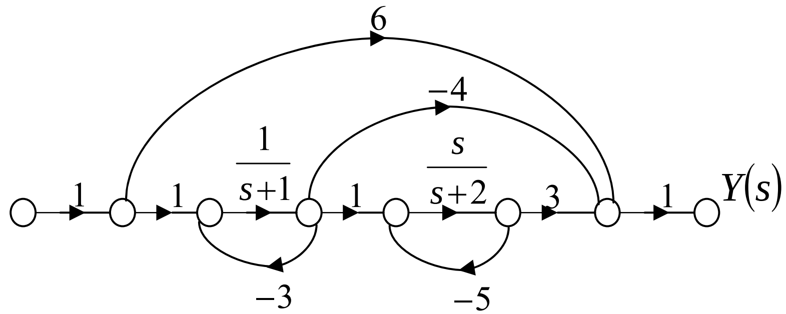
$$\ell_1 = \frac{-3}{s+1}$$

$$\ell_2 = \frac{-5s}{s+2}$$

$$\Delta_1 = 1 - (\ell_1 + \ell_2) + \ell_1 \ell_2$$

$$= \frac{s^2 + 3s + 2 + 3s + 6 + 5s^2 + 5s + 15s}{s^2 + 3s + 2}$$

$$= \frac{6s^2 + 26s + 8}{s^2 + 3s + 2}$$



$$\Delta_2 = 1 - \ell_2 = \frac{6s+2}{s+2} \qquad \Delta_3 = 1$$

$$\Rightarrow T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

$$= \frac{6 \cdot \frac{6s^2 + 26s + 8}{s^2 + 3s + 2} + \left(\frac{-4}{s+1}\right)\left(\frac{6s+2}{s+2}\right) + \frac{3s}{(s+1)(s+2)} \cdot 1}{\frac{6s^2 + 26s + 8}{s^2 + 3s + 2}}$$

$$= \frac{36s^2 + 156s + 48 - 24s - 8 + 3s}{6s^2 + 26s + 8}$$

$$= \frac{36s^2 + 135s + 40}{6s^2 + 26s + 8}$$

#

$$P_1 = 6$$

$$P_2 = \frac{-4}{s+1}$$

$$P_3 = \frac{3s}{(s+1)(s+2)}$$

5.6 狀態方程式之信號流程圖

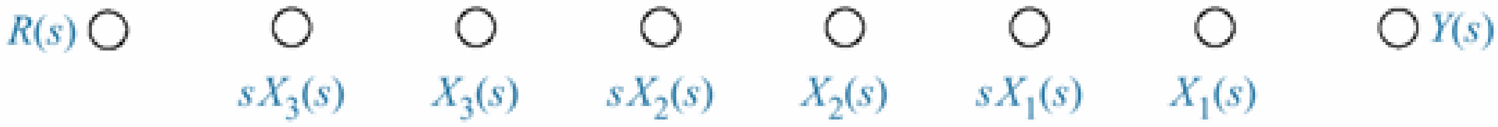
- 考慮

$$\dot{x}_1 = 2x_1 - 5x_2 + 3x_3 + 2r$$
$$\dot{x}_2 = -6x_1 - 2x_2 + 2x_3 + 5r$$
$$\dot{x}_3 = x_1 - 3x_2 - 4x_3 + 7r$$

$$y = -4x_1 + 6x_2 + 9x_3$$

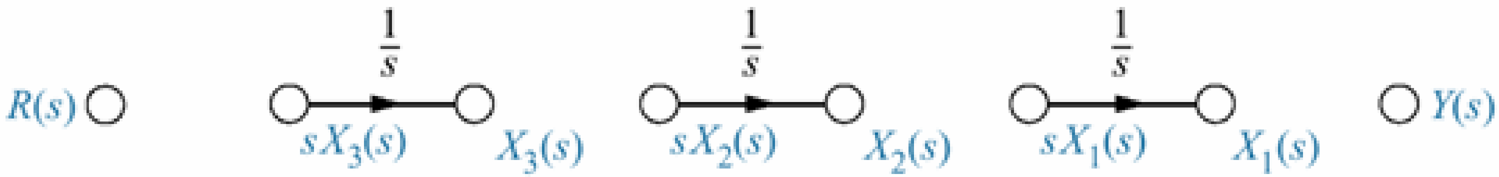
步驟

a.定出節點代表狀態變數並在每一節點左邊另放一個節點表示各狀態變數之微分量



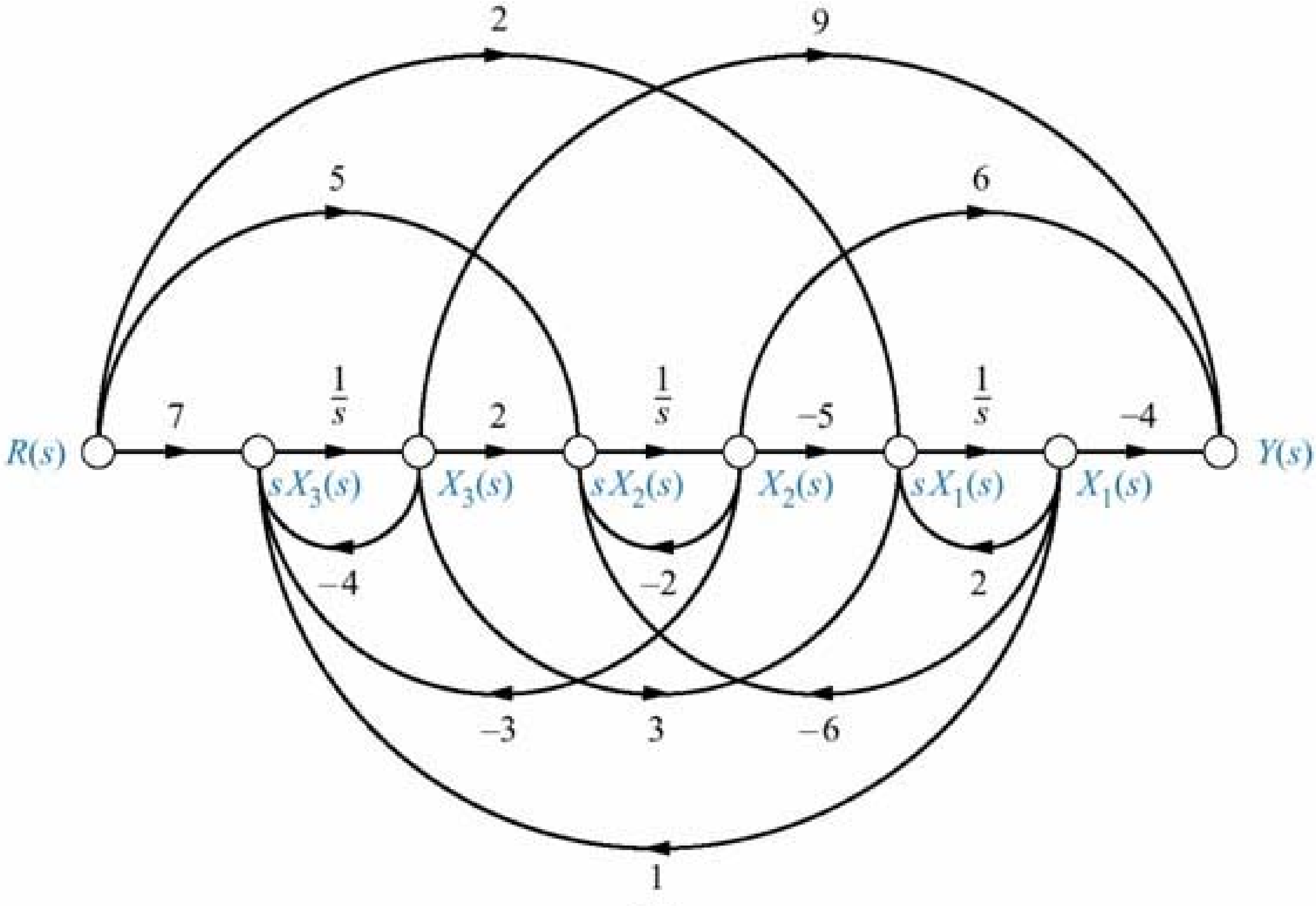
(a)

b.將狀態變數和其微分量以積分 $\frac{1}{s}$ 連接



(b)

c. 完成 $\dot{x}_k$ 與 y 之連接圖



(f)

- EX5.5

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 0 & 10 \end{bmatrix} \mathbf{x}$$

$$\dot{x}_1 = -2x_1 + x_2$$

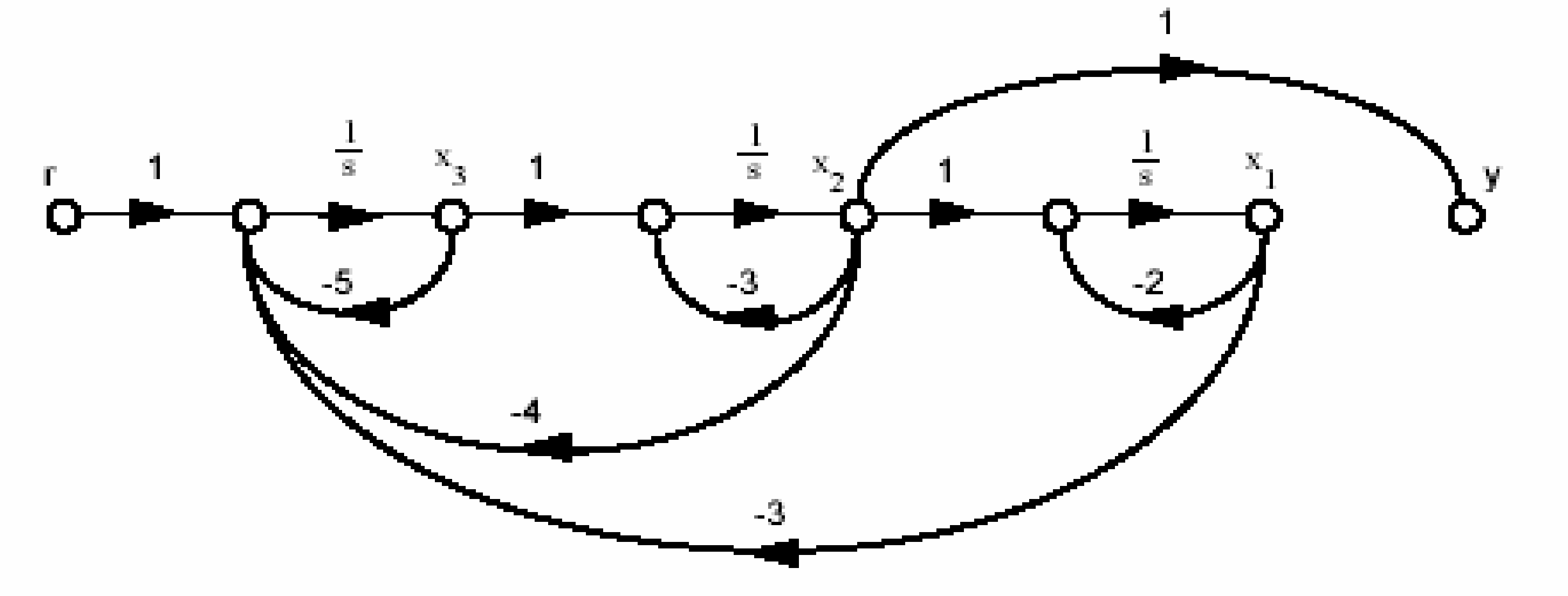
$$\dot{x}_2 = \qquad\quad -3x_2 + x_3$$

$$\dot{x}_3 = -3x_1 - 4x_2 - 5x_3 + r$$

$$y = x_2$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ -3 & -4 & -5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \mathbf{x}$$



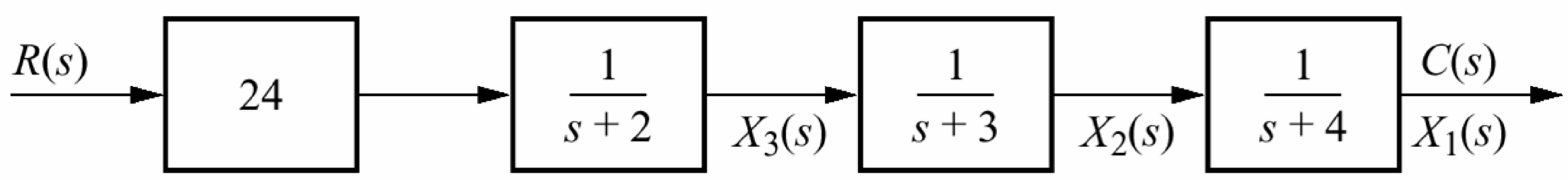
5.7 狀態空間中之另一種表示式

- 系統在狀態空間僅使用相位變數形成表示，實際上系統在狀態空間有很多不同於相位變數的表示式。
- Why？較易模型—方便將子系統轉換到狀態變數表示式。

- 串聯模式（Cascade Form）

Ex

$$\frac{C(s)}{R(s)}=\frac{24}{(s+2)(s+3)(s+4)}$$

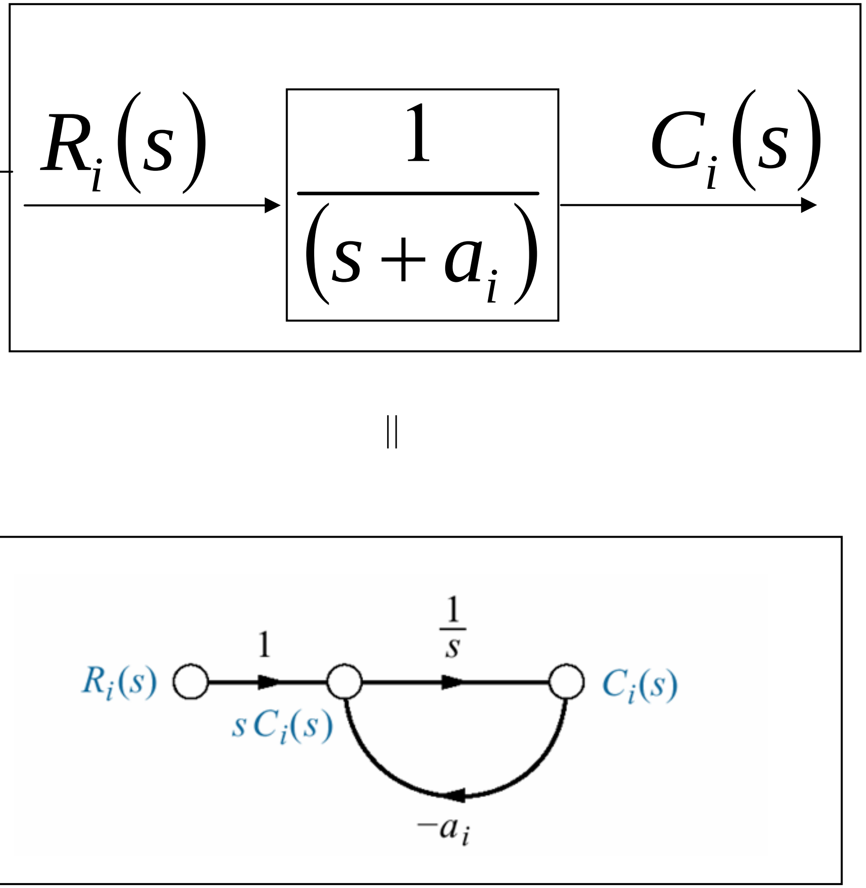


$$\frac{C_i(s)}{R_i(s)} = \frac{1}{(s + a_i)}$$

$$C_i(s)(s + a_i) = R_i(s)$$

$$\frac{dc_i(t)}{dt} + a_i c_i(t) = r_i(t)$$

$$\frac{dc_i(t)}{dt} = -a_i c_i(t) + r_i(t)$$



$$\frac{1}{s+4} \longrightarrow \dot{x}_1 = -4x_1 + x_2$$

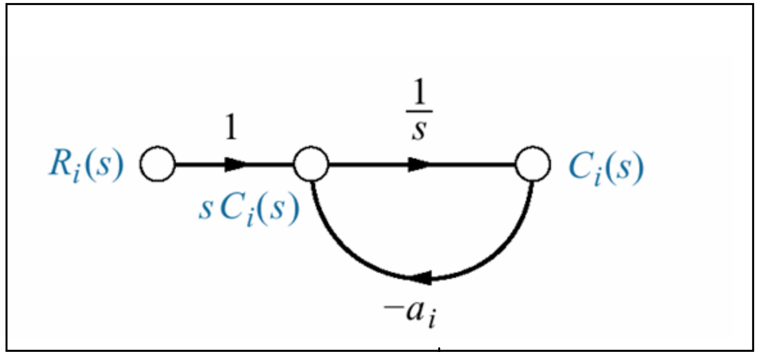
$$\frac{1}{s+3} \longrightarrow \dot{x}_2 = \qquad -3x_2 + x_3$$

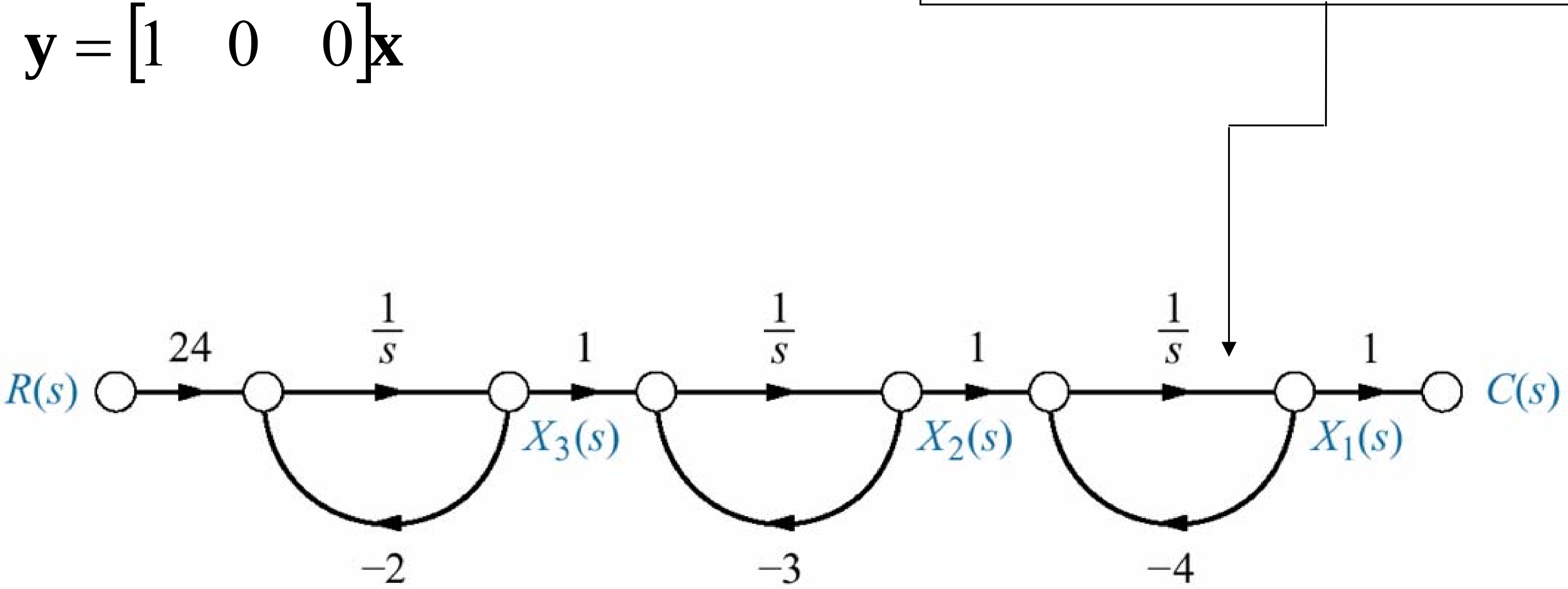
$$\frac{1}{s+2} \longrightarrow \dot{x}_3 = \qquad\qquad -2x_3 + 24r$$

$$y = c(t) = x_1$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

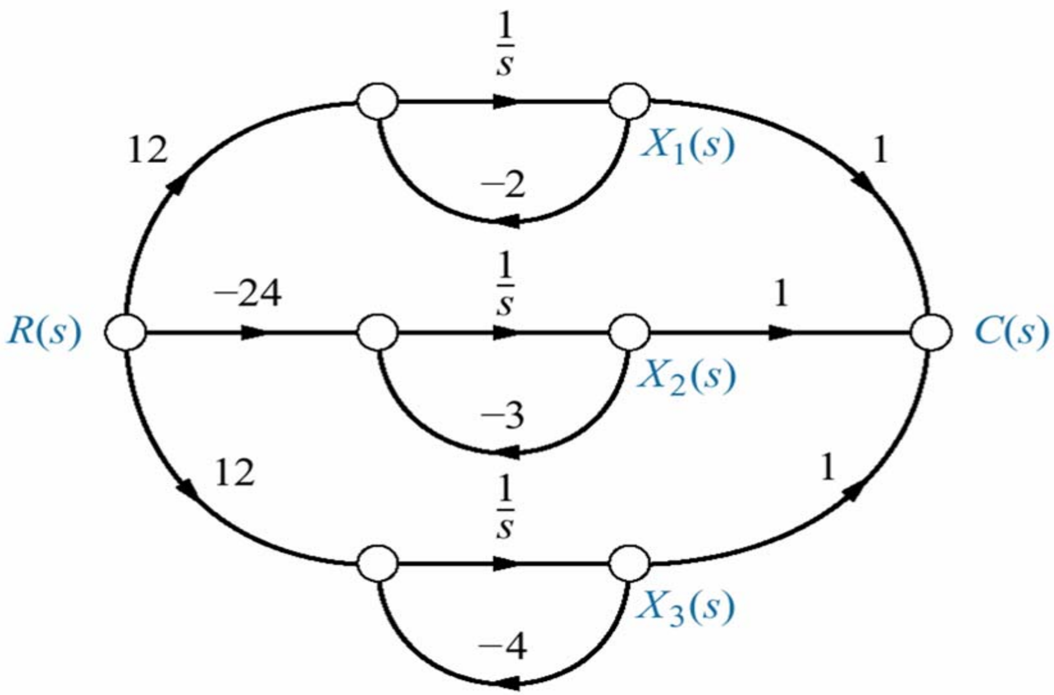




- 並聯模式（Parallel Form）

$$\frac{C(s)}{R(s)} = \frac{24}{(s+2)(s+3)(s+4)} = \frac{12}{s+2} + \frac{-24}{s+3} + \frac{12}{s+4}$$

$$C(s) = R(s)\frac{12}{s+2} + R(s)\frac{-24}{s+3} + R(s)\frac{12}{s+4}$$



$$\dot{x}_1 = -2x_1 \qquad\qquad\qquad +12r$$
$$\dot{x}_2 = \qquad -3x_2 \qquad -24r$$
$$\dot{x}_3 = \qquad\qquad\qquad -4x_3 + 12r$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -2 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 12 \\ -24 \\ 12 \end{bmatrix} r$$

$$y = c(t) = x_1 + x_2 + x_3$$

$$\mathbf{y} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \mathbf{x}$$

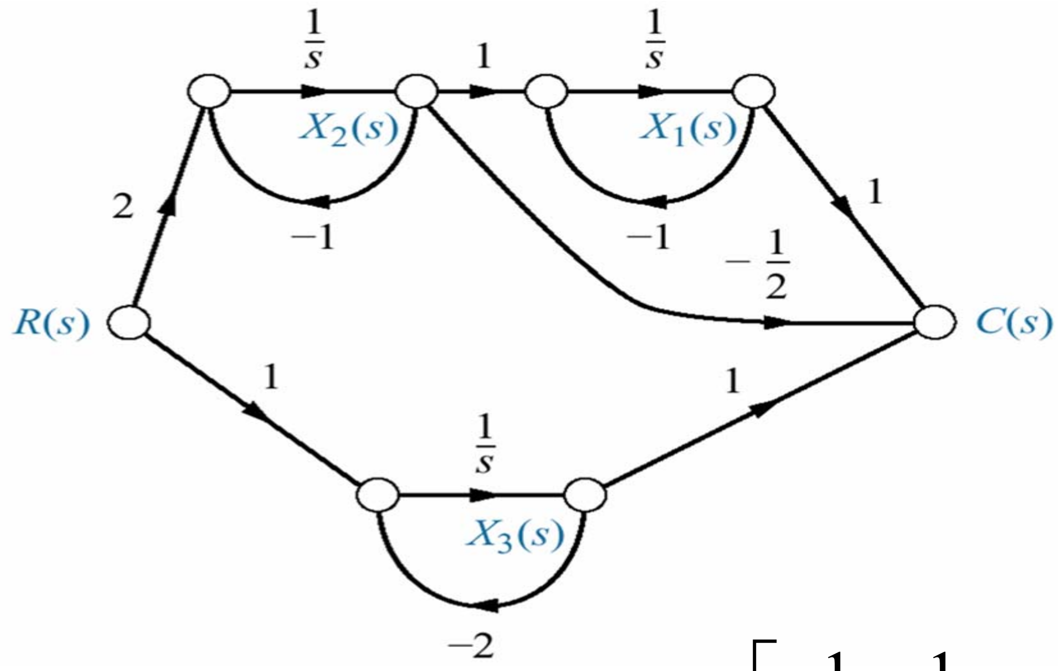
- 優點：每一方程式只有一個變數的一階微分方程，可以獨立求解。（去耦合的 decoupled）

- 重根：

Ex

$$\frac{C(s)}{R(s)} = \frac{(s+3)}{(s+1)^2(s+2)}$$

$$= \frac{2}{(s+1)^2} + \frac{-1}{(s+1)^2} + \frac{1}{(s+2)^2}$$



$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = \qquad -x_2 \qquad +2r$$

$$\dot{x}_3 = \qquad -2x_3 + r$$

$$y = c(t) = x_1 - \frac{1}{2}x_2 + x_3$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} r$$

$$\mathbf{y} = \begin{bmatrix} 1 & -\frac{1}{2} & 1 \end{bmatrix}\mathbf{x}$$

控制標準形式（Controller Canonical Form）

＊可用來設計控制器（Ch.12），可從反順序的相位

Ex：

$$G(s)=\frac{C(s)}{R(s)}=\frac{s^2+7s+2}{s^3+9s^2+26s+24}$$

相位變數：

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}r \qquad \mathbf{y}=\begin{bmatrix}2 & 7 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

將相位變數以反順序編號得

$$\begin{bmatrix}\dot{x}_3\\ \dot{x}_2\\ \dot{x}_1\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -24 & -26 & -9\end{bmatrix}\begin{bmatrix}x_3\\ x_2\\ x_1\end{bmatrix}+\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}r \qquad \mathbf{y}=\begin{bmatrix}2 & 7 & 1\end{bmatrix}\begin{bmatrix}x_3\\ x_2\\ x_1\end{bmatrix}$$

重寫（按順序）

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-9 & -26 & -24\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}r \qquad \mathbf{y}=\begin{bmatrix}1 & 7 & 2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

觀測器標準形式

＊用來設計觀測器

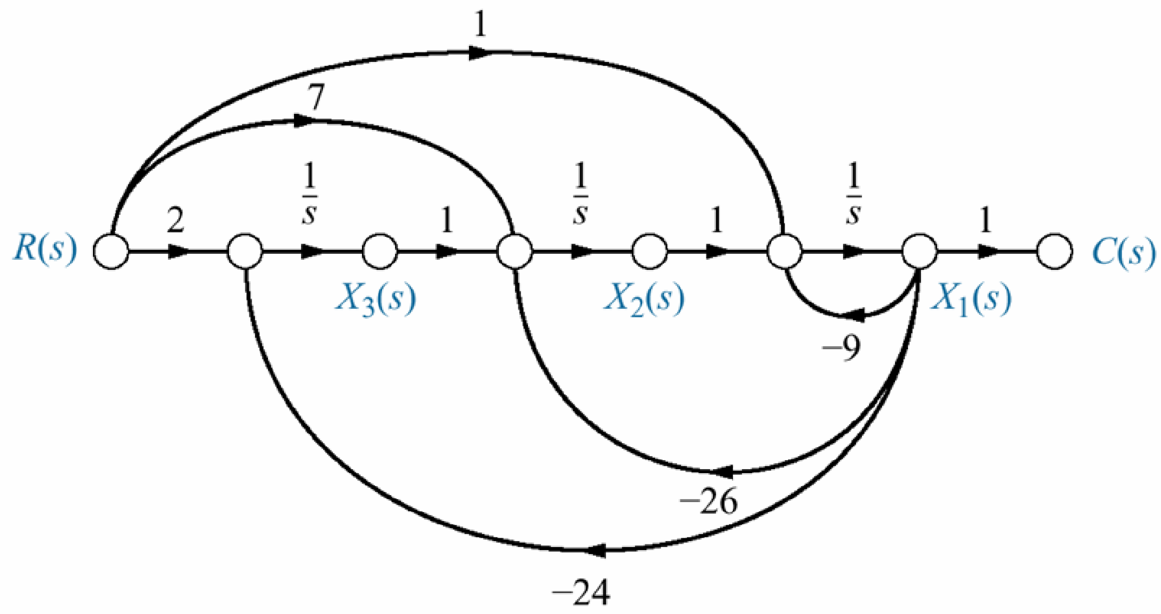
Ex：

$$G(s)=\frac{C(s)}{R(s)}=\frac{s^2+7s+2}{s^3+9s^2+26s+24}=\frac{\frac{1}{s}+\frac{7}{s^2}+\frac{2}{s^3}}{1+\frac{9}{s}+\frac{26}{s^2}+\frac{24}{s^3}}$$

$$C(s)\left[1+\frac{9}{s}+\frac{26}{s^2}+\frac{24}{s^3}\right]=R(s)\left[\frac{1}{s}+\frac{7}{s^2}+\frac{2}{s^3}\right]$$

$$C(s)=\frac{1}{s}\left(R(s)-9C(s)\right)+\frac{1}{s^2}\left(7R(s)-26C(s)\right)+\frac{1}{s^3}\left(2R(s)-24C(s)\right)$$

$$=\frac{1}{s}\left[\underbrace{\left(R(s)-9C(s)\right)}_{x_1}+\frac{1}{s}\underbrace{\left[\left(7R(s)-26C(s)\right)+\frac{1}{s}\underbrace{\left(2R(s)-24C(s)\right)}_{x_3}\right]}_{x_2}\right]$$

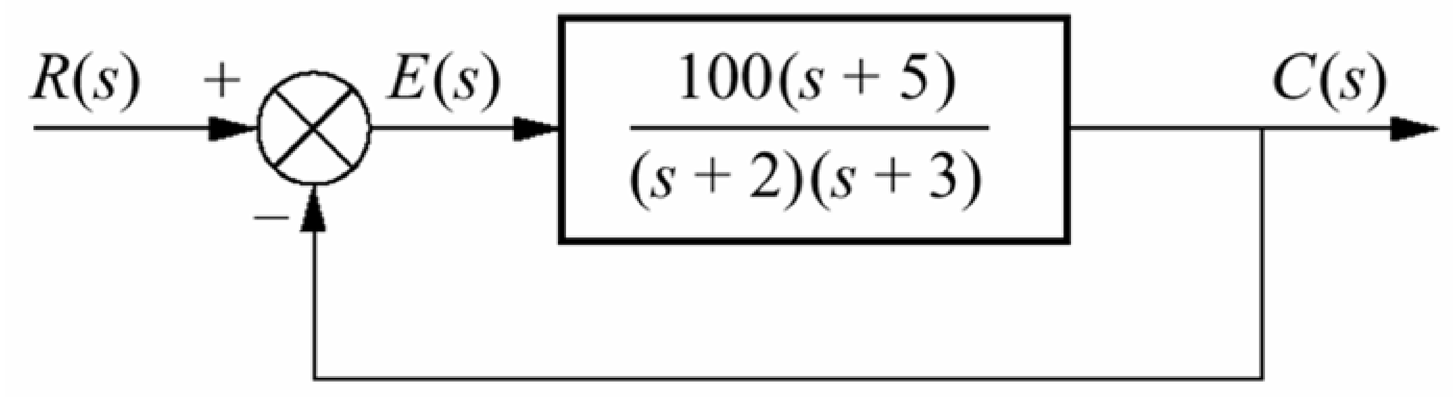


$$\dot{x}_1 = -9x_1 + x_2 + r$$
$$\dot{x}_2 = -26x_1 + x_3 + 7r$$
$$\dot{x}_3 = -24x_1 + 2r$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -9 & 1 & 0 \\ -26 & 0 & 1 \\ -24 & 0 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix} r$$

$$y = c(t) = x_1 \qquad \mathbf{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}\mathbf{x}$$

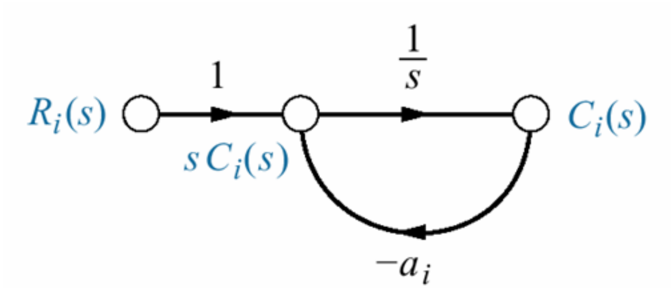
- EX



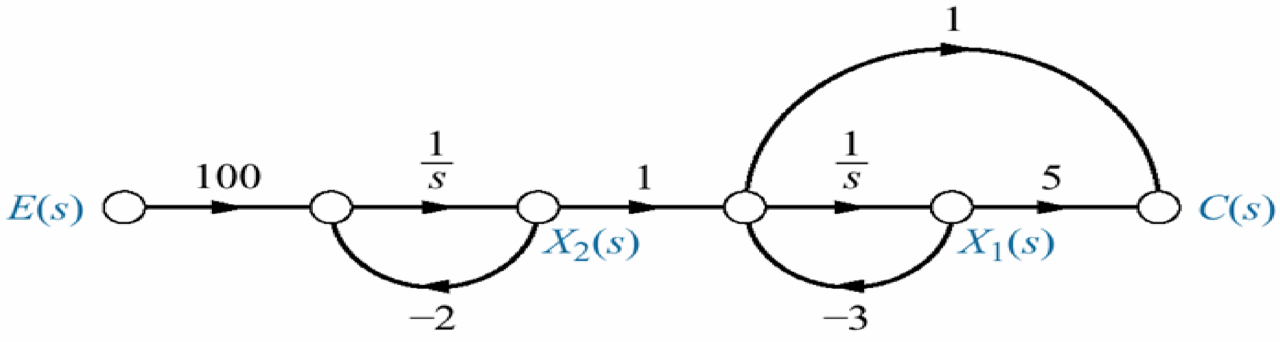
以串聯形式之狀態空間表示回授系統

$$s+5=\frac{C}{R} \qquad \frac{dr}{dt}+5r=c$$

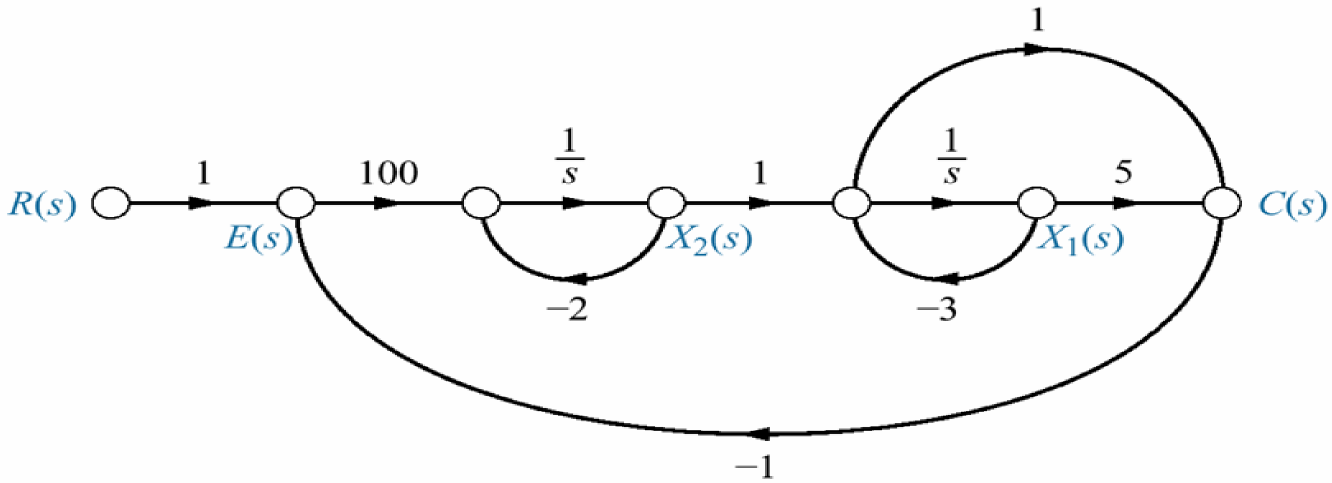
$$R(s+5)=C \qquad \frac{d\,x_1}{dt}+5\,x_1=c$$



1.順向轉移函數



2.加回授



$$\dot{x}_1 = -3x_1 + x_2$$

$$\dot{x}_2 = \qquad -2x_2 + 100(r - c)$$

$$c = 5x_1 + (x_2 - 3x_1)$$

$$= 2x_1 + x_2$$

$\Rightarrow$

$$x_1 = -3x_1 + x_2$$

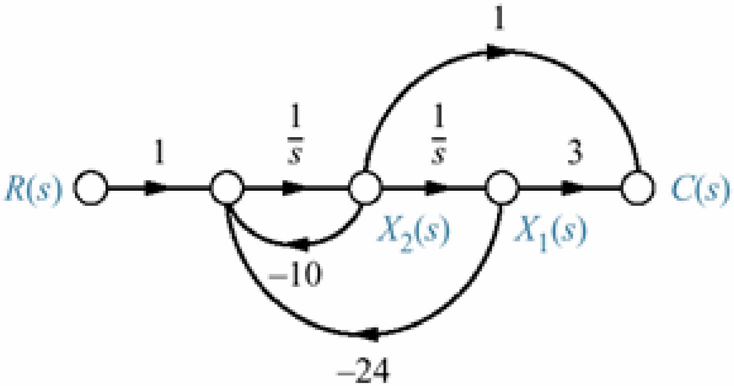
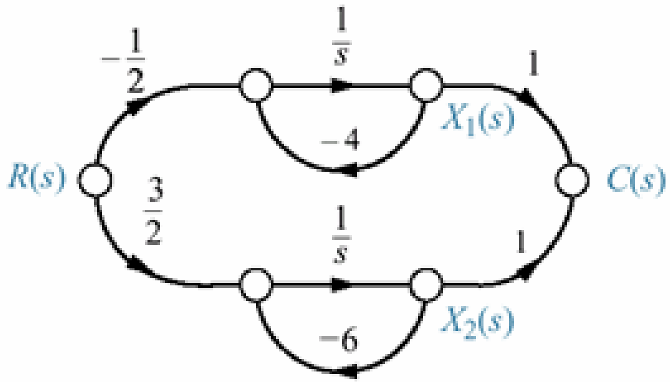
$$x_2 = -200x_1 - 102x_2 + 100r$$

$$y = c(t) = 2x_1 + x_2$$

$$\dot{\mathbf{x}} = \begin{bmatrix} -3 & 1 \\ -200 & -102 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 100 \end{bmatrix} r$$

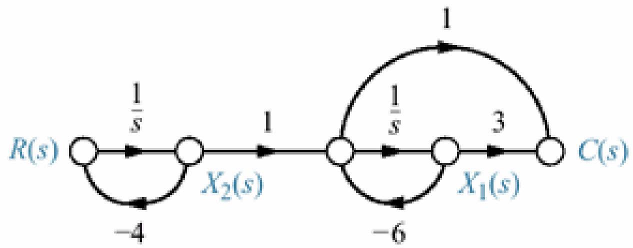
$$\mathbf{y} = \begin{bmatrix} 2 & 1 \end{bmatrix} \mathbf{x}$$

5.8 相似轉換

Phase variable	$\frac{1}{(s^2+10s+24)} * (s+3)$		$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -24 & -10 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$ $y = [3 \quad 1]\mathbf{x}$
Parallel	$\frac{-1/2}{(s+4)} + \frac{3/2}{s+6}$		$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 0 \\ 0 & -6 \end{bmatrix}\mathbf{x} + \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} r$ $y = [1 \quad 1]\mathbf{x}$

Cascade

$$\frac{1}{(s+4)} * \frac{(s+3)}{(s+6)}$$

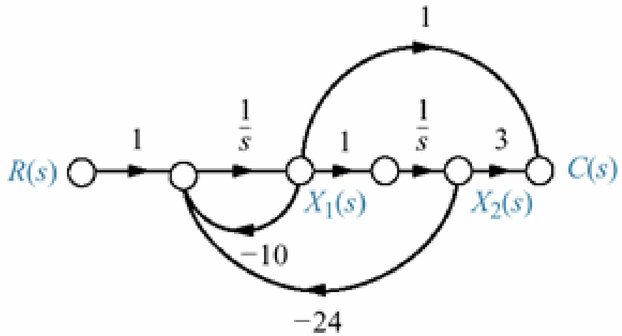


$$\dot{\mathbf{x}} = \begin{bmatrix} -6 & 1 \\ 0 & -4 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} -3 & 1 \end{bmatrix}\mathbf{x}$$

Controller canonical

$$\frac{1}{(s^2 + 10s + 24)} * (s+3)$$

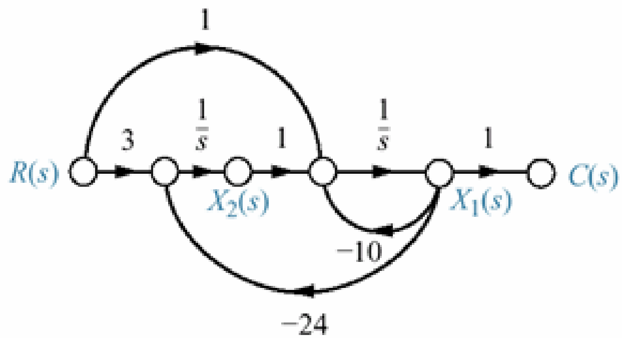


$$\dot{\mathbf{x}} = \begin{bmatrix} -10 & -24 \\ 1 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 3 \end{bmatrix}\mathbf{x}$$

Observer canonical

$$\frac{\frac{1}{s} + \frac{3}{s^2}}{1 + \frac{10}{s} + \frac{24}{s^2}}$$



$$\dot{\mathbf{x}} = \begin{bmatrix} -10 & 1 \\ -24 & 0 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ 3 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix}\mathbf{x}$$